**Martingale problems and stochastic equations**

Beginning with Lévy’s characterization of Brownian motion, the martingale properties of stochastic processes have proven to be fundamental in the formulation and analysis of stochastic models, in particular, in the study of Markov processes. Stochastic equations of various types provide a second fundamental approach to defining stochastic models. These lectures will explore aspects of both of these approaches and their relationship. The martingale problem of Stroock and Varadhan will be formulated and some of its properties described. The notion of a weak solution of a stochastic differential equation will be introduced and the fact that solutions of appropriate martingale problems give weak solutions of stochastic differential equations will be discussed. Time change equations for Markov chains and diffusions will be given and the equivalence of these equations to corresponding martingale problems shown.

**Background**

Students interested in following the lectures are encouraged to review of the basics of stochastic processes and martingales in continuous time and of stochastic integration. This background is summarized on several slides that are posted at

<http://www.math.wisc.edu/~kurtz/Lectures/NELDLECT.pdf>

Professor Kurtz will be more than happy to answer questions on any of this material.

More detail can be found in the following:

The first five chapters of the notes *Lectures on Stochastic Analysis* which has the advantage of being free. <http://www.math.wisc.edu/~kurtz/m735.htm>

Chapter 2 of Ethier and Kurtz, *Markov Processes: Characterization and Convergence*

The first two chapters of Protter, *Stochastic Integration and Differential Equations*, Second Edition

**Lectures**

**Monday, April 27, 2015**

**Characterizing stochastic processes by their martingale properties**

 Lévy’s characterization of Brownian motion

 Watanabe’s characterization of the Poisson process

 Intensities for counting processes

**Markov processes and their generators**

 Markov chains

 Diffusion processes

 Building Markov generators (sums of generators are generators--usually)

**Tuesday, April 28, 2015**

**Forward equations**

 Weak form

 Equivalence with martingale problems

**Other types of martingale problems**

 Local martingale problems

 Constrained martingale problems

**Wednesday, April 29, 2015**

**Martingale problems for conditional distributions**

 A Markov mapping theorem

 Applications to lookdown constructions

**Filtering**

 Derivation of filtering equations

 Markov property of filters

 Uniqueness

**Thursday, April 30, 2015**

**Stochastic equations for Markov processes**

 Classic models in Rd

 Equations for spatial birth and death processes

**Equivalence of stochastic equations and martingale problems**

 Classic models in Rd

 Equations for spatial birth and death processes

**Friday, May 1, 2015**

**Time change equations**

 Continuous time Markov chains

 Doeblin and time-change equations for diffusions

**Particle representation**

 Measure-valued processes

 Stochastic partial differential equations