Introduction to Rheology of complex fluids
Brief Lecture Notes

Kinematics and material functions for extensional flows

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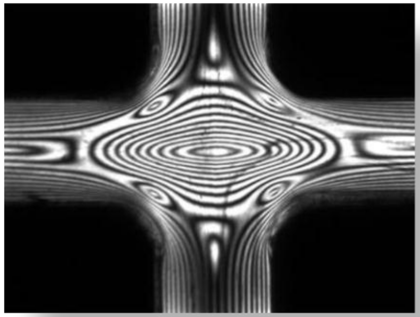
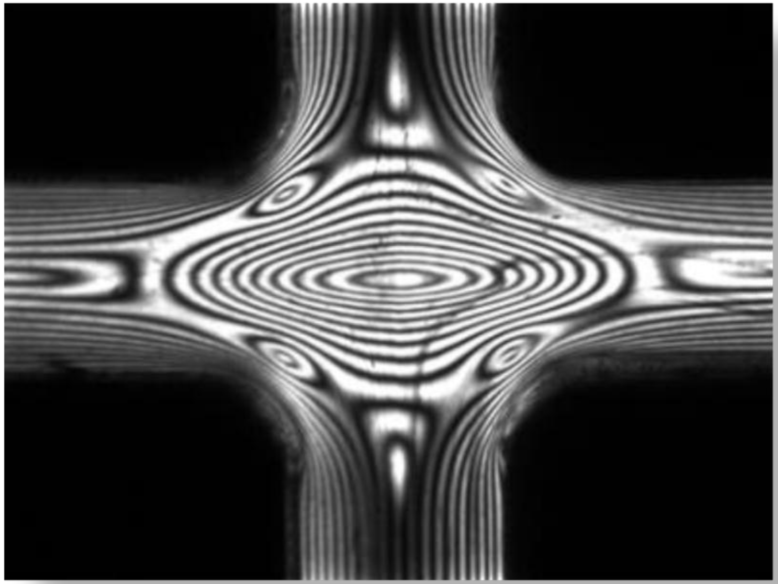
Contents

- Introductory Lecture
- **Simple Flows**
- **Material functions & Rheological Characterization**
- **Experimental Observations**
- Generalized Newtonian Fluids
- Generalized Linearly viscoelastic Fluids
- Nonlinear Constitutive Models





Kinematics of extensional flow

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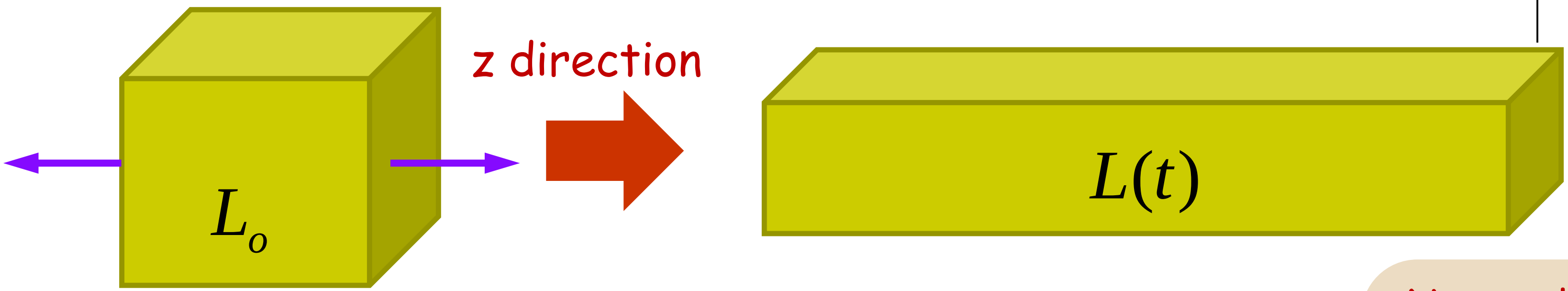
Deformation in extensional flow



Uniaxial Stretching

$$\varepsilon(t_{ref}, t) = \int_{t_{ref}}^{t} \dot{\varepsilon}(t')dt'$$

Hencky Strain

$$= \dot{\varepsilon}_o t$$

The deformation is proportional to time, when $\dot{\varepsilon}_o$= const.

$$= \ln\left(\frac{L(t)}{L_o}\right)$$

The ratio of lengths is an exponential function of time

Nomenclature For Strain

$$\varepsilon \leftrightarrow \gamma$$

$$\varepsilon \leftrightarrow \varepsilon_{zz}$$

Rate of Strain

$$\dot{\varepsilon}_o \leftrightarrow \dot{\gamma}_o$$

Extensional Flow Characteristics

- **Strong**

  In comparison to simple shear flow which is a weak flow.

$$L(t) = L_o \exp(\dot{\varepsilon} t)$$

Exponential Change

$$z(t) = z(t=0) + \dot{\gamma}\, t$$

Linear Change

- **Irrotational**

  Recirculations are not formed, while the deformation results from the stretching and the orientation of the macromolecules.

- **Non-viscometric flow**

  The third invariant of the rate of deformation is non-zero.

- **Three types**

  Uniaxial, Biaxial and 2D Elongation

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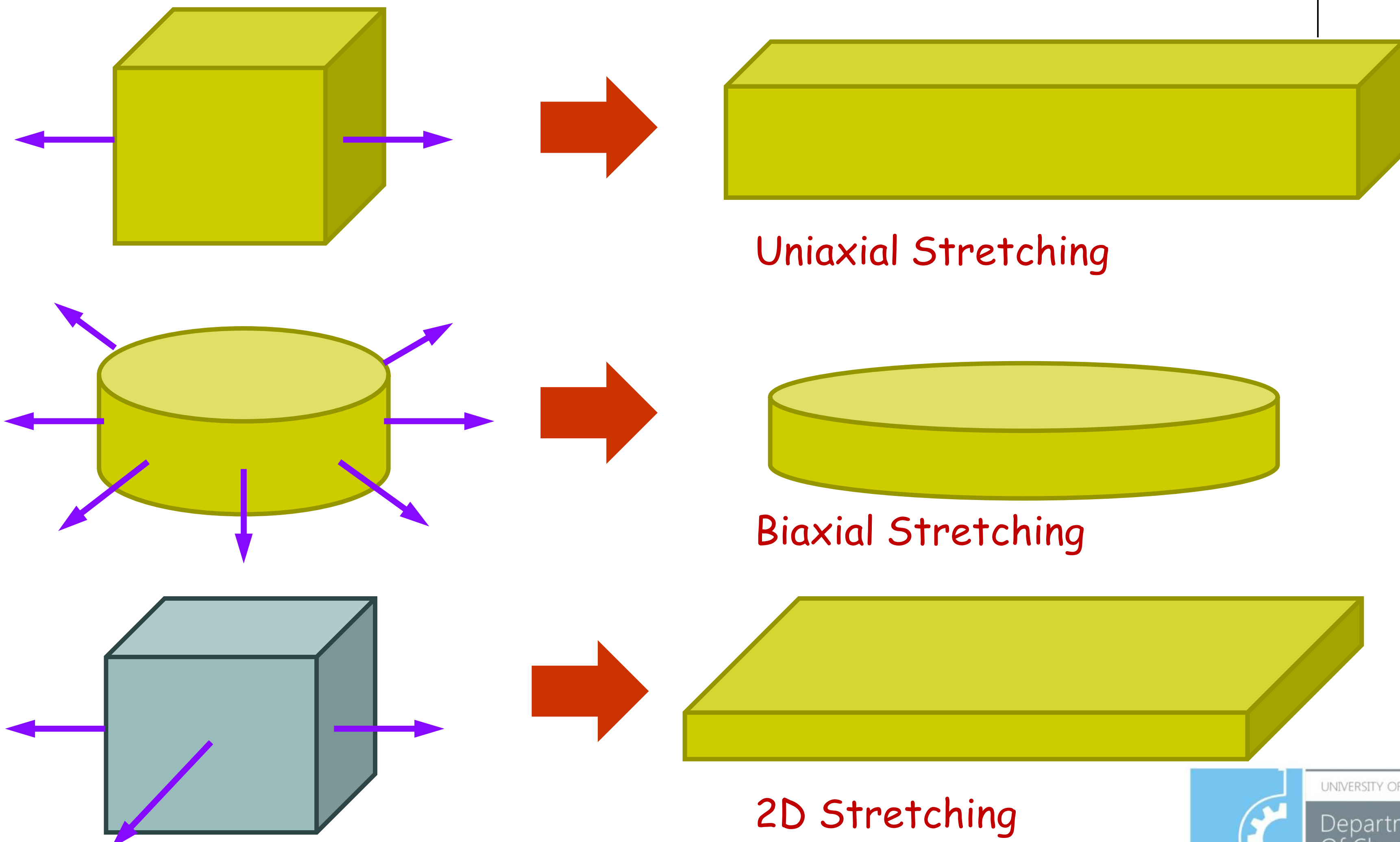
Three Types of Flow



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Three Types of Flow

$$\dot{\varepsilon}(t) = \dot{\varepsilon}_o = \text{constant}$$

Kinematics

$$v_x(x, y, z) = -\frac{1}{2}\dot{\varepsilon}(t)(1+b)x$$

$$v_y(x, y, z) = -\frac{1}{2}\dot{\varepsilon}(t)(1-b)y$$

$$v_z(x, y, z) = \dot{\varepsilon}(t)z$$

Exrension in z direction

→

Nomenclature

Uniaxial Stretching:

$$b = 0, \quad \dot{\varepsilon} > 0$$

Biaxial Stretching:

$$b = 0, \quad \dot{\varepsilon} < 0$$

2D Stretching:

$$b = 1, \quad \dot{\varepsilon} > 0$$

Three Steady Extensional flows

Rate of Deformation Tensor

$$\underline{\underline{\nabla v}} = \dot{\varepsilon}_o \begin{pmatrix} -\frac{1}{2}(1+b) & 0 & 0 \\ 0 & -\frac{1}{2}(1-b) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underline{\underline{\dot{\gamma}}} = \underline{\underline{\nabla v}} + \left(\underline{\underline{\nabla v}}\right)^T = 2\dot{\varepsilon}_o \begin{pmatrix} -\frac{1}{2}(1+b) & 0 & 0 \\ 0 & -\frac{1}{2}(1-b) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Nomenclature

σ ⟷ τ

Uniaxial Stretching:

$b = 0, \quad \dot{\varepsilon} > 0$

Biaxial Stretching:

$b = 0, \quad \dot{\varepsilon} < 0$

2D Stretching:

$b = 1, \quad \dot{\varepsilon} > 0$

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Three Steady Extensional flows

Material Properties

$$\bar{\eta} \equiv \frac{\tau_{zz} - \tau_{xx}}{\dot{\varepsilon}_o}$$

$$\bar{\eta}_{P_2} \equiv \frac{\tau_{yy} - \tau_{xx}}{\dot{\varepsilon}_o}$$

$$\bar{\eta} \Leftrightarrow \bar{\eta}_B \Leftrightarrow \bar{\eta}_{P_1}$$

Uniaxial or Biaxial or 1st level viscosity

2nd level viscosity

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Application of Uniaxial Extension In Newtonian Fluids

$$\underline{\underline{\tau}} = \eta \underline{\underline{\dot{\gamma}}}$$

$$\tau_{zz} = 2\eta\dot{\varepsilon}$$

$$\tau_{yy} = \tau_{xx} = -\eta\dot{\varepsilon}$$

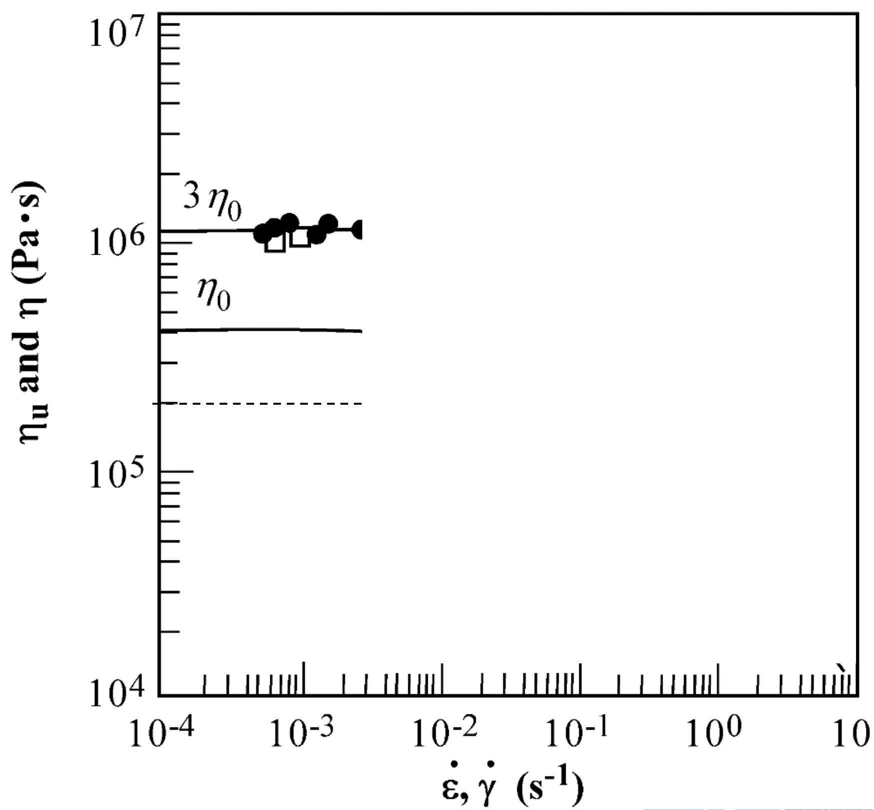
$$\underline{\underline{\dot{\gamma}}} = \underline{\underline{\nabla v}} + \left(\underline{\underline{\nabla v}}\right)^T = 2\dot{\varepsilon}\begin{pmatrix} -1/2 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Extensional Viscosity Definition

$$\eta_E(t) = \frac{\tau_{zz} - \tau_{yy}}{\dot{\varepsilon}} = 3\eta(t)$$

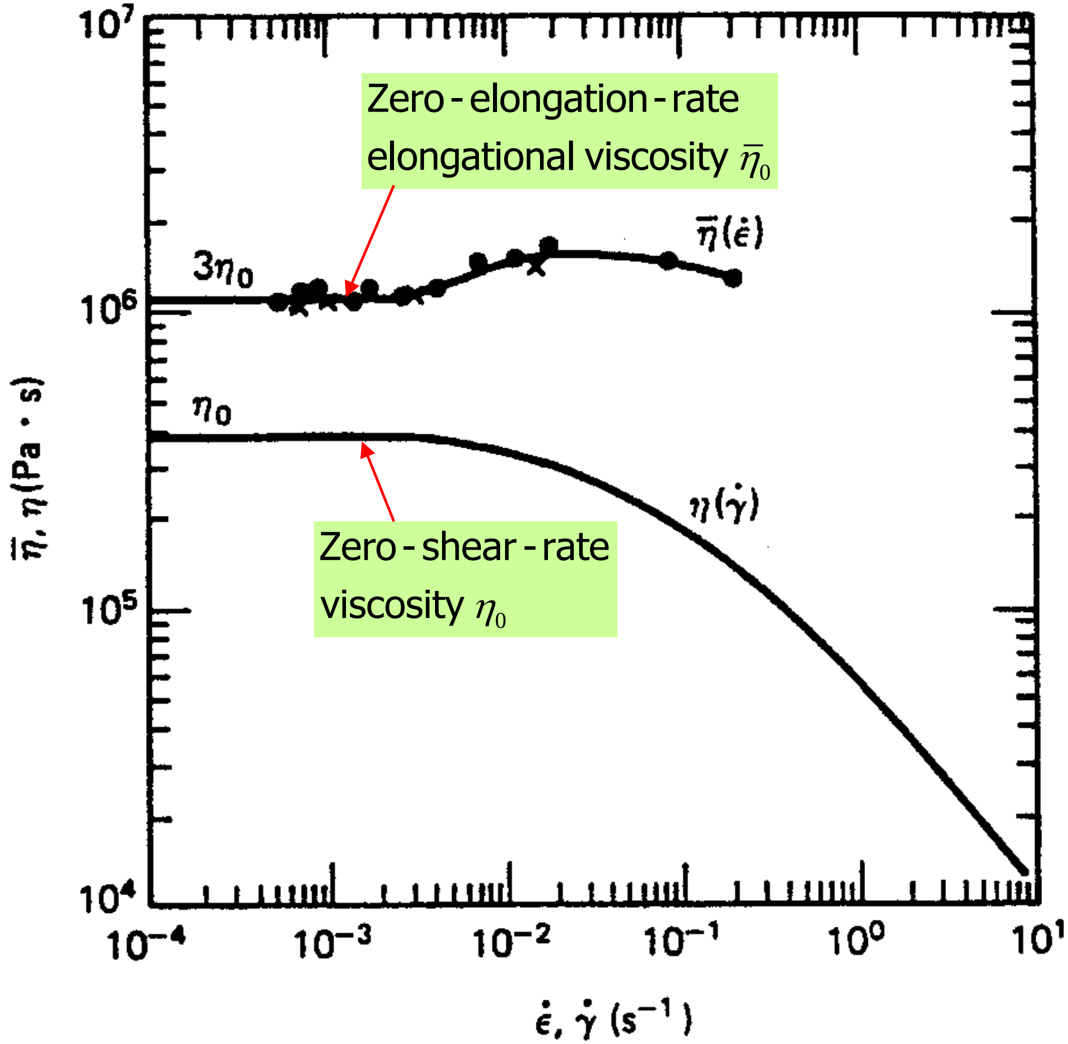
For low rate of deformation:

$$\eta_{E,o} = 3\eta_o$$



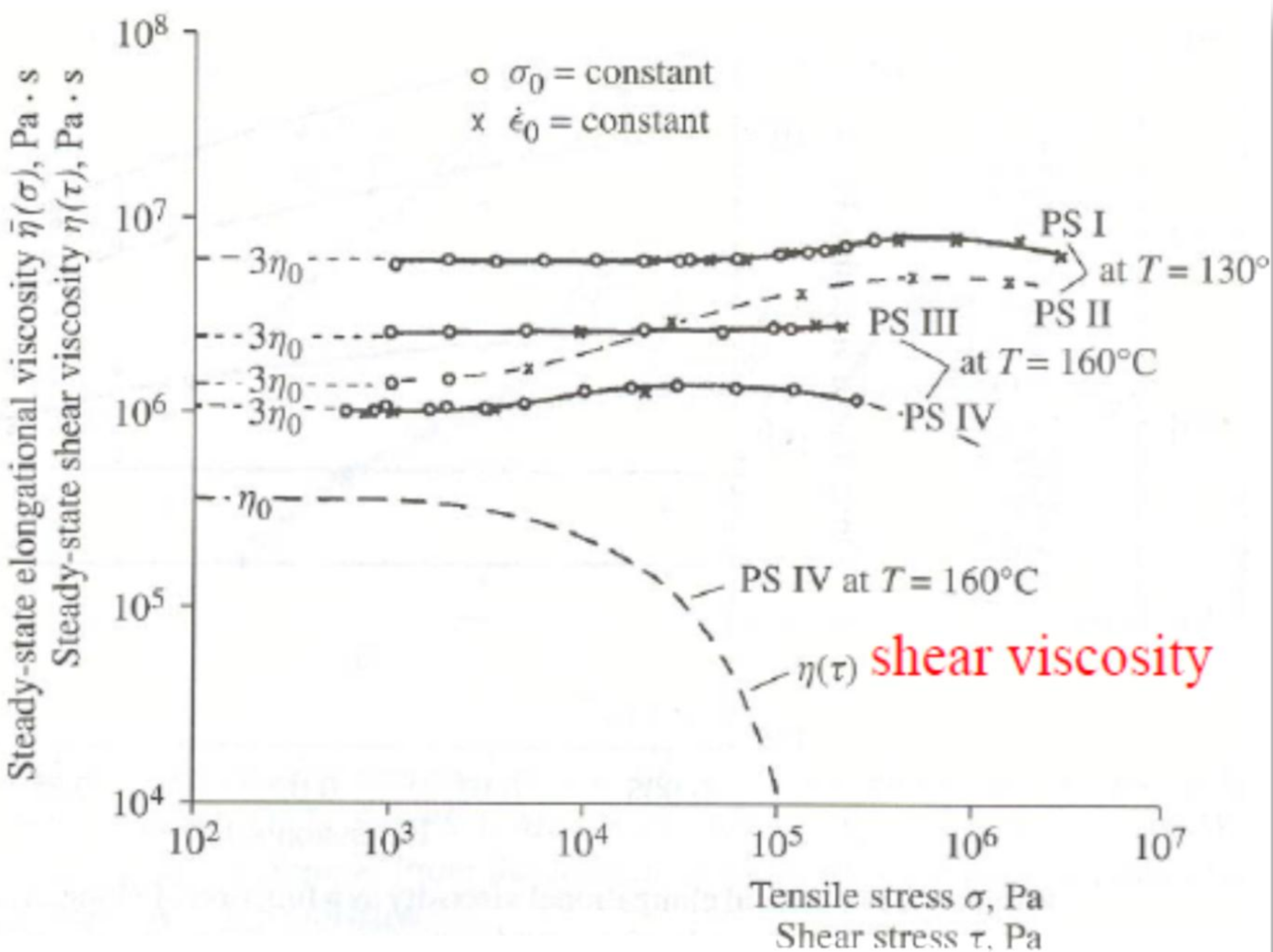
A lot of materials diverge from this observation

Experimental Observations



Steady elongational and shear viscosity of a PS melt vs. extensional and shear rate of deformation, respectively

Experimental Observations

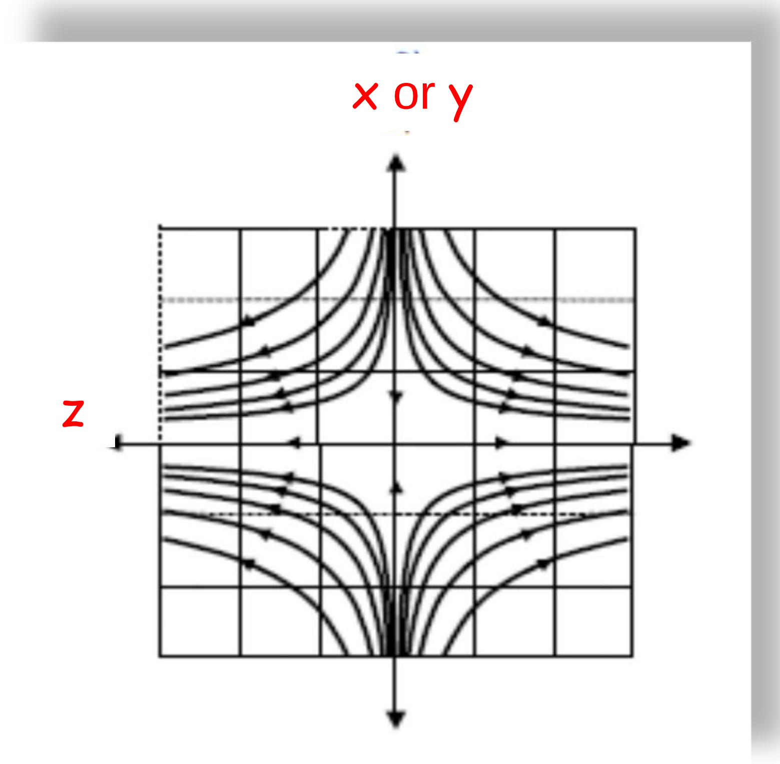


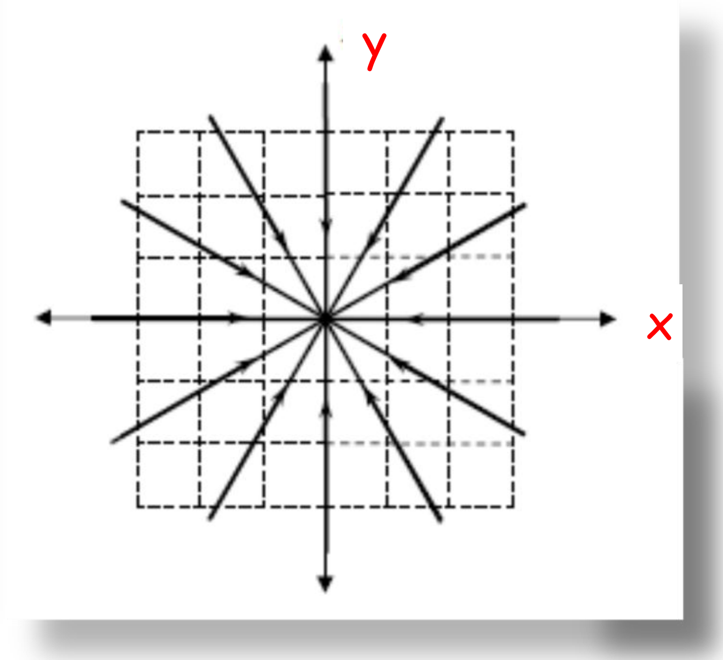
The extensional viscosity first increases and then decreases with increasing $\dot{\varepsilon}$.

Trouton Ratio

$$Tr \equiv \frac{\bar{\eta}}{\eta_o}$$

Uniaxial Extensional Flow





Fluid Streaklines

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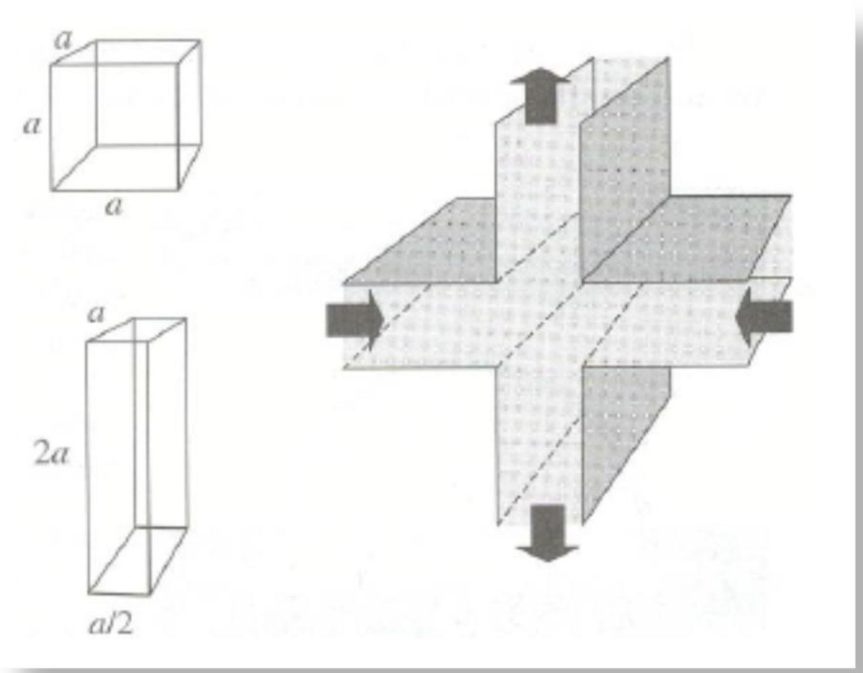
Biaxial Extensional Flow

Velocity Field

$$v_x(x,y,z) = -\dot{\varepsilon}(t)x$$
$$v_y(x,y,z) = 0$$
$$v_z(x,y,z) = \dot{\varepsilon}(t)z$$

Fluid element



Transient Extensional Flows

As in shear flows, in extensional flows a variety of basic and interesting transient patterns have been defined

- Start-up of Extensional Flows
- Cessation of Extensional Flows (almost impossible)
- Extensional Creep
- Extensional Ramp Deformation
- SAOE oscillation of small amplitude

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Extensional Material Properties

Start-up of uniaxial elongation

$$\eta_E^+(t,\dot{\varepsilon}) = \frac{\tau_E(t,\dot{\varepsilon})}{\dot{\varepsilon}}$$

$$\tau_E = \tau_{11} - \tau_{22} = \tau_{11} - \tau_{33}$$

In LVE:

$$\lim_{\dot{\varepsilon}\to 0}\left\{\eta_E^+(t,\dot{\varepsilon})\right\} = \eta_{E,o}^+(t) = 3\eta_o^+(t)$$

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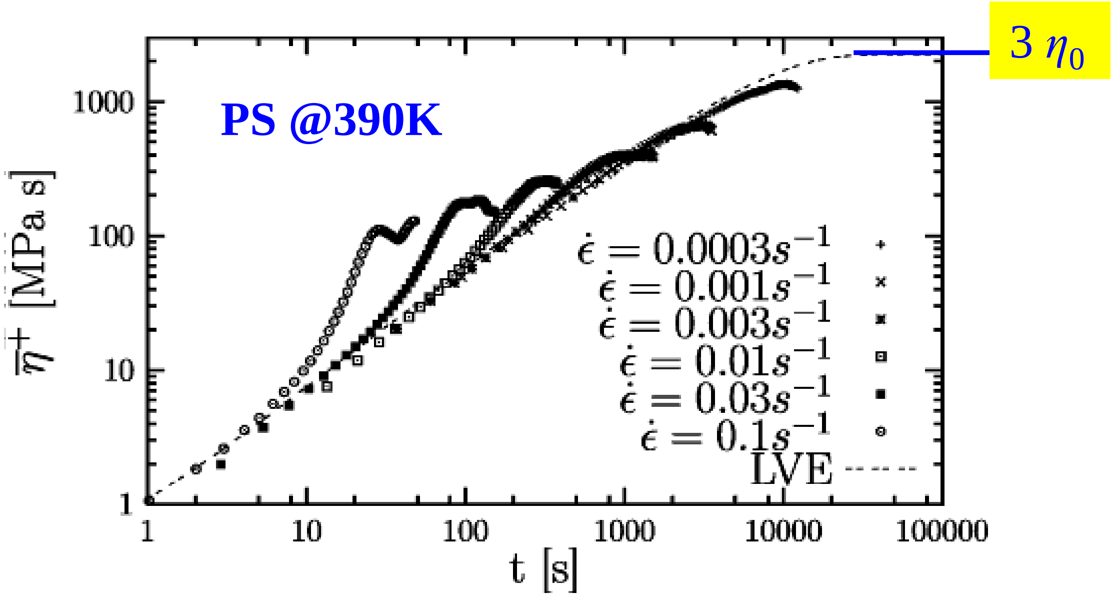
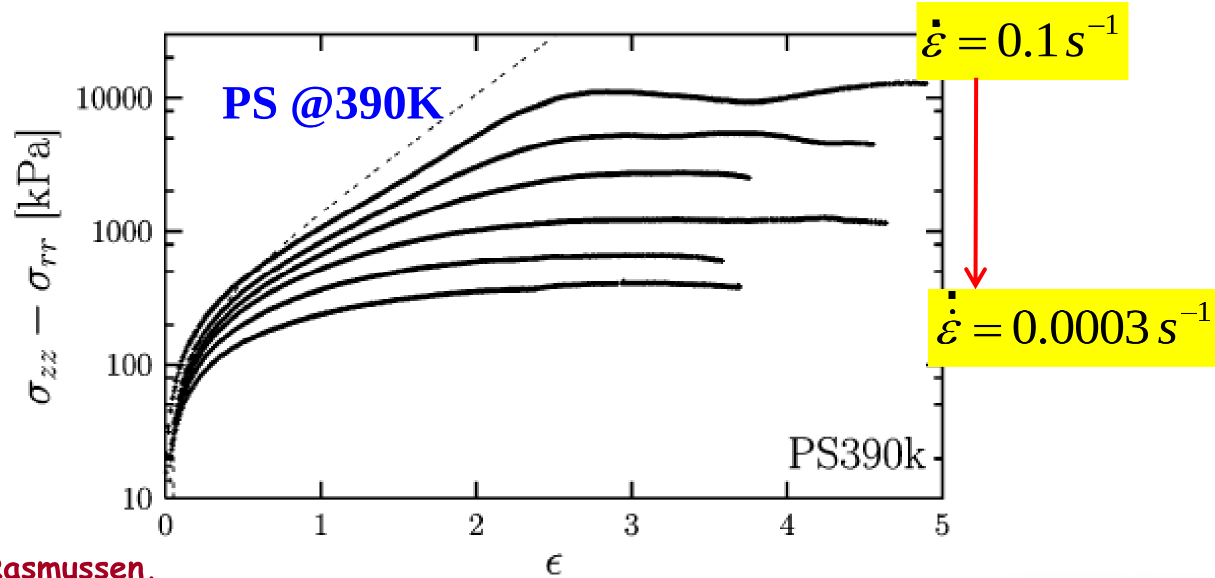
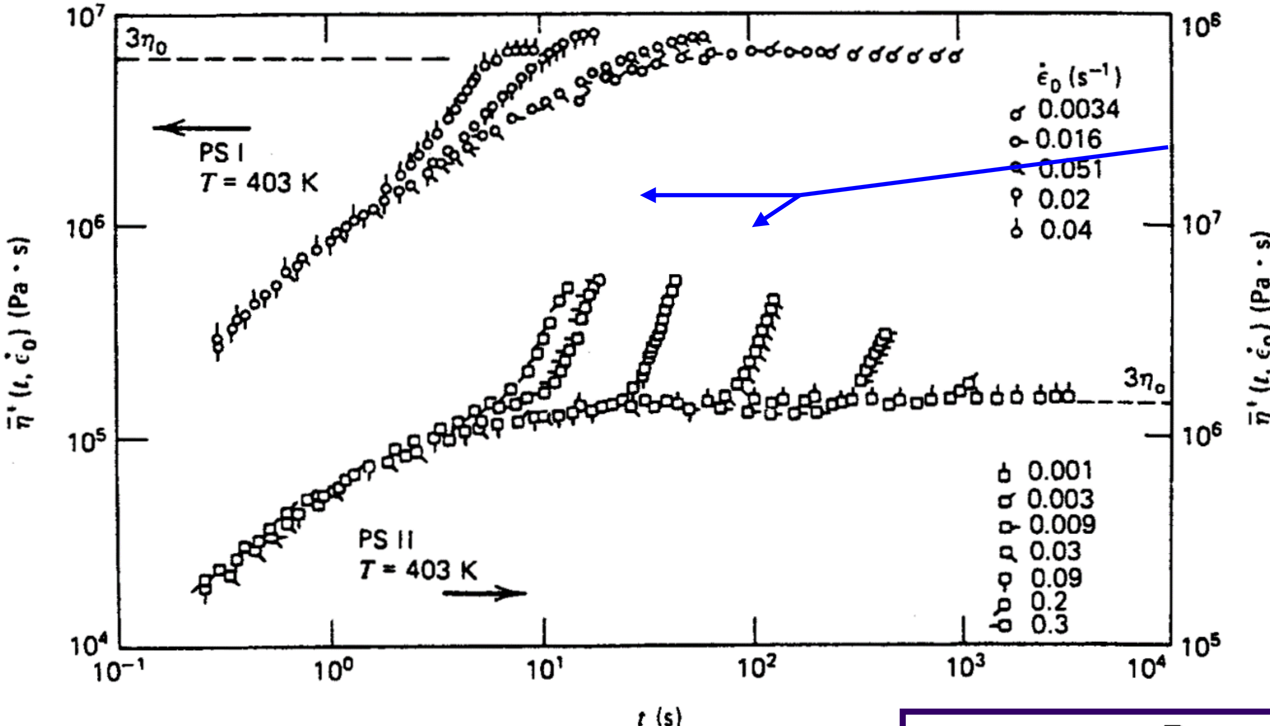
Startup of extensional flow





A. Bach, K. Almdal, H. K. Rasmussen, O. Hassager, *Macromolecules* 36, 5174-5179 (2003)

Start-up of Extensional Flow



Extensional increase of the viscosity in constant value of Hencky strain:

$$\varepsilon(0,t) = \dot{\varepsilon}_o t$$

Molecular Weight of the samples:

	$\bar{M}_w$	$\bar{M}_w/\bar{M}_n$
PS I	7.4×10^4	1.2
PS II	3.9×10^4	1.1
PS III	2.53×10^5	1.9
PS IV	2.19×10^5	2.3

Monodisperse

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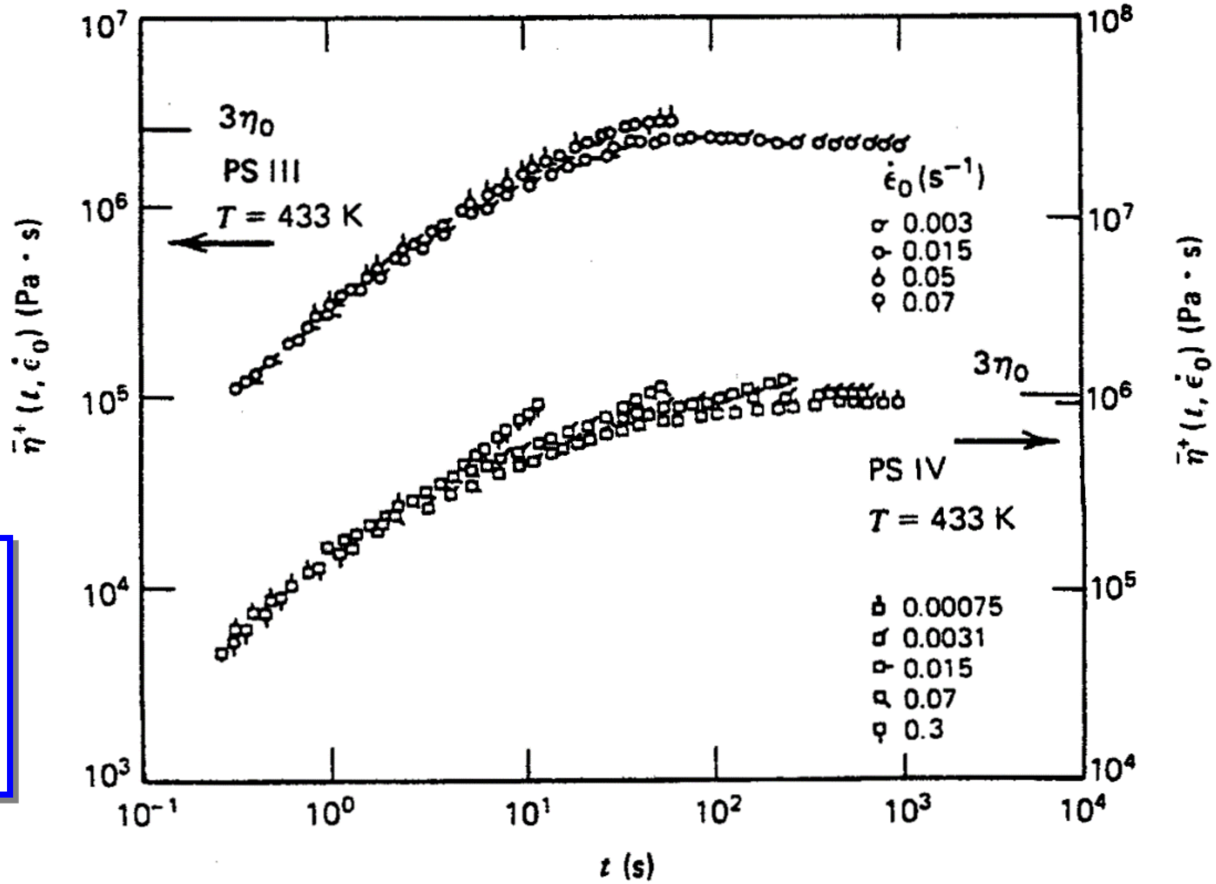
Start-up of Extensional Flow



	$\bar{M}_w$	$\bar{M}_w/\bar{M}_n$
PS I	7.4×10^4	1.2
PS II	3.9×10^4	1.1
PS III	2.53×10^5	1.9
PS IV	2.19×10^5	2.3

Extensional Material Properties

Start-up of biaxial elongation

$$\eta_B^+(t,\dot{\varepsilon}_B) = \frac{\tau_B(t,\dot{\varepsilon}_B)}{\dot{\varepsilon}} \qquad \tau_\mathrm{B} = \tau_{11} - \tau_{22} = \tau_{11} - \tau_{33}$$

In LVE:

$$\lim_{\dot{\varepsilon}_B \to 0}\left\{\eta_B^+(t,\dot{\varepsilon}_B)\right\} = \eta_{B,o}^+(t) = 6\eta_o^+(t)$$

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Extensional Material Properties

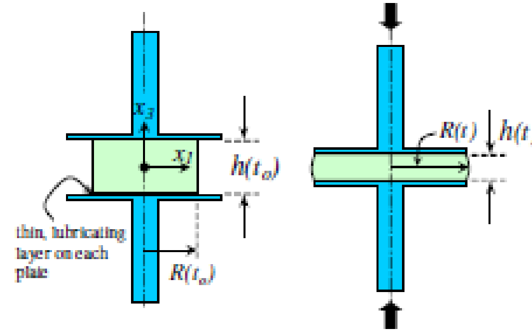
Start-up of 2D elongation

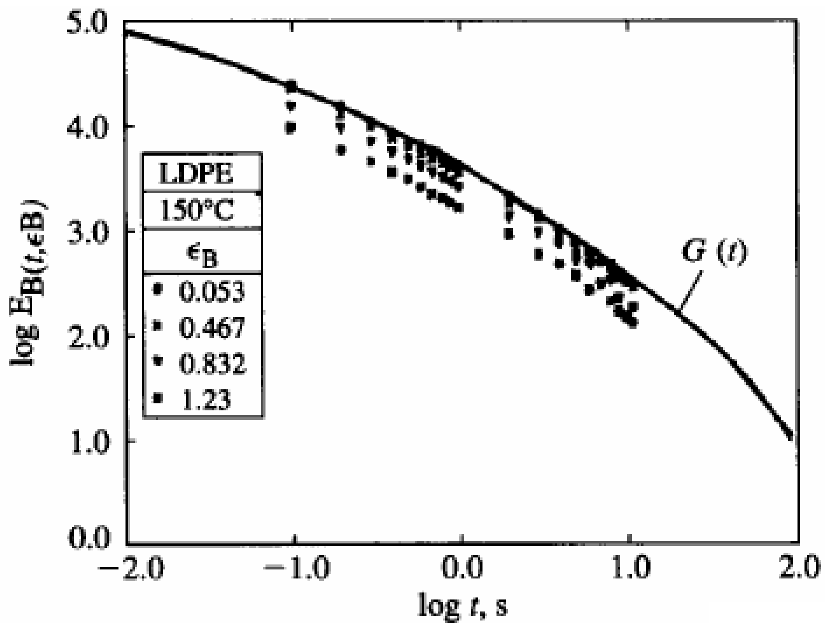
$$\eta^+_{p_1}(t,\dot{\varepsilon}_B)=\frac{\tau_{11}-\tau_{33}}{\dot{\varepsilon}} \qquad \eta^+_{p_2}(t,\dot{\varepsilon}_B)=\frac{\tau_{22}-\tau_{33}}{\dot{\varepsilon}}$$

In LVE:

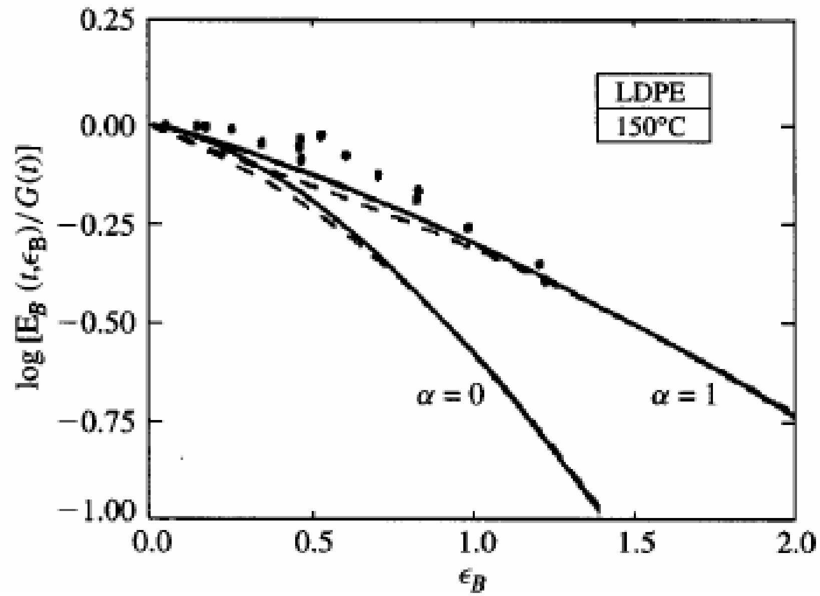
$$\eta^+_{p_1,o}(t,\dot{\varepsilon}_B)=4\eta^+_o(t_B) \qquad \eta^+_{p_2,o}(t,\dot{\varepsilon}_B)=4\eta^+_o(t_B)$$

Extensional Flow due to Step Deformation





Deformation function for biaxial flow



Damping function for biaxial flow

Transient Rheological Parameters

Parameter	Shear	Elongation	Units
Strain	$\gamma = \gamma_0 \sin(\omega t)$	$\varepsilon = \varepsilon_0 \sin(\omega t)$	---
Stress	$s = s_0 \sin(\omega t + \delta)$	$t = t_0 \sin(\omega t + \delta)$	Pa
Storage Modulus (Elasticity)	$G' = (s_0/\gamma_0)\cos\delta$	$E' = (t_0/\varepsilon_0)\cos\delta$	Pa
Loss Modulus (Viscous Nature)	$G'' = (s_0/\gamma_0)\sin\delta$	$E'' = (t_0/\varepsilon_0)\sin\delta$	Pa
Tan δ	G''/G'	E''/E'	---
Complex Modulus	$G^* = (G'^2 + G''^2)^{0.5}$	$E^* = (E'^2 + E''^2)^{0.5}$	Pa
Complex Viscosity	$\eta^* = G^*/\omega$	$\eta_E^* = E^*/\omega$	Pa-sec

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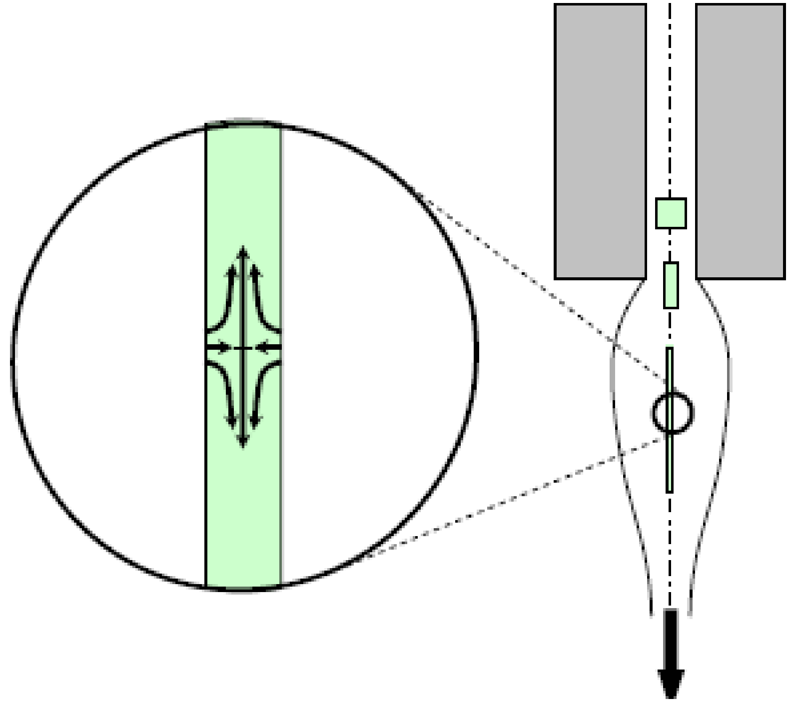
Why Extensional flow is a Rheological Flow?

- **Simple Flow Field**
- **Represents many similar, but more complex flows**
- **Simple expression of the stress tensor**



Comments on the Extensional Flows

Steady Deformation

- Difficulties in reproducing the experiments
- Difficulties even in the steady extensional flows
- Important for many practical processes

Non Steady Deformation

- Difficulties in reproducing the experiments
- Open Question: What is the quantitative increase of stresses with the deformation (strain hardening)

Why have we chosen these flows?

Because they are **symmetric.**

Symmetry facilitates reaching conclusions for the stress tensor that produces these well-described flow fields **for each different fluid.**

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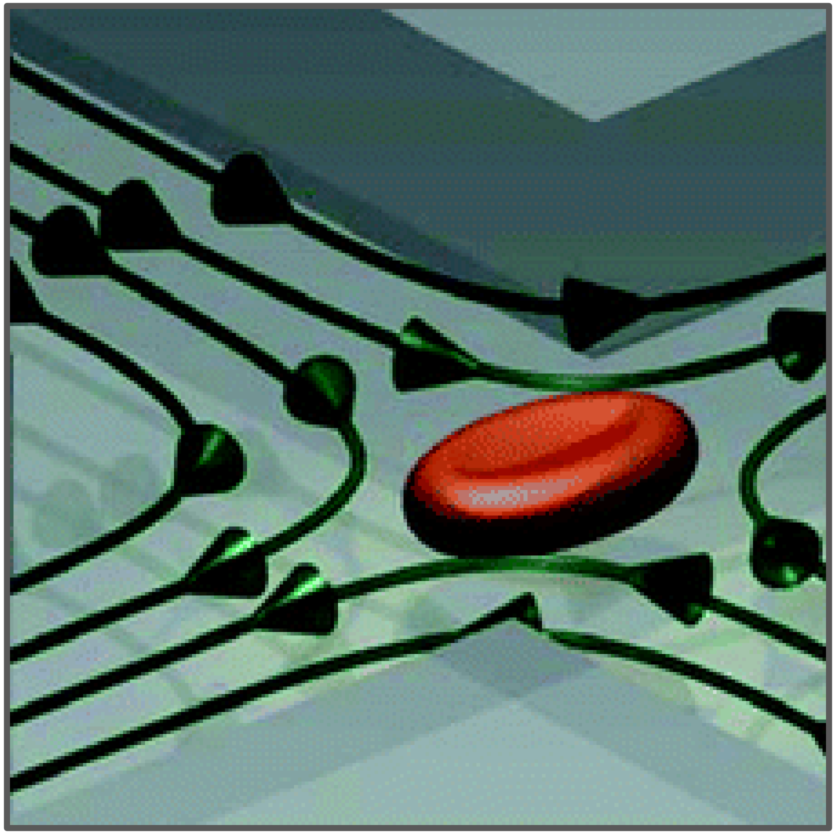

End of lecture

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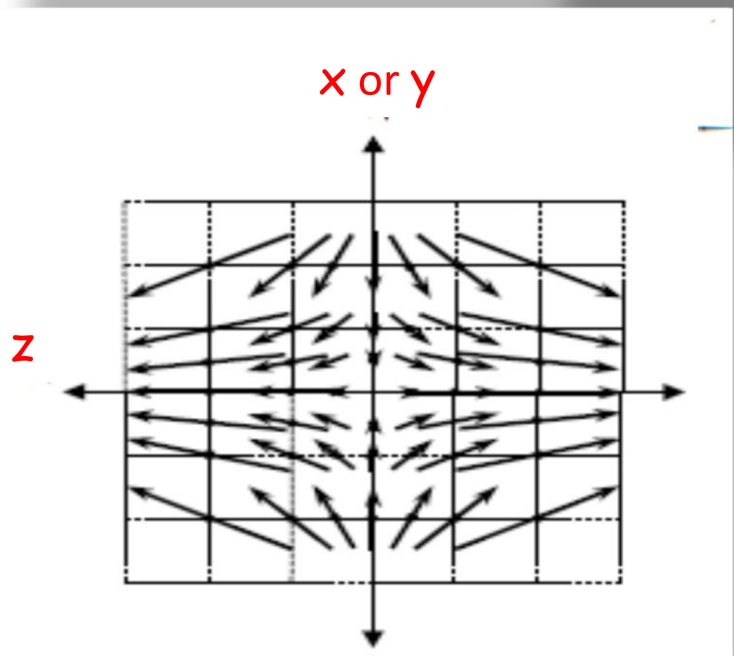
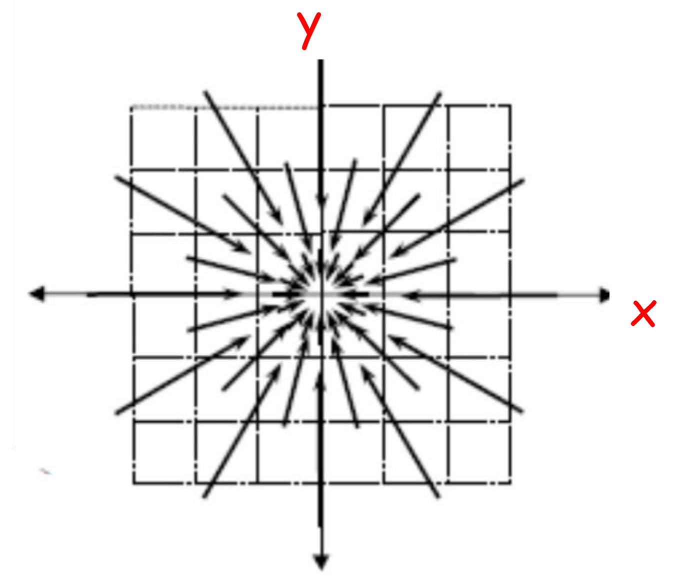
Uniaxial Extensional Flow



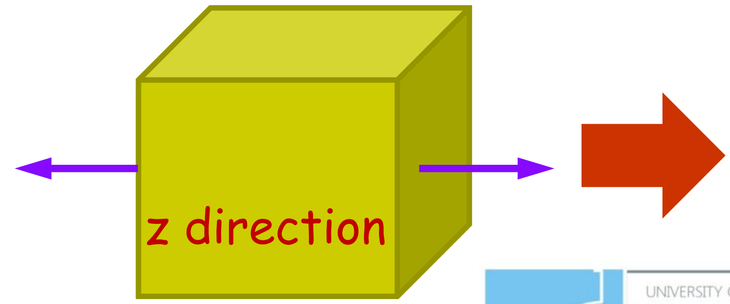


Velocity Field

$$v_x(x,y,z) = -\frac{\dot{\varepsilon}(t)}{2}x$$

$$v_y(x,y,z) = -\frac{\dot{\varepsilon}(t)}{2}y$$

$$v_z(x,y,z) = \dot{\varepsilon}(t)z$$



Uniaxial Extension

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Start-up of Extensional Flow

Extensional Increase of the viscosity

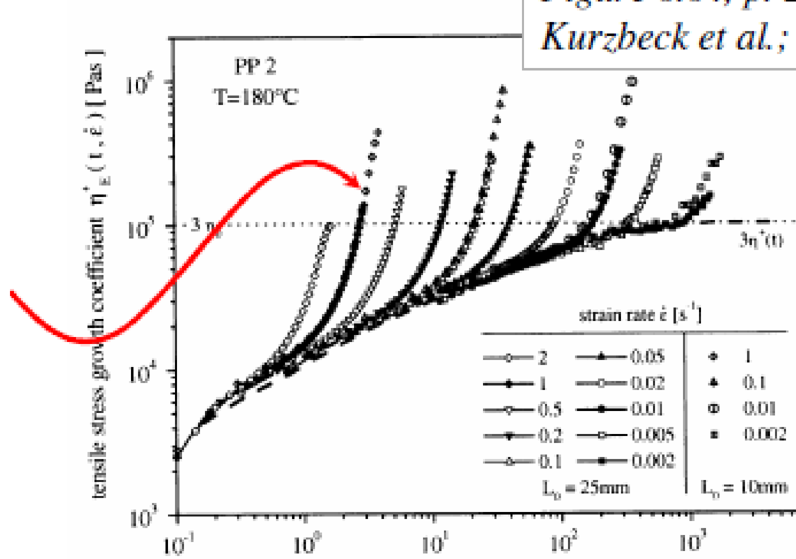

Figure 6.64, p. 218 Kurzbeck et al.; PP

Fitting with Pom-Pom model

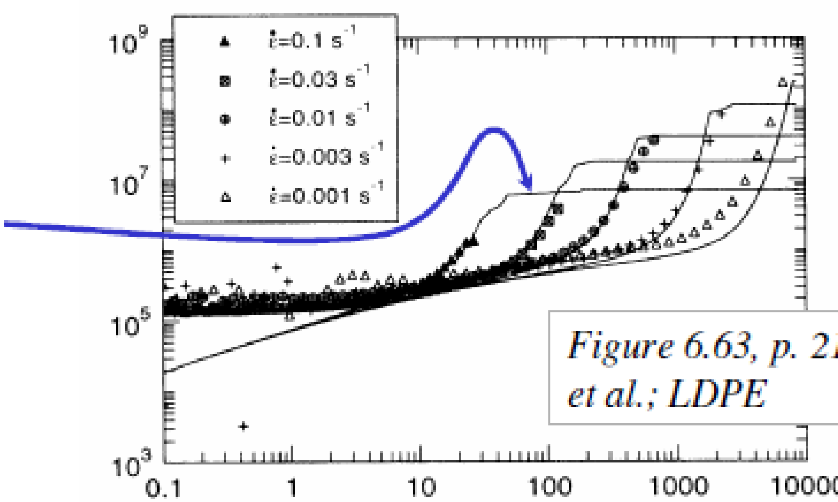

Figure 6.63, p. 217 Inkson et al.; LDPE

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