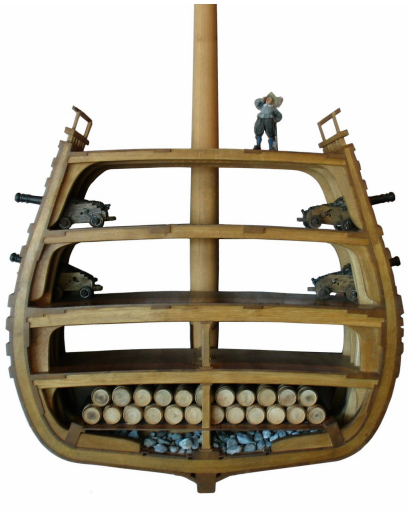
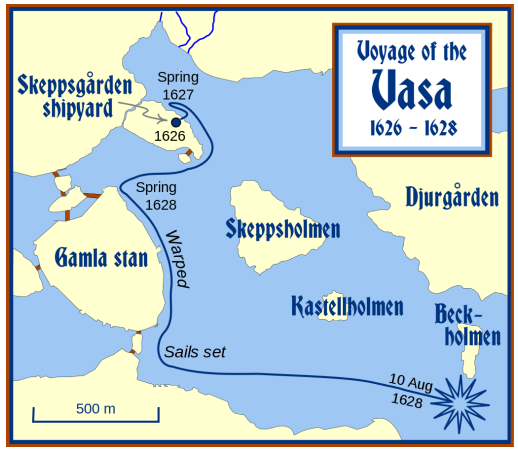
Random Systems: How Can They Be Stable?

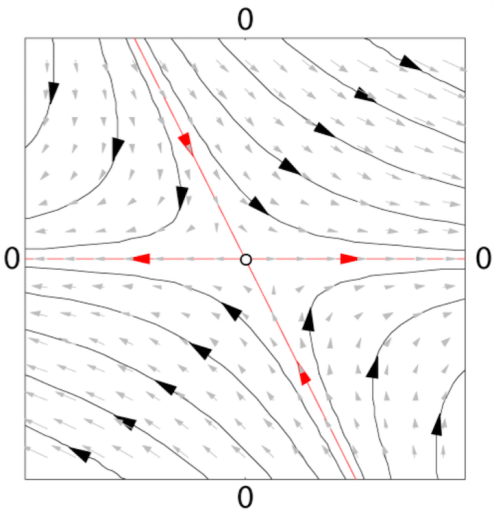


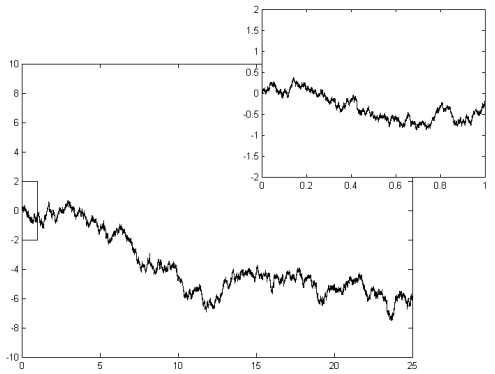


Components of Stability:

- deterministic (e.g. initial balance of Vasa)
- random / uncertain (e.g. wind & waves)

Models for Uncertainty?

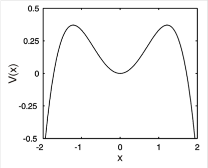


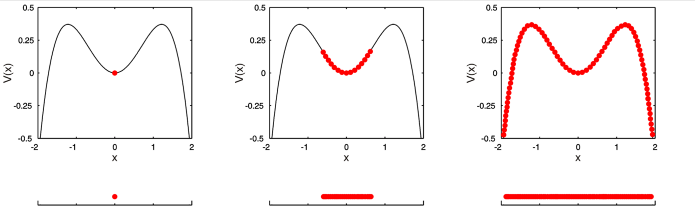


Concept of stability in systems with randomness:

- What do we mean by stable?
- In Brownian Motion model for randomness, increments may be unbounded (arbitrarily large noise amplitude!)

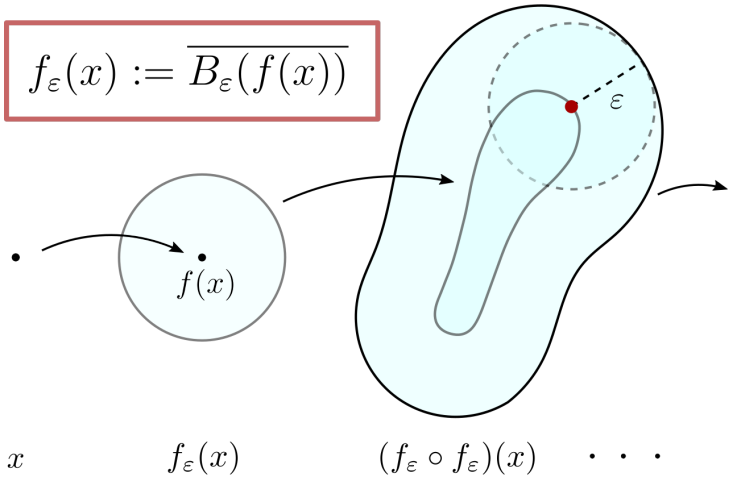
Stability Under Bounded Noise





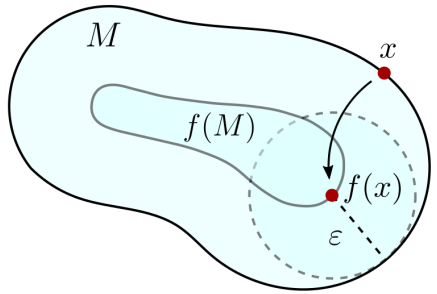
Studying Bounded Noise via Set-valued Dynamics

$$f_\varepsilon(x) := \overline{B_\varepsilon(f(x))}$$

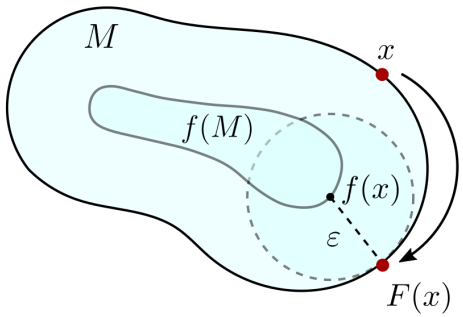


Invariant (stable) sets satisfy the equation

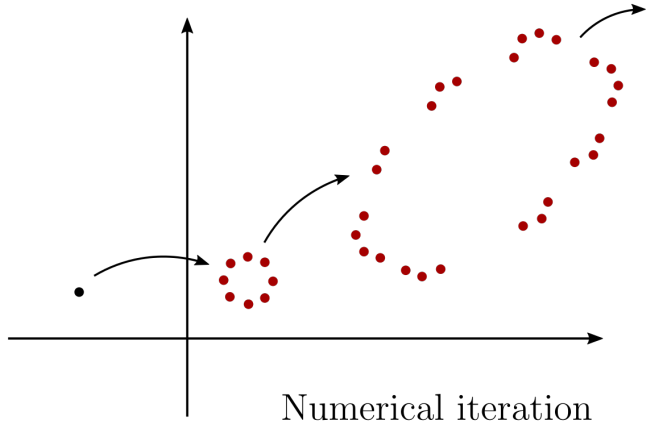
$$M = f_\varepsilon(M)$$



Set-valued approach allows one to track dynamics on the boundary ∂M

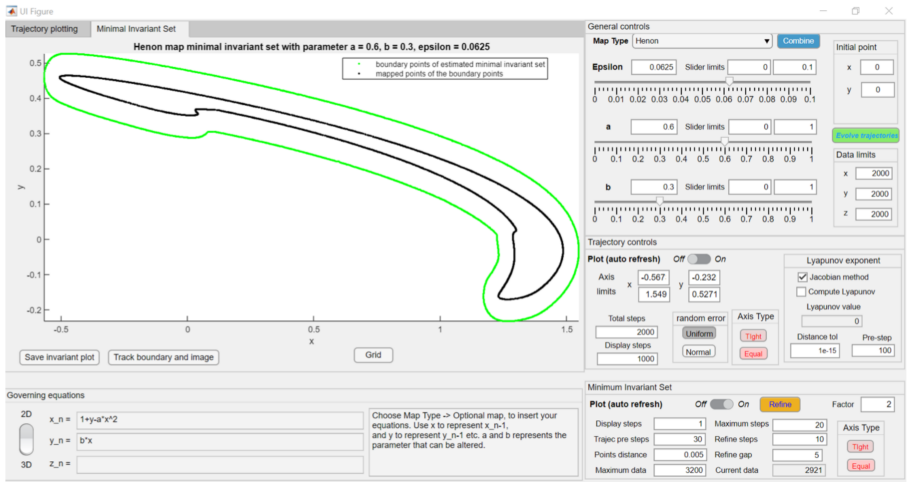


Possible clues for early warning signals!



Numerical iteration of set-valued map f_ε

Overview of Project



Aspects of the project include:

- To understand the properties of stable sets and the corresponding boundary maps. Early warning signals for loss of stability?
- Several possibilities: state-dependent or time-dependent uncertainty, different distributions for noise, etc.
- Many questions open: how to deal with high dimensional systems, how to devise efficient numerical algorithms, etc.
- Objective: to build a novel, efficient and flexible tool for numerical stability analysis for real-world systems