Admissions Test Sample Questions and Solutions

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1. The expression $\dfrac{2+2\sqrt{2}}{2+\sqrt{2}}$ is equal to

   A. $2\sqrt{2}$

   B. $2+\sqrt{2}$

   C. 4

   D. $\sqrt{2}$

   E. 2

Solution

Multiply both the numerator and the denominator by $(2-\sqrt{2})$ as follows:

$$\frac{(2+2\sqrt{2})}{(2+\sqrt{2})} = \frac{(2+2\sqrt{2})(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}.$$

This is to remove the surd in the denominator. Then multiply out the brackets to obtain:

$$\frac{(2+2\sqrt{2})(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})} = \frac{4-2\sqrt{2}+4\sqrt{2}-4}{4-2\sqrt{2}+2\sqrt{2}-2}.$$

Finally, simplify the result to get:

$$\begin{aligned}\frac{4-2\sqrt{2}+4\sqrt{2}-4}{4-2\sqrt{2}+2\sqrt{2}-2} &= \frac{2\sqrt{2}}{2} \\ &= \sqrt{2}.\end{aligned}$$

Therefore the correct answer is D.

2. The derivative with respect to x of $\tan x$ is

   A. $\sec^2 x$

   B. $\operatorname{cosec}^2 x$

   C. $\cot x$

   D. $\sin^2 x$

   E. $\cos^2 x$

Solution

The first step is to realise than $\tan x = \frac{\sin x}{\cos x}$, and then apply the product rule as follows:

$$\begin{aligned}
\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x} &= \frac{d}{dx}\sin x(\cos x)^{-1} \\
&= \left(\frac{d}{dx}\sin x\right)(\cos x)^{-1} + \sin x\left(\frac{d}{dx}(\cos x)^{-1}\right) \\
&= \cos x(\cos x)^{-1} + \sin x(-1)(\cos x)^{-2}(-\sin x) \\
&= 1 + \frac{\sin^2 x}{\cos^2 x} \\
&= \frac{\sin^2 x + \cos^2 x}{\cos^2 x} \\
&= \frac{1}{\cos^2 x} = \sec^2 x,
\end{aligned}$$

where we have used the fact that $\frac{d}{dx}\sin x = \cos x$ and $\frac{d}{dx}\cos x = -\sin x$, and the identity $\sin^2 x + \cos^2 x = 1$. Therefore the correct answer is A.

3. A cylinder 6 inches long is drilled straight through the centre of a solid sphere of radius R, as shown below. What is the radius of the cylinder?

A. $\sqrt{R^2 - 9}$
B. $\sqrt{R^2 - 3}$
C. $\sqrt{R - 9}$
D. $R - 3$
E. Cannot determine – more information needed

Solution
Once a suitable diagram has been set up, it is straightforward to see that the answer is obtained by using Pythagoras' theorem. The diagram below shows the radius R of the sphere (pointing from the centre of the sphere to the edge of the sphere where it meets the corner of the cylinder), the radius of the cylinder (which we will call x), and half of the length of the cylinder ($6/2 = 3$). Using Pythagoras:

$$\begin{aligned} R^2 &= 3^2 + x^2 \\ x^2 &= R^2 - 9 \\ x &= \sqrt{R^2 - 9}. \end{aligned}$$

Therefore the correct answer is A.

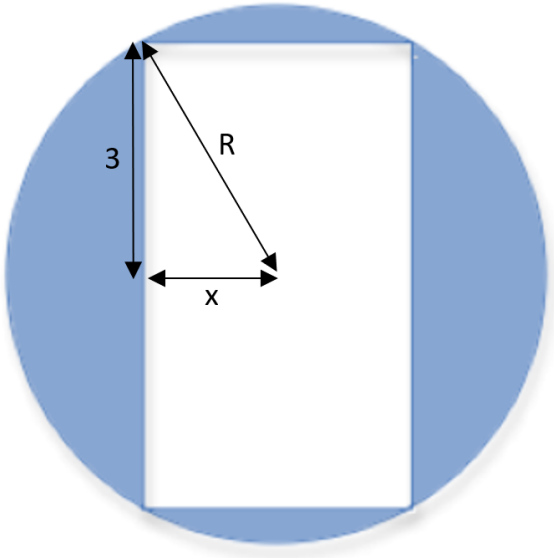

Figure 1: Labeled diagram of the cylinder in question 3 to show how to set it up to obtain the answer.

4. The Poynting vector is a useful quantity in the study of electromagnetism. The magnitude of the Poynting vector, S, in vacuum is defined as

$$S = \frac{1}{\mu_0} E \times B.$$

The units of the vacuum permeability, μ_0, are Newtons per square Ampere. The units of the electric field, E, are Newtons per Coulomb. The units of the magnetic field, B, are Joules per Ampere per square metre.

By considering the equation and the units given, what is the physical quantity represented by the Poynting vector?

A. Power per unit area
B. Energy per unit area
C. Energy per unit charge
D. Force per unit area
E. Force per unit charge

Solution

This question has been designed to test how you deal with unfamiliar equations. All the information that is needed to answer the question is provided in the question, or is in the GCSE or A-level Year 12 syllabus.

You are not expected to know what the Poynting vector is. The equation is given and the first step is to substitute the units for each quantity in the right-hand side of the equation and then simplify to find the units of the unknown quantity S. Let us do that

$$\begin{aligned}
\text{The units of } S \text{ are} &= \left(\frac{1}{N/A^2}\right)\left(\frac{N}{C}\right)\left(\frac{J}{A.m^2}\right) \\
&= \left(\frac{A^2}{N}\right)\left(\frac{N}{C}\right)\left(\frac{J}{A.m^2}\right) \\
&= \frac{A.J}{C.m^2} \\
&= \frac{J}{m^2.s} \\
&= \frac{W}{m^2}
\end{aligned}$$

where to get to the second-to-last line we have used the definition of current as the rate of flow of charge, such that 1 Ampere = 1 Coulomb per second. And to get to the final line we have used the definition of power as the rate of change of energy, such that 1 Watt = 1 Joule per second.

Considering these units, we see that a Watt is the unit of the quantity Power, and a square metre is the unit of the quantity area. Therefore, the quantity represented by the Poynting vector is Power per unit area, giving answer A.

5. The below figure shows a person pushing a plug into a wall socket using their fingers. The plug is fully inserted and the person remains applying a horizontal force to the plug for a short time to make sure that it has been inserted properly. Which one of the following is the force that is paired with the force being applied by the person's fingers on the plug, according to Newton's third law?

A. The reaction force from the wall on the plug

B. The force from the plug on the person's fingers

C. The weight of the plug

D. The vertical frictional force preventing the fingers from slipping off the plug

E. The force from the plug on the wall

Solution
Newton's third law states that the forces result from an interaction between two objects such that the force one object exerts on another is equal in magnitude and opposite in direction to the force the second object exerts on the first.
The question statement refers to the force of the person's fingers on the plug. Therefore, the two objects we are considering are the fingers and the plug. Reversing the order of these two objects in the question statement gives the solution: the force from the plug on the person's fingers, which is choice B.