Robustness of Systemic Risk Analyses

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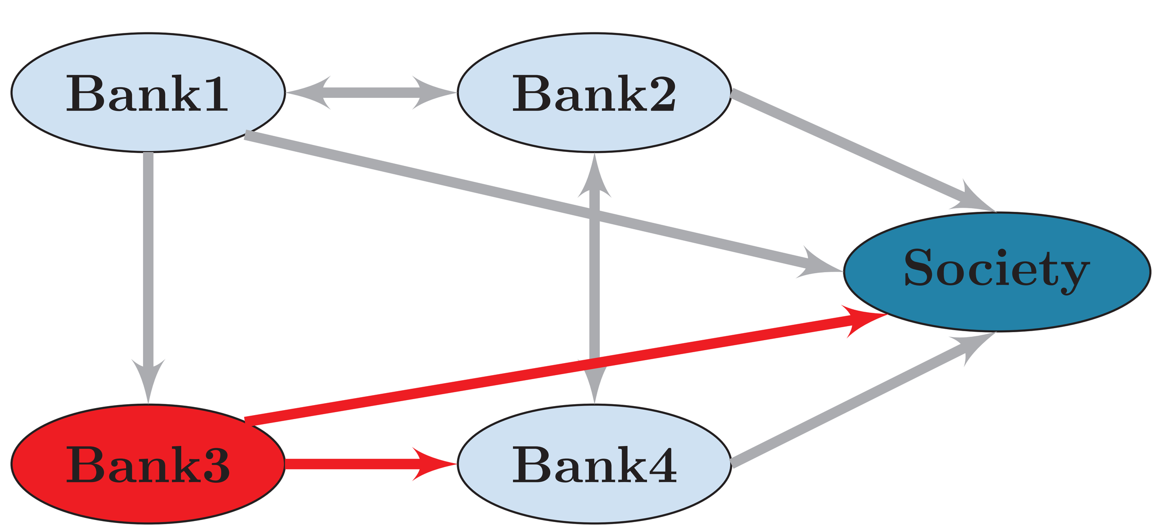
Motivation

- Regulators use the Eisenberg-Noe algorithm to quantify contagion in macro stress tests.
- When accurate data on interbank liabilities is not available, this needs to be estimated.
- Goal: We want to compute the maximal error in the estimation of the clearing payments for a given average estimation error in interbank liabilities.

Financial networks $(A, \bar{p}, x)$

Financial network:
- $\mathcal{N} = \{1, \ldots, N\}$: banks;
- x_i: capital of bank i;
- $l_{ij} \geq 0$: liability of bank i to bank j;
- $\bar{p}_i$: total liabilities of i;
- A: relative liability matrix $a_{ij} := \frac{l_{ij}}{\bar{p}^i}$.

Network with society: Society is modelled by adding a node 0 to the network.



Clearing vector

Assumptions: A solvent bank repays all of its obligations. A defaulting bank repays obligations pro-rata. These rules yield a *clearing payment* as a solution of the fixed point problem:

$$p(A) = \bar{p} \wedge (x + A^T p(A)).$$

Existence and Uniqueness: A fixed point always exists, and under mild technical assumptions it is unique. A simple sufficient condition for uniqueness is that every bank has some positive equity $x_i > 0$ for all i to begin with.

Regular Financial Network: *Surplus set* $S \subset \mathcal{N}$: $\forall (i,j) \in S \times S^c : a_{ij} = 0$ and $\sum_{i \in S} x_i > 0$. *Risk orbit* of bank i: $o(i) = \{j \in \mathcal{N} \mid$ there exists a directed path from i to $j\}$. *Regular network*: each bank's risk orbit $o(i)$ is a surplus set. Ω set of all regular networks.

Estimation Errors

Let A be the true relative liability matrix. We model the estimation error as $A + \epsilon B$ for some $\epsilon > 0$, such that $A + \epsilon B$ is still an admissible relative liability matrix.

Sensitivity

We prove continuity of $p(A)$ wrt A. This allows us to compute the directional derivative of the fixed point (clearing vector) in direction of B:

$$\mathcal{D}_B(p(A)) = \lim_{\epsilon \to 0} \frac{p(A + \epsilon B) - p(A)}{\epsilon}.$$

The derivative is given by a fixed point equation:

$$\mathcal{D}_B(p(A)) = \Lambda A^\top \mathcal{D}_B(p(A)) + \Lambda B^\top p(A).$$

Results: Sensitivity

Theorem: First order sensitivity.
In a regular financial network $(\mathcal{A}, \bar{p}, x) \in \Omega$, the directional derivative of the clearing vector in direction of an error matrix B is given, (except on the measure zero set $\{(A, \bar{p}, x) \in \Omega \,|\, \exists i \text{ s.t. } x_i + \sum_{j=1}^n a_{ji} p_j(A) = \bar{p}_i\}$), by

$$\mathcal{D}_B(p(A)) = \left(I - \Lambda A^\top\right)^{-1} \Lambda B^\top p(A),$$

where Λ is the diagonal matrix with entries $\mathbb{1}_{\{x_i + \sum_{j=1}^n a_{ji} p_j(A) < \bar{p}_i\}}$.

Theorem: Higher order derivatives.
In a regular financial network $(\mathcal{A}, \bar{p}, x) \in \Omega$, higher order derivatives of the clearing vector in direction of the matrix B are given for all $m \geq 1$ by

$$\mathcal{D}_B^{(m)}(p(A)) = m(I - \Lambda A^\top)^{-1} \Lambda B^\top \mathcal{D}_B^{(m-1)}(p(A)) = m! \left(\left(I - \Lambda A^\top\right)^{-1} \Lambda B^\top\right)^m p(A).$$

Hence, for all $\epsilon < \epsilon^*$, we get the exact Taylor expansion

$$p(A + \epsilon B) = \left(I - \epsilon (I - \Lambda A^\top)^{-1} \Lambda B^\top\right)^{-1} p(A),$$

where ϵ^* describes the first change in Λ, which happens when a solvent bank becomes insolvent (or vice versa): $\epsilon^* := \inf\{t > 0 \,|\, \exists i : x_i + \ldots \left[A^\top (I - t(I - \Lambda A^\top)^{-1} \Lambda B^\top)^{-1} p(A)\right]_i \in \{0, \bar{p}_i\}\}$.

Empirical application: Robustness & worst case error

Let Ω be a regular financial network with society.

Question: *How far off can the estimated payout to society be for a given average estimation error in the relative liability matrix?*

Problem formulation: We need to solve the LP

$$\min_B a_0^\top p(A + \epsilon B) \rightsquigarrow \min_B a_0^\top \mathcal{D}_B(p(A)) \qquad (1)$$

A unique solution is obtained by solving the LP $\min_{b \in \mathcal{B}_I(A), c^\top b \leq z^*} b^\top b$, where z^* is the value of the objective function achieved by a minimizer of LP (1). This corresponds to finding the maximum perturbation that has minimum Frobenius norm.

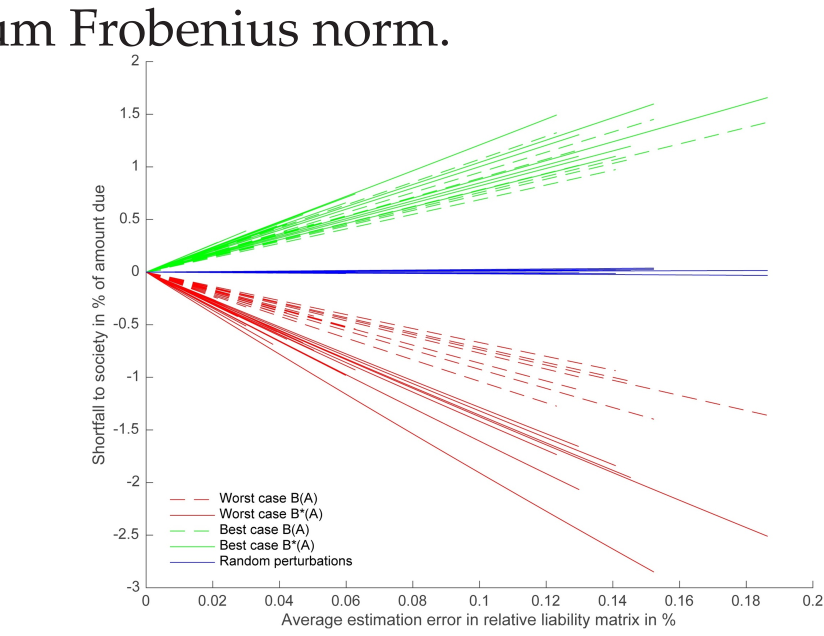


Figure 1: Influence of errors on shortfall.

Empirical application to European Banks: We use bank balance sheet data from the EBA and construct a relative liability network consistent with this data using the Gandy-Veraart MCMC algorithm. Next, we compute the payout to society under a variety of stress tests and for each scenario, we compute the worst-case shortfalls to society given an average misspecification of the relative liability matrix.

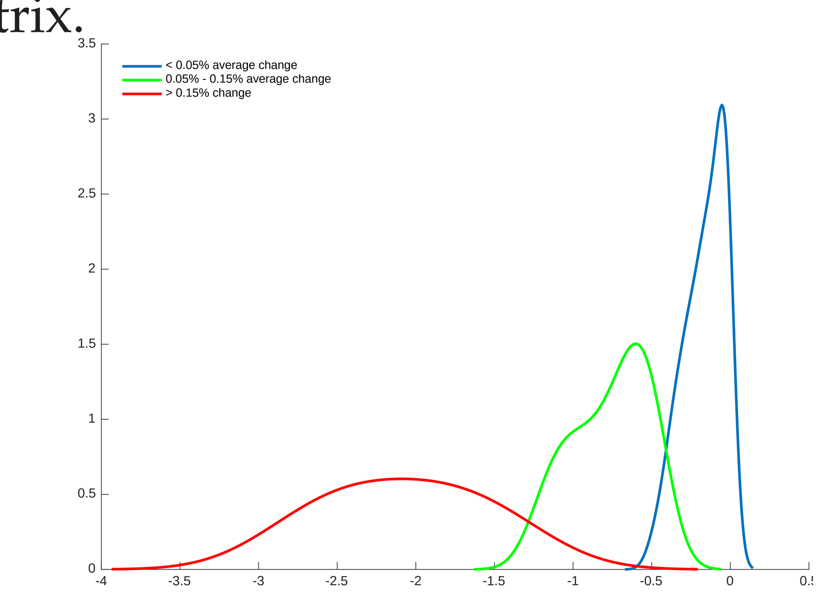


Figure 2: Kernel density estimates of the shortfall to society for different average estimation errors along the worst-case perturbation.

We find that a 0.15% error of the relative interbank liabilities could in the worst case lead to an additional shortfall of 2.5% to society.

Conclusions and future research

Structure of worst-case errors: The worst case estimation errors occur when one overestimates the liabilities of well-capitalised banks, and underestimates the liabilities of weak banks.

Robustness of systemic risk assessments: The results and conclusions of a stress test can deviate substantially even for small average estimation errors, if they cluster in the direction of the worst case estimation errors. This highlights the importance and benefits of precise data collection compared to estimating interbank exposures.

Probability of worst case error: Quantify the probability of being "close" to a worst case shortfall error estimation for different relative liability matrix estimation-error distributions.

References

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2. A. Gandy, L. Veraart, A Bayesian methodology for systemic risk assessment in financial networks, 2015.

Funding

Luxembourg Research Fund: AFR PhD Grant
National Science Foundation under Grant No. 1321794.

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