Consistent Recalibration of Yield Curve Models

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joint work with

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Imperial-ETH Workshop, March 2015

ETH
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Swiss Federal Institute of Technology Zurich

FNSNF
Fonds national suisse
Schweizerischer Nationalfonds
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Swiss National Science Foundation

- Absence of arbitrage.
- Robust calibration: the model is selected simultaneously from time series and prevailing market yields.
- Consistent recalibration: tomorrow's market yield curve does not imply a rejection of today's model.
- Analytic tractability: yield curve increments can be simulated accurately and efficiently.

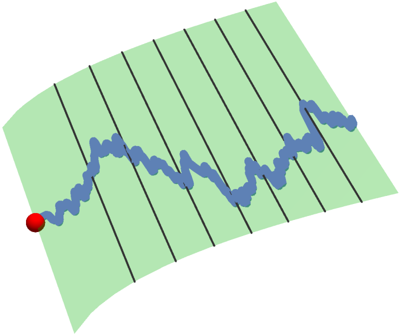
- Factor models: do not allow for robust calibration and consistent recalibration.
- HJM models: lack of analytic tractability.
- PCA models: absence of arbitrage and analytic tractability are issues.
- Filtered historical simulation: ditto.

- Use well-understood affine factor models as “tangent” models.
- The infinitesimal increments of our model belong to affine models with different coefficients.
- This allows us to fit the market dynamics better than in the case of affine models with fixed coefficients.
- The resulting models are called consistent recalibration (CRC) models.

CRC models

Construction 1/2

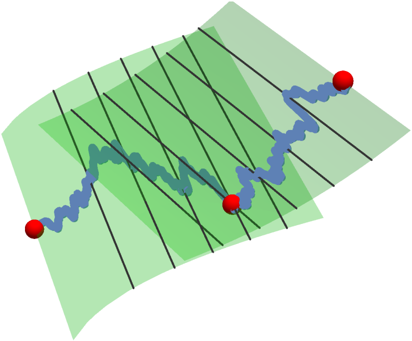
- For each parameter vector y, consider a Hull-White extended affine factor model for the short rate.
- Each factor model admits a finite dimensional realisation in the space of yield curves.



CRC models

Construction 2/2

- Concatenate yield curve increments, each belonging to a Hull-White extended affine factor model with possibly different y.
- CRC models are continuous-time limits of such concatenations.



CRC models

Setup

- $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ is a stochastic basis where $\mathbb{P}$ is a risk-neutral probability measure;
- W is $(\mathcal{F}_t)_{t\geq 0}$-Brownian Motion;
- for each parameter y and $\theta \in C(\mathbb{R}_+)$ consider the factor model

$$dX(t) = (\theta(t) - b_y(X(t)))\,dt + \sqrt{a_y(X(t))}dW(t), \quad t \geq 0,$$

  where a_y and b_y are admissible affine functions; and
- each factor model defines a short rate process by $r = \ell(X)$, where ℓ is a fixed affine map.

CRC models

Heath-Jarrow-Morton (HJM) equation

- The HJM equation for the factor model with fixed parameter y is

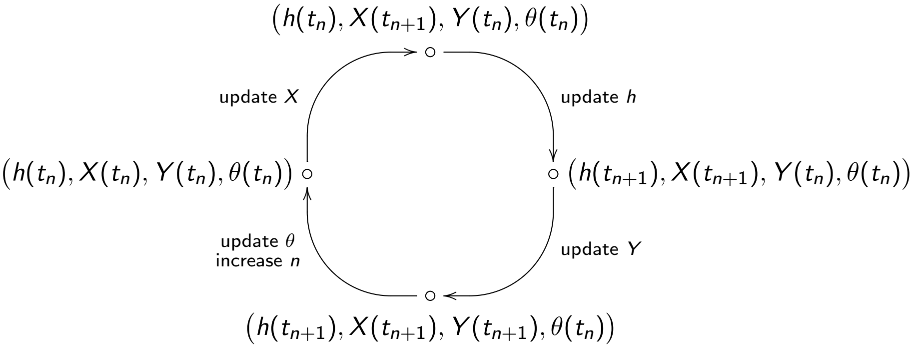
$$dh(t) = \left(h'(t) + \mu_y^{\mathrm{HJM}}(X(t))\right) dt + \sigma_y^{\mathrm{HJM}}(X(t))\, dW(t),$$
$$dX(t) = (\mathcal{C}_y h(t)(0) + b_y(X(t)))\, dt + \sqrt{a_y(X(t))} dW(t),$$

  where C_y is an operator which calibrates θ to the prevailing term structure.

- CRC models replace y by a Markov process Y. Thus, they are described by an SPDE for (h, X, Y).

CRC models

Analytic tractability



- By semigroup methods, one obtains convergence of the simulation scheme to solutions of the HJM equation.
- Increments of the HJM equation can be sampled accurately and efficiently.

- Quadratic covariations of forward rates satisfy

$$d\left[h(\cdot,\tau_i),h(\cdot,\tau_j)\right]=\sigma_Y^{\mathrm{HJM}}(X)(\tau_i)\sigma_Y^{\mathrm{HJM}}(X)(\tau_j)dt,\quad \tau_{i,j}\geq 0.$$

- Estimate some of the components of Y fitting CRC covariation matrices to the dynamics of market yields.
- Calibrate the remaining components of Y to the prevailing market yield curve by regression methods.
- Select and fit a model for the estimated time series of Y.

CRC Models

Consistent recalibration property

- The process h does not leave a pre-specified set $\mathcal{I}$ of possible curves.
- The set $\mathcal{I}$ includes a large portion of possible market observables.
- The process h reaches any neighbourhood of any curve in $\mathcal{I}$ with positive probability.

- Let $\mathcal{I}$ be the space of all possible forward rate curves. For each parameter $y \in \mathbb{R}$ consider the one-factor Vasiček model

$$dh(t) = \left(h'(t) + \mu_y^{\mathrm{HJM}}\right) dt + \sigma_y^{\mathrm{HJM}} dW(t),$$

where

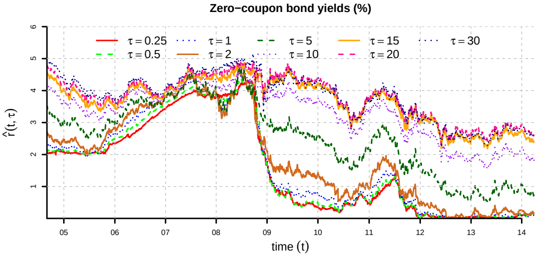
$$\mu_y^{\mathrm{HJM}}(\tau) = -\frac{a}{\beta(y)} e^{\beta(y)\tau} \left(1 - e^{\beta(y)\tau}\right),$$
$$\sigma_y^{\mathrm{HJM}}(\tau) = \sqrt{a} e^{\beta(y)\tau},$$

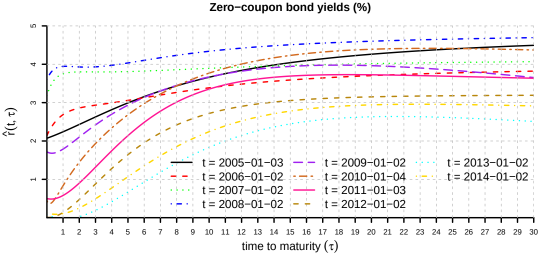
for $a > 0$ fixed and mapping $y \mapsto \beta(y)$.

- Parameter process: $Y = \sigma \widetilde{W}$ for $\sigma > 0$ and $\widetilde{W}$ independent of W.
- Choose $\beta \in C_b^\infty$ such that $\sup_y \beta(y) < 0$ and $\beta'(y) \neq 0$ for all y.

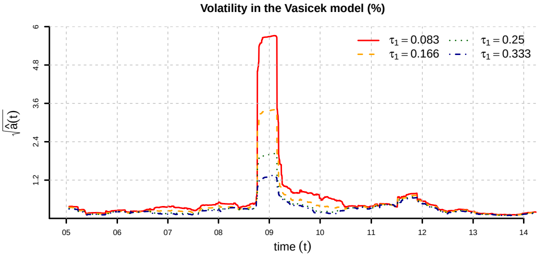
Numerical example

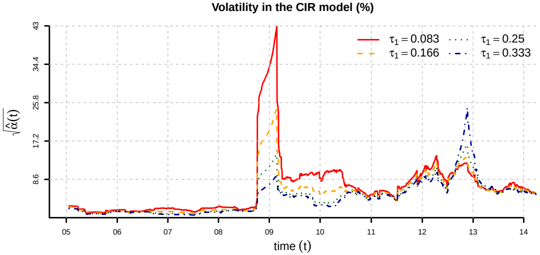
Zero-coupon yields estimated from Euro area government bonds by the ECB





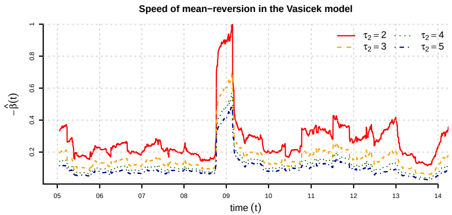
Calibration in the Vasiček and CIR cases: a_Y estimated from the market dynamics

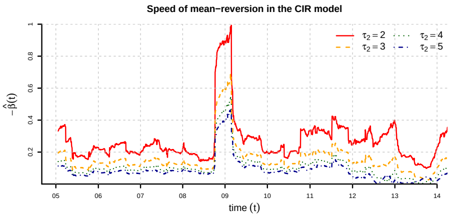




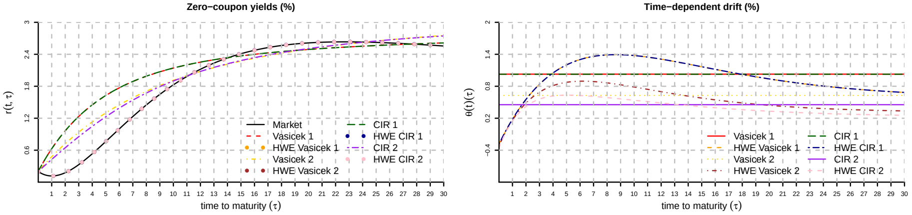
Numerical Example

Calibration in the Vasiček and CIR cases: b_Y estimated from the market dynamics



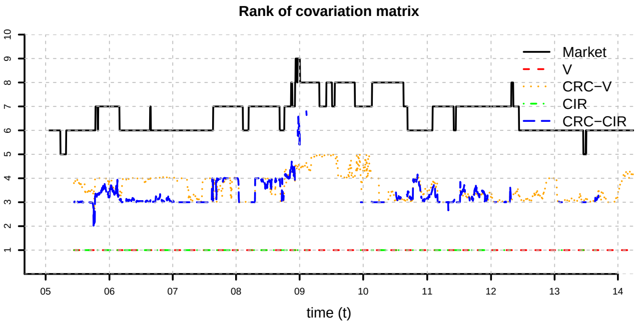


- a_Y and b_Y vary significantly over time.
- Models with constant parameters y do not satisfy the requirement of robust calibration.



- Vasiček 1 and CIR 1: b_Y and a_Y are estimated from the yield curve dynamics.
- Vasiček 2 and CIR 2: b_Y and a_Y are fitted to the prevailing yield curve.
- θ is calculated so that the initial model yield curve exactly matches the prevailing market yield curve.

Numerical Example
Covariation matrices



- V and CIR: Hull-White extended Vasiček and CIR models.
- CRC-V and CRC-CIR: CRC versions of V and CIR.
- The consistent recalibration property of CRC models is reflected in the higher ranks of the covariation matrices.