Mathematical Problems in Climate Dynamics, Nelder Fellow Lectures

24 February 2014

Lecture IV
The Wind-Driven Ocean Circulation

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Please visit these sites for more info.
http://www.atmos.ucla.edu/tcd/
http://www.environnement.ens.fr/

Motivation

- The ***North Atlantic Oscillation (NAO)*** is a leading mode of variability of the Northern Hemisphere and beyond.
- It affects ***the atmosphere and oceans*** on several time and space scales.
- Its ***predictive understanding*** could help interannual and decadal-scale climate prediction over and around the North Atlantic basin.
- The ***hierarchical modeling*** approach allows one to give proper weight to the understanding provided by the models *vs*. their realism, respectively.
- Back-and-forth between ***"toy"*** (conceptual) and ***detailed*** ("realistic") ***models***, and between ***models*** and ***data***.

Joint work with *F. Codron, H. A. Dijkstra, Y. Feliks, S. Jiang, F.-F. Jin, H. Le Treut, E. Simonnet, S. Speich*, and *S. Wang*

Outline, Tipping Points II

- The NAO and the oceans' wind-driven circulation
- The low-frequency variability of the double-gyre circulation
  - bifurcations in a toy model
    - ➔multiple equilibria, periodic and chaotic solutions
  - some intermediate model results
- Atmospheric impacts
  - simple and intermediate models + GCMs
- Some data analysis – atmospheric and oceanic
- Some very promising NAO results
- Conclusions and bibliography

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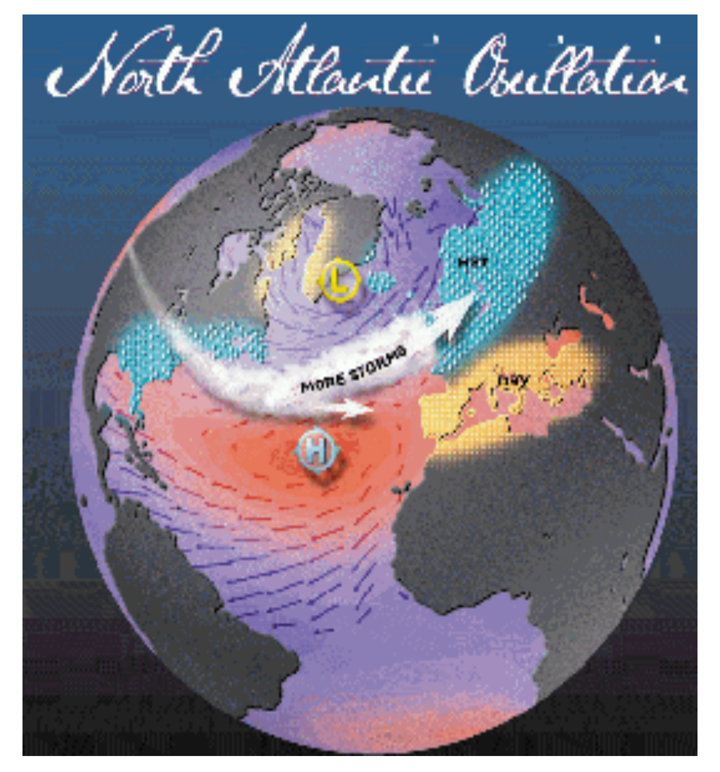
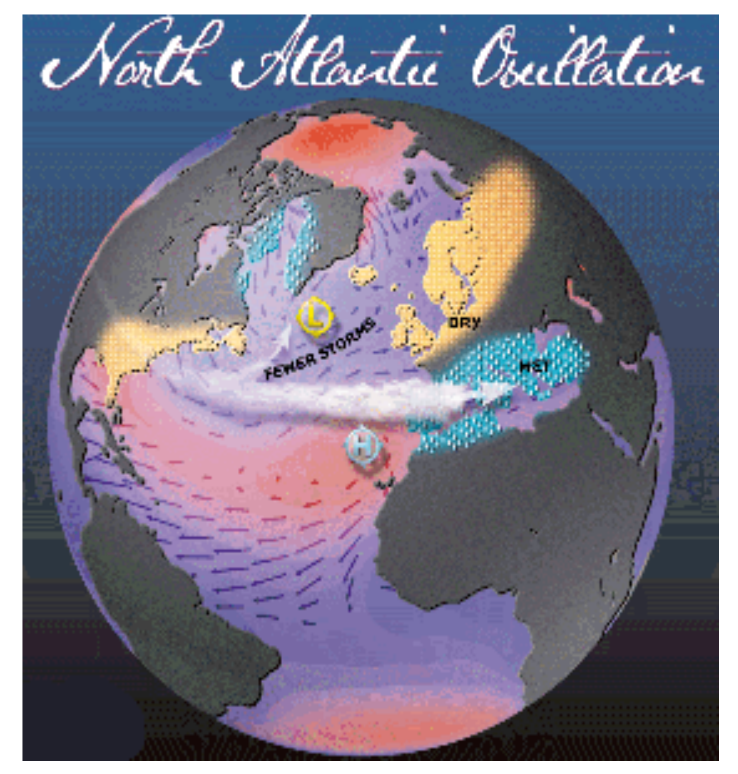
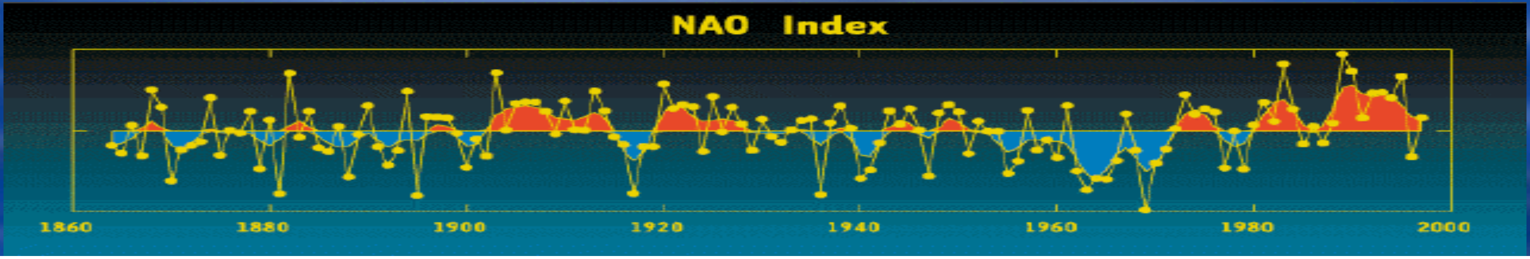
The North Atlantic Oscillation (NAO)

Positive phase

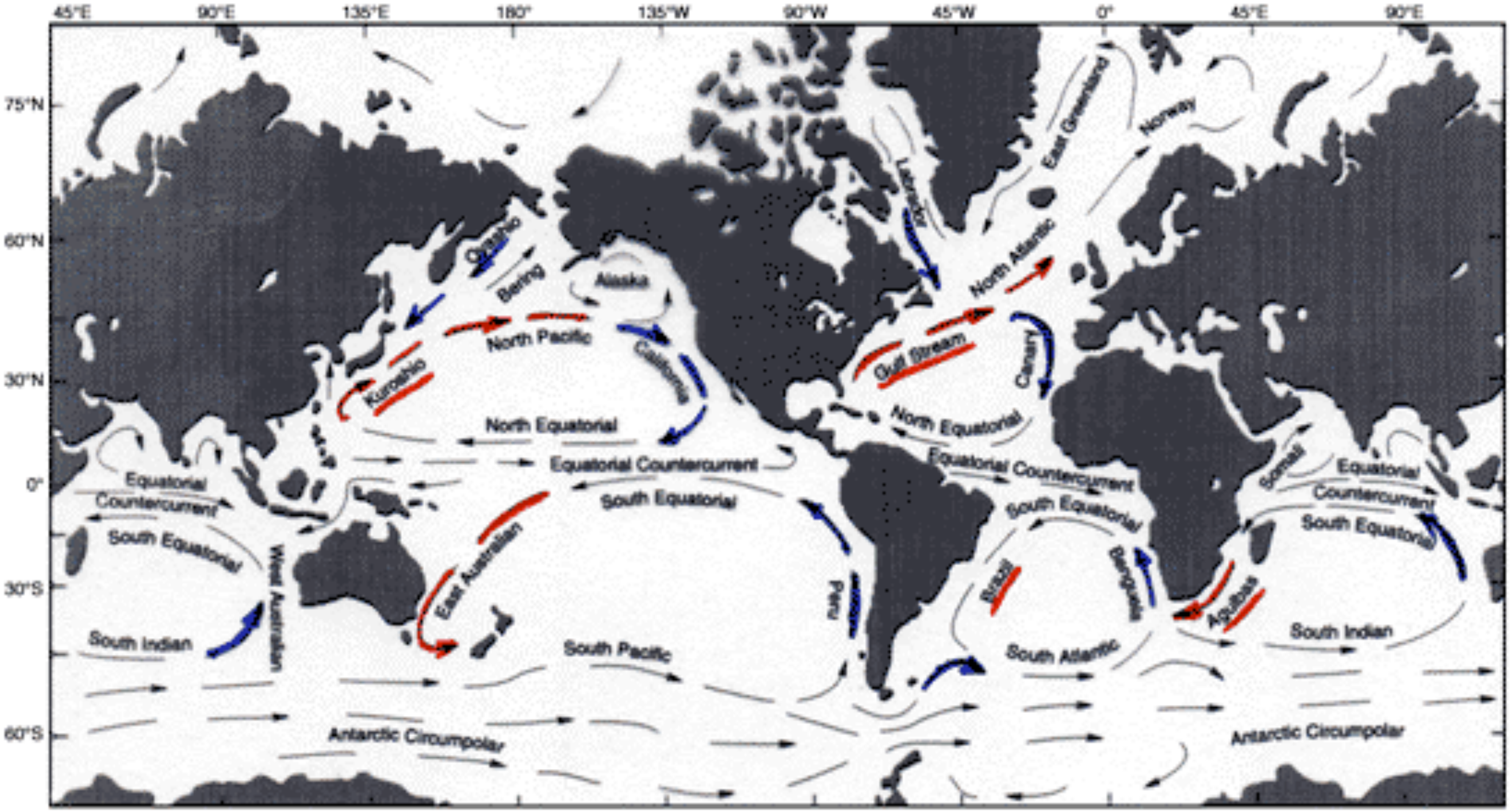


Negative phase





An example of bifurcations and hierarchical modeling: The oceans’ wind-driven circulation



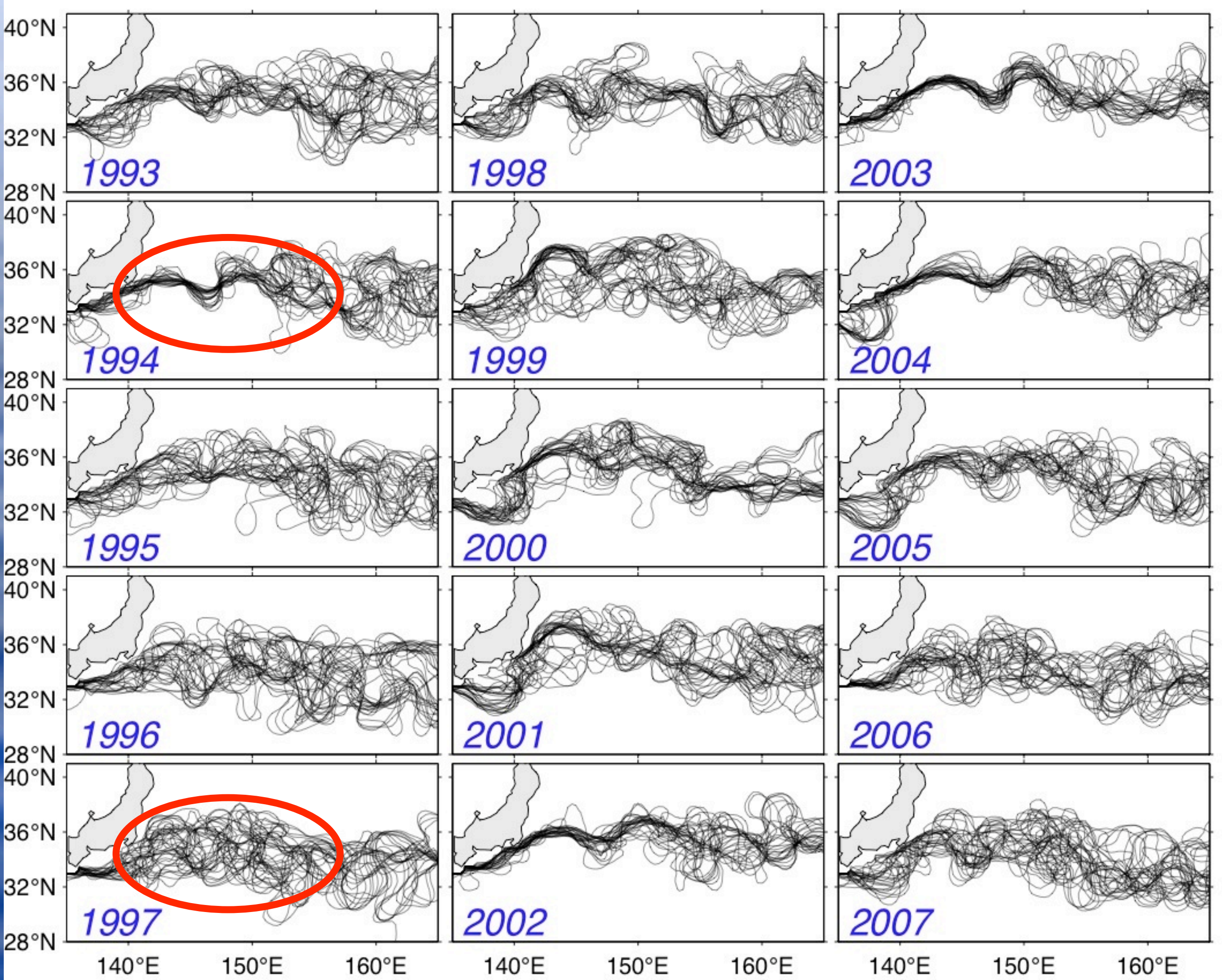
J. Apel (1987), Principles of Ocean Physics

The mean surface currents are (largely) wind-driven

Kuroshio Extension (KE) Path Changes

Monthly paths from altimeter:

Stable vs. unstable periods



Qiu & Chen
(*Deep-Sea Res.*, 2009)

"Limited-contour" analysis for atmospheric low-frequency variability

10-day sequences of
subtropical jet paths:
blocked vs. zonal
flow regimes

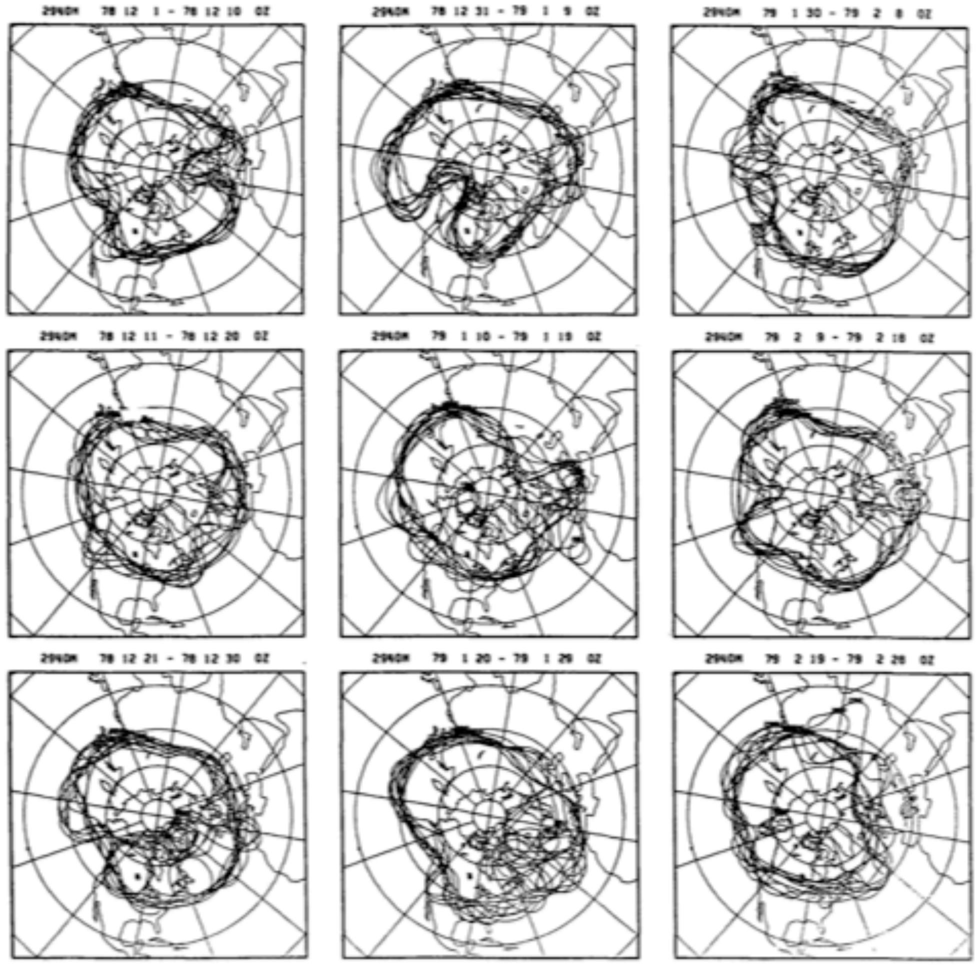


FIG. 1. Limited contour analysis of Northern Hemisphere (NH) flows. Daily contours of a prescribed height (2940 m in this case—roughly corresponding to the jet axis) are superimposed for successive 10-day intervals during NH winter 1978/79. Persistence is illustrated by some of the panels (see text for details).

Kimoto & Ghil, *JAS*, 1993a

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Modeling Hierarchy for the Oceans

Ocean models

- 0-D: box models – chemistry (BGC), paleo
- 1-D: vertical (mixed layer, thermocline)
- 2-D – meridional plane – THC
    - → also 1.5-D: a little longitude dependence
  - – horizontal – wind-driven
    - → also 2.5-D: reduced-gravity models (n.5)
- 3-D: OGCMs - simplified
  - - with bells & whistles ("kitchen sink")

Coupled 0-A models

- Idealized (0-D & 1-D): intermediate couple models (ICM)
- Hybrid (HCM) - diagnostic/statistical atmosphere

  - highly resolved ocean
- Coupled GCM (3-D): CGCM

The double-gyre circulation and its low-frequency variability

An “intermediate” model of the mid-latitude, wind-driven ocean circulation: 20-km resolution, about 15 000 variables

Shallow-water model

$$\begin{aligned}
\frac{\partial U}{\partial t} + \nabla\cdot(\mathbf{u}U) &= -g'h\frac{\partial h}{\partial x} + fV + \underline{\alpha_A} A\nabla^2 U - RU - \underline{\alpha_\tau}\frac{\tau^x}{\rho}\\
\frac{\partial V}{\partial t} + \nabla\cdot(\mathbf{u}V) &= -g'h\frac{\partial h}{\partial y} - fU + \underline{\alpha_A} A\nabla^2 V - RV\\
\frac{\partial h}{\partial t} &= -(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y})
\end{aligned}$$

where

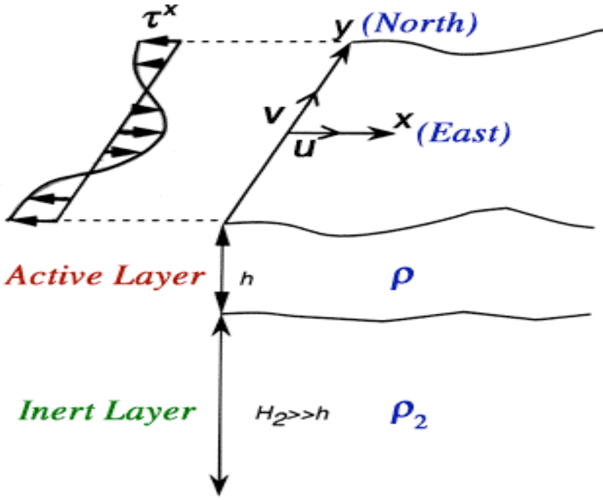
$$U\hat{e_x} + V\hat{e_y} = h\mathbf{u} = h(u\hat{e_x} + v\hat{e_y})$$

g': reduced gravity $(= g(\rho_2 - \rho)/\rho)$

A: viscosity coefficient $(= 300\ \mathrm{m^2 s^{-1}})$

R: Rayleigh coefficient $(= 1/200\ \mathrm{day^{-1}})$

τ^x: wind stress $= \tau_o \cos 2\pi/L (\tau_o = 1\ \mathrm{dyn\ cm^{-2}}\ \&L = 2000\mathrm{km})$

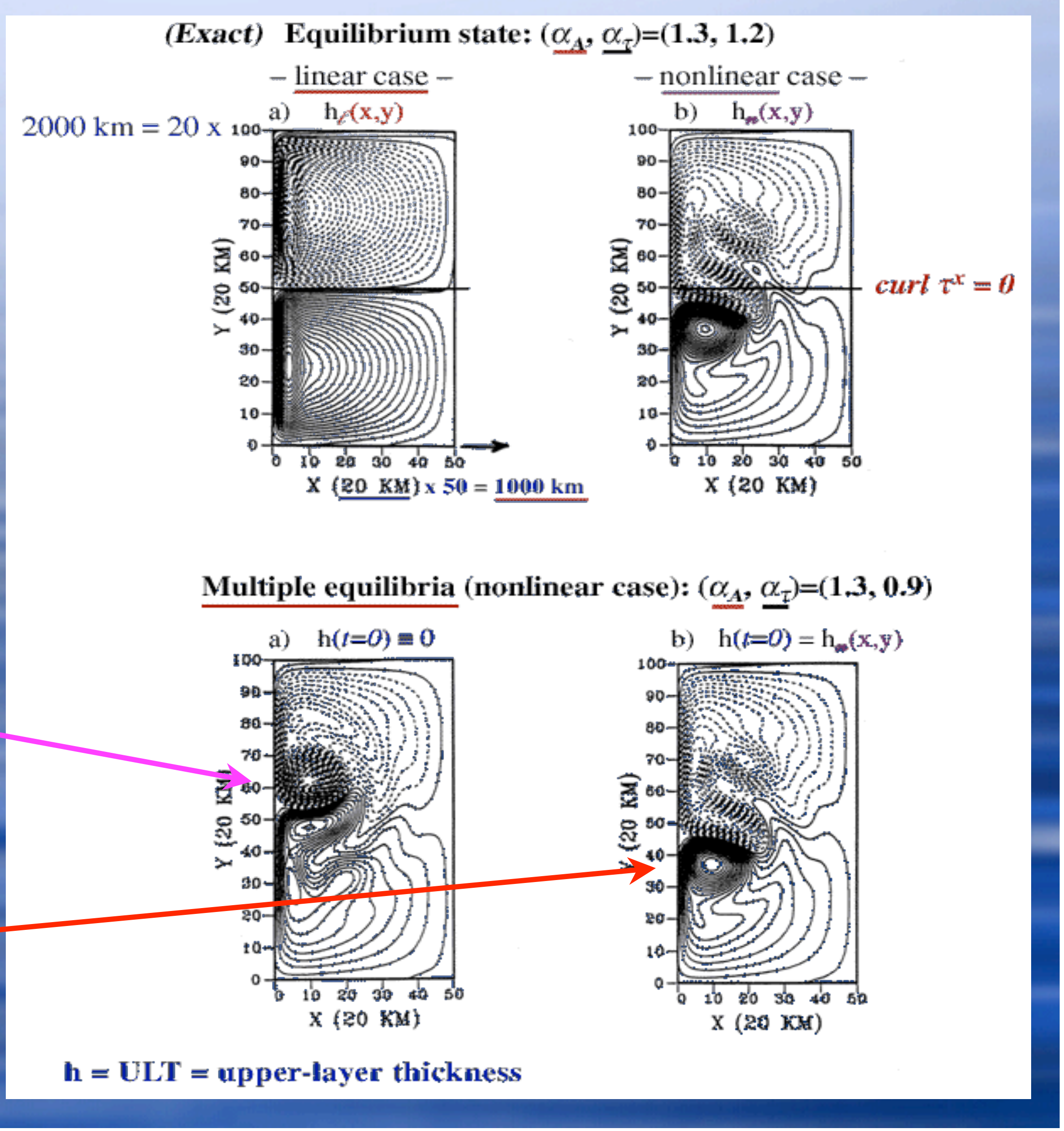


Reduced gravity (1.5 - layer) model

S. Jiang, F.-F. Jin & M. Ghil (1995)
J. Phys. Oceanog., 25: 764-786

The JJG model's equilibria

Nonlinear (advection) effects break the (near) symmetry: (perturbed) pitchfork bifurcation?



(Exact) Equilibrium state: (α_A, α_τ)=(1.3, 1.2)

– linear case – – nonlinear case –

Multiple equilibria (nonlinear case): (α_A, α_τ)=(1.3, 0.9)

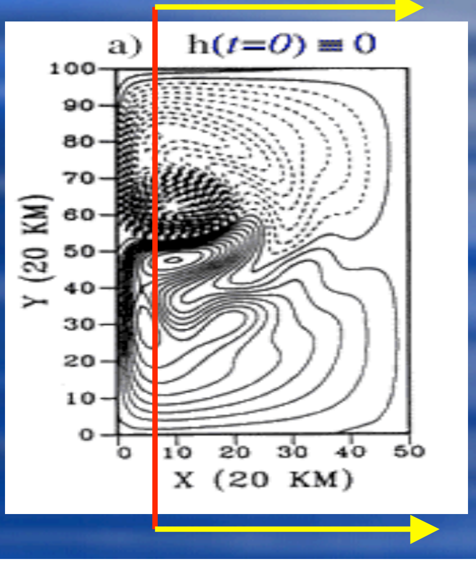
h = ULT = upper-layer thickness

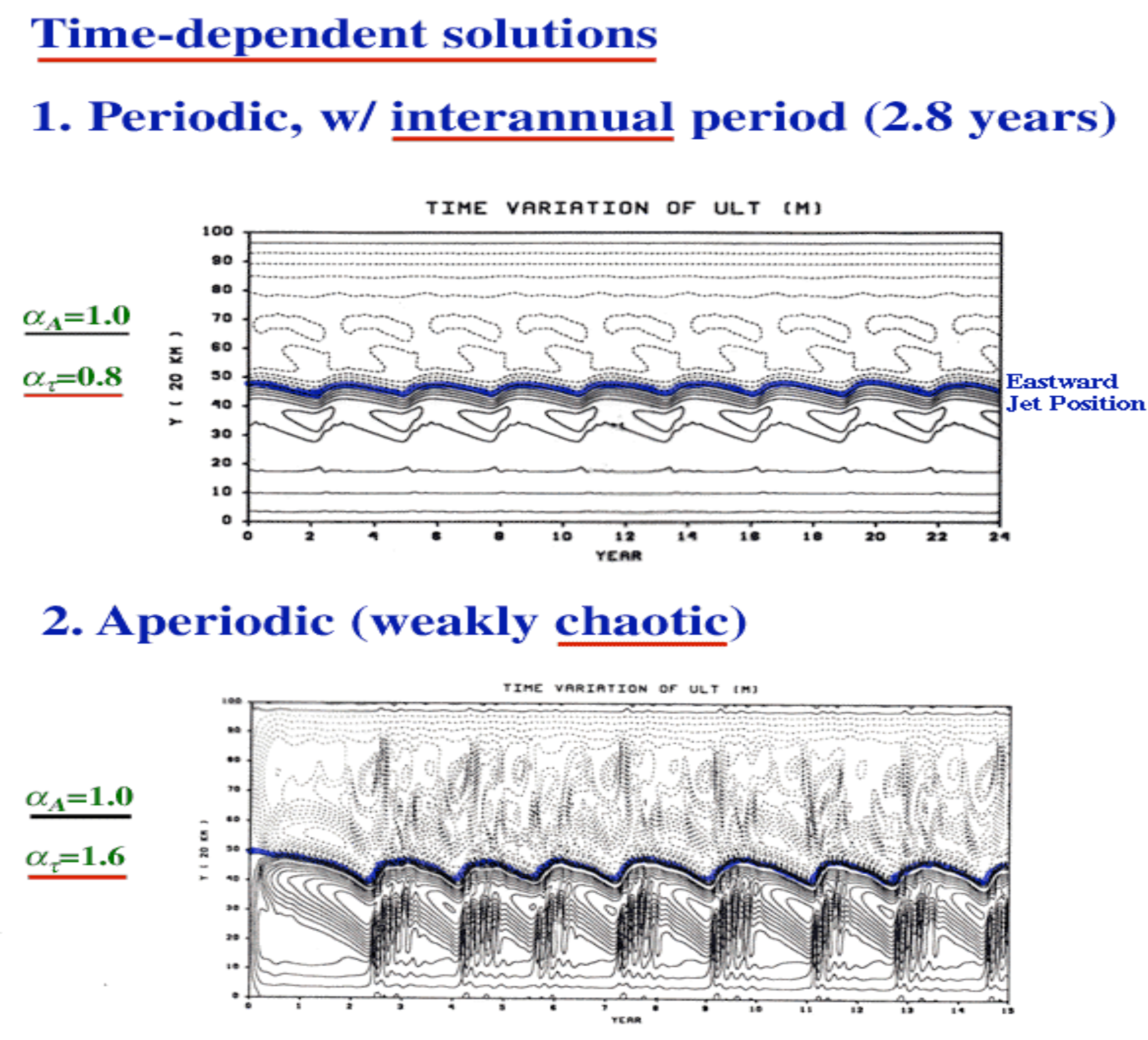
Subpolar gyre dominates

Subtropical gyre dominates

Time-dependent solutions: periodic and chaotic

To capture space-time dependence, meteorologists and oceanographers often use Hovmöller diagrams





Time-dependent solutions

1. Periodic, w/ interannual period (2.8 years)

$\alpha_A=1.0$

$\alpha_\tau=0.8$

Eastward Jet Position

2. Aperiodic (weakly chaotic)

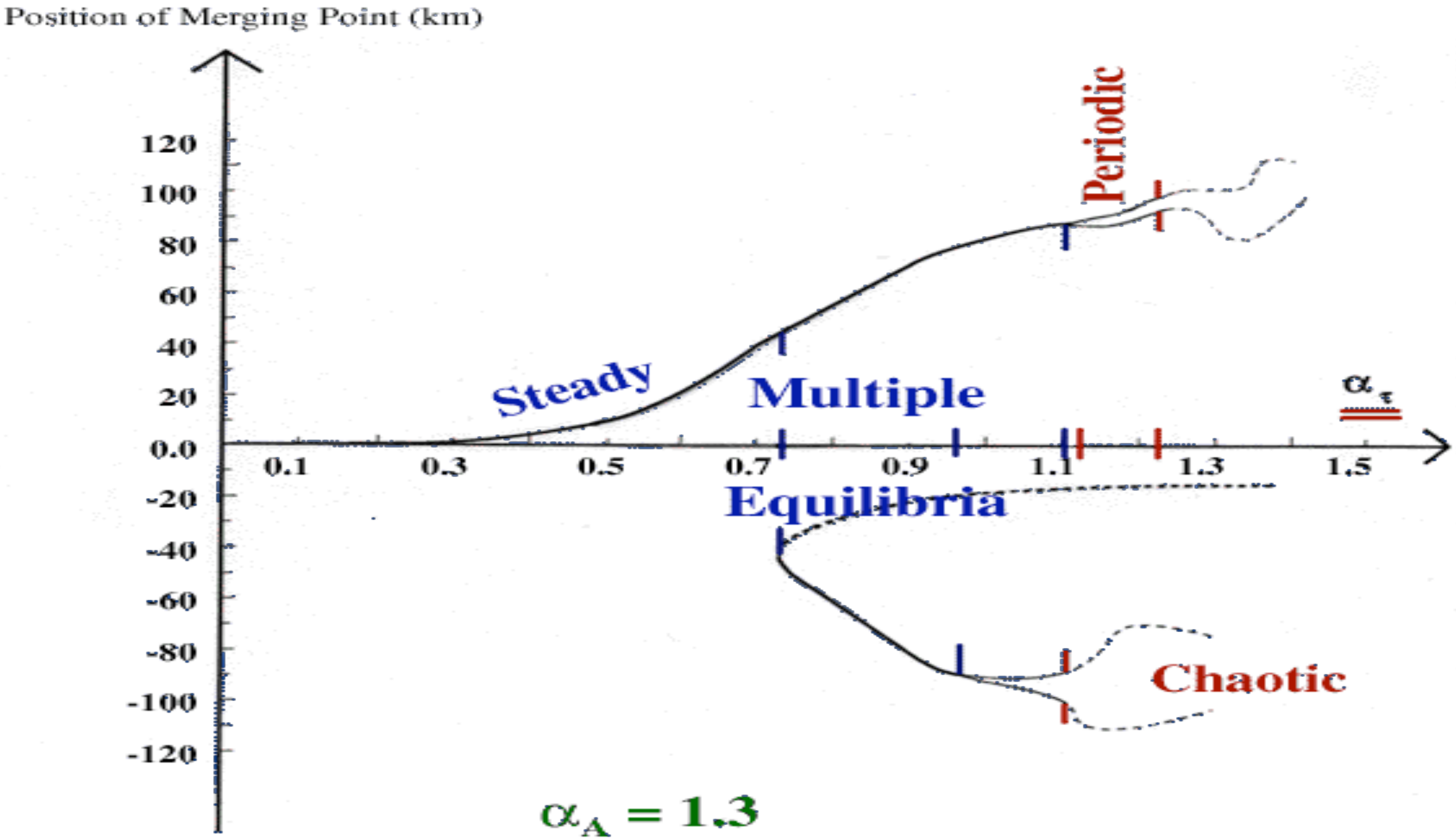
$\alpha_A=1.0$

$\alpha_\tau=1.6$

Poor man's continuation method

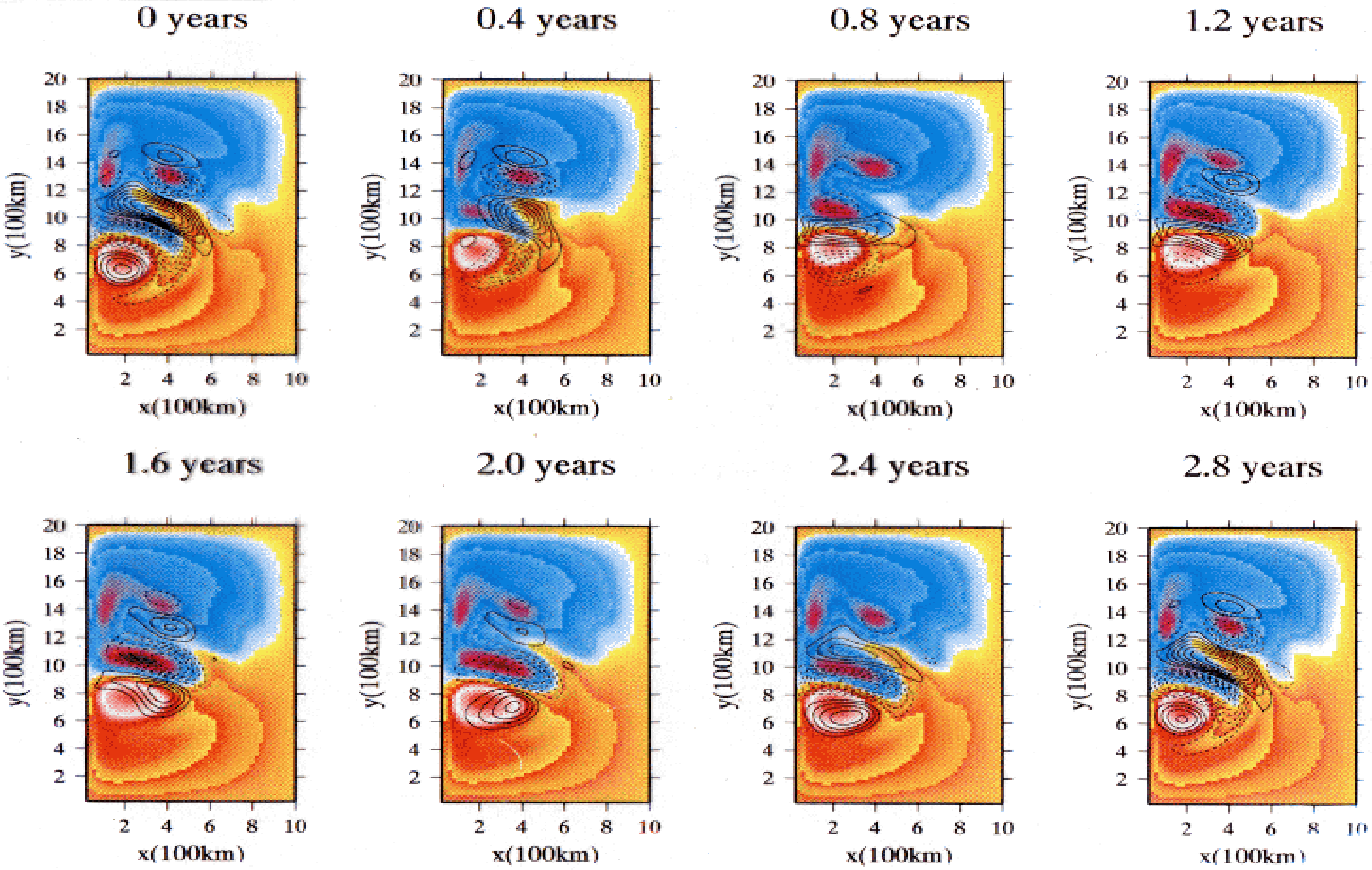
Bifurcation diagram

Perturbed pitchfork + Hopf + transition to chaos



$\alpha_A = 1.3$

Interannual variability: relaxation oscillation



0 years

0.4 years

0.8 years

1.2 years

1.6 years

2.0 years

2.4 years

2.8 years

Global bifurcations in “intermediate” models

Bifurcation tree in a QG, equivalent-barotropic, high-resolution (10 km) model: pitchfork, mode-merging, Hopf, and homoclinic

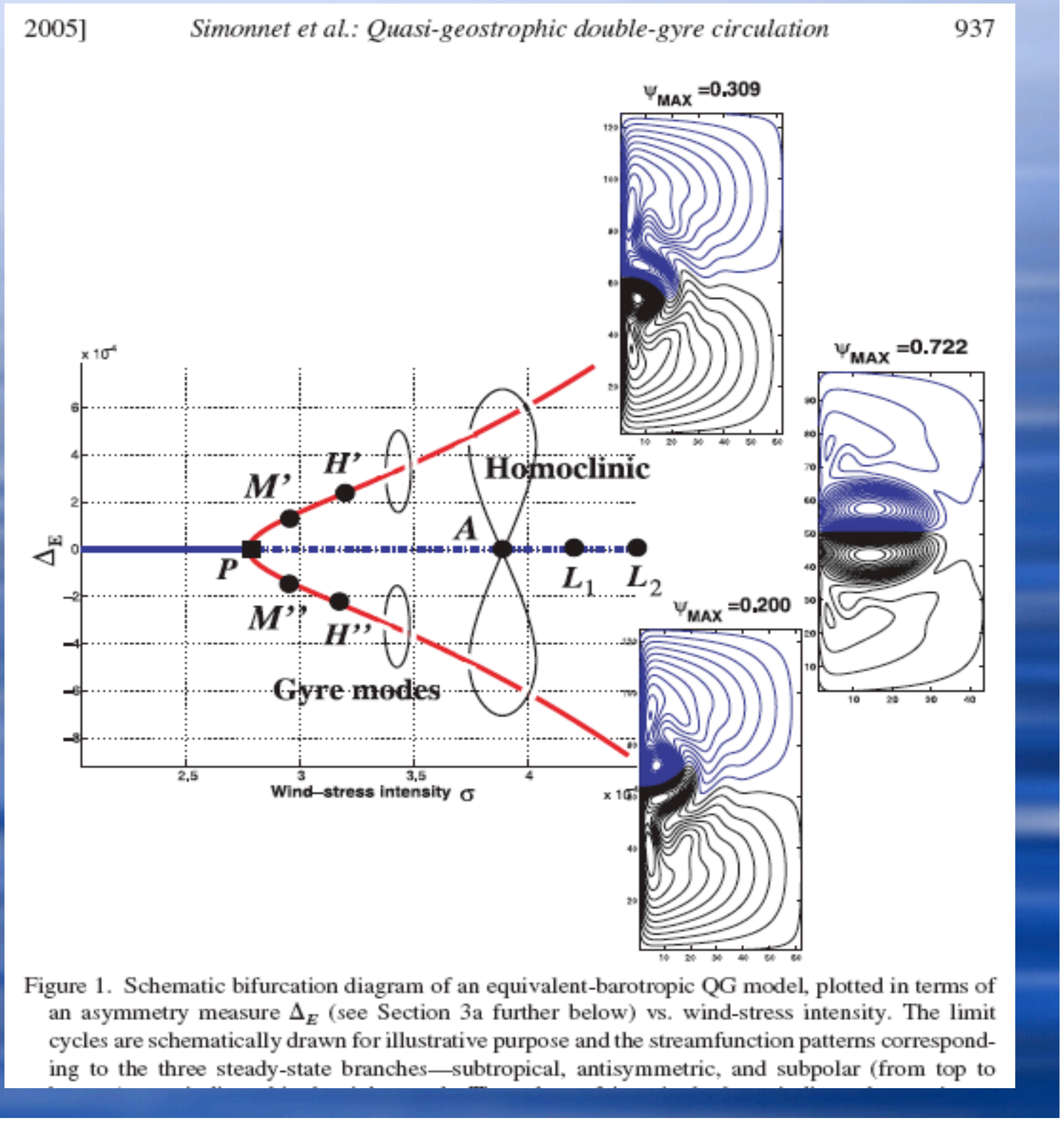
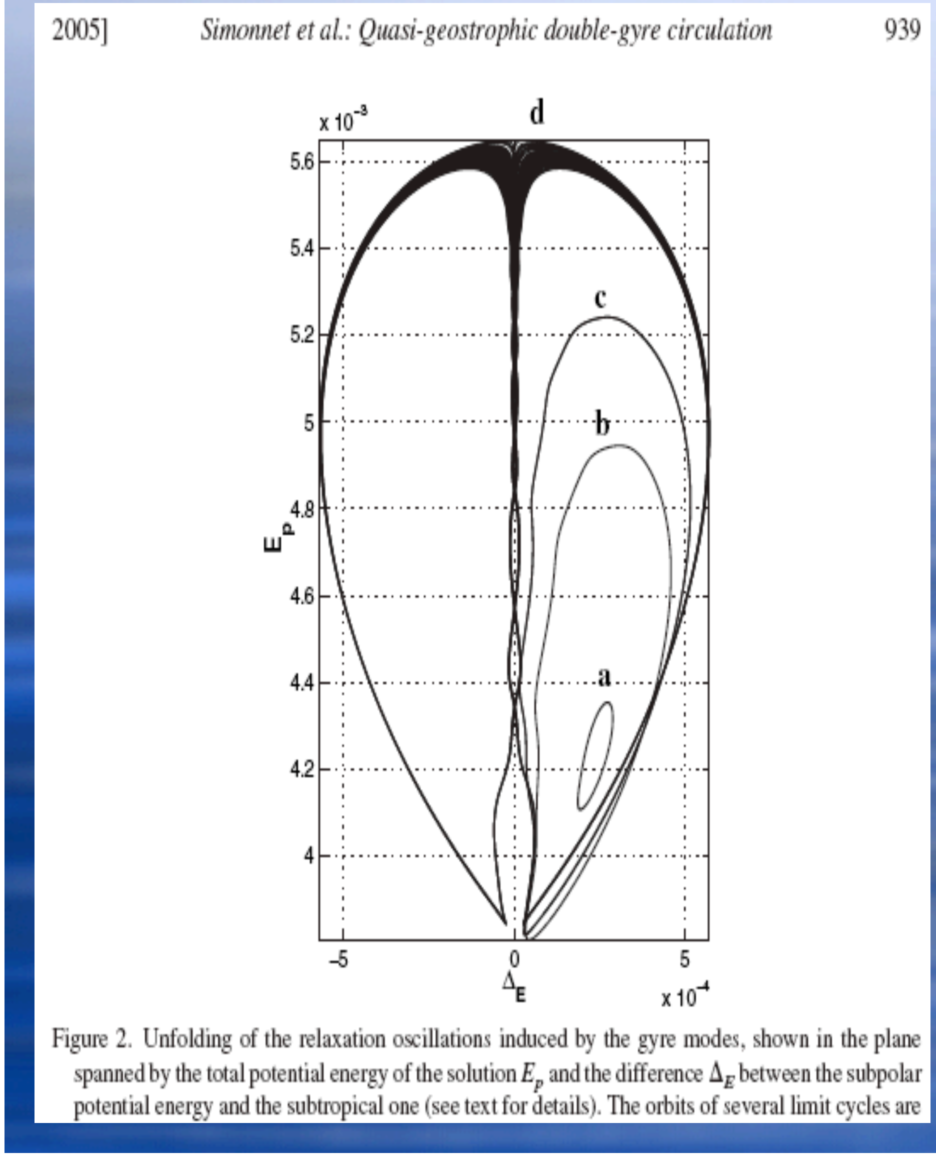


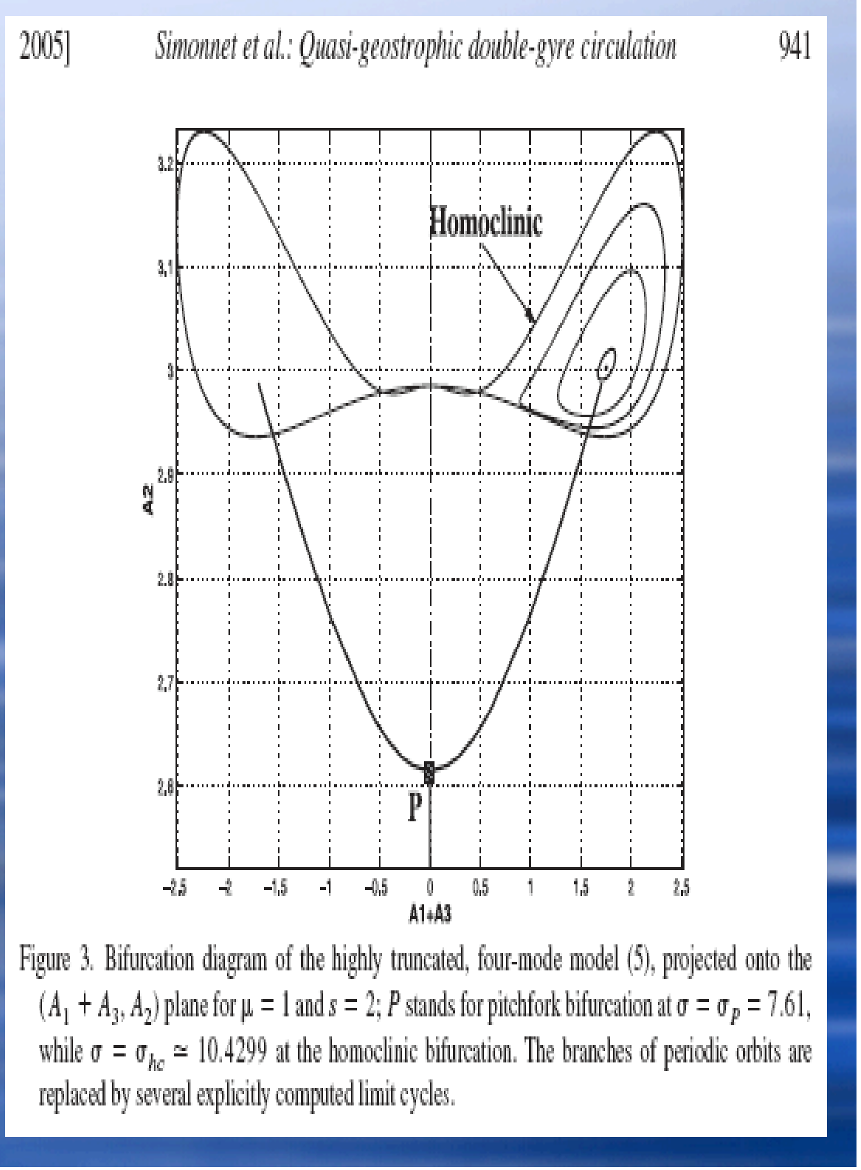
Figure 1. Schematic bifurcation diagram of an equivalent-barotropic QG model, plotted in terms of an asymmetry measure Δ_E (see Section 3a further below) vs. wind-stress intensity. The limit cycles are schematically drawn for illustrative purpose and the streamfunction patterns corresponding to the three steady-state branches—subtropical, antisymmetric, and subpolar (from top to

Homoclinic orbit: numerical and analytical



2005] *Simonnet et al.: Quasi-geostrophic double-gyre circulation* 939

Figure 2. Unfolding of the relaxation oscillations induced by the gyre modes, shown in the plane spanned by the total potential energy of the solution E_p and the difference Δ_E between the subpolar potential energy and the subtropical one (see text for details). The orbits of several limit cycles are

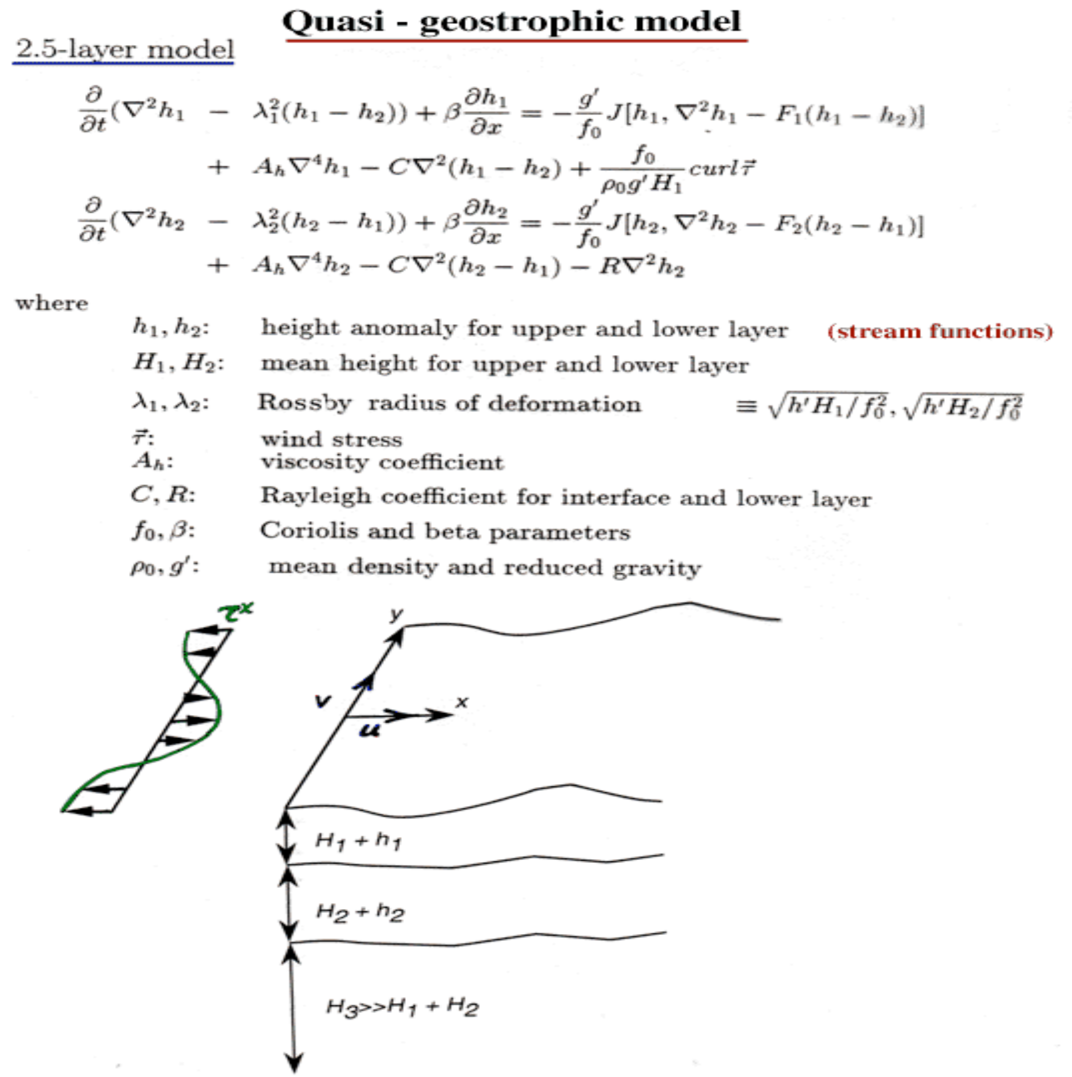


2005] *Simonnet et al.: Quasi-geostrophic double-gyre circulation* 941

Figure 3. Bifurcation diagram of the highly truncated, four-mode model (5), projected onto the $(A_1 + A_3, A_2)$ plane for $\mu = 1$ and $s = 2$; P stands for pitchfork bifurcation at $\sigma = \sigma_p = 7.61$, while $\sigma = \sigma_{hc} \simeq 10.4299$ at the homoclinic bifurcation. The branches of periodic orbits are replaced by several explicitly computed limit cycles.

The double-gyre circulation: A different rung of the hierarchy

Another “intermediate” model of the double-gyre circulation: slightly different physics, higher resolution – down to 10 km in the horizontal and more layers in the vertical, much larger domain, …



Quasi - geostrophic model

2.5-layer model

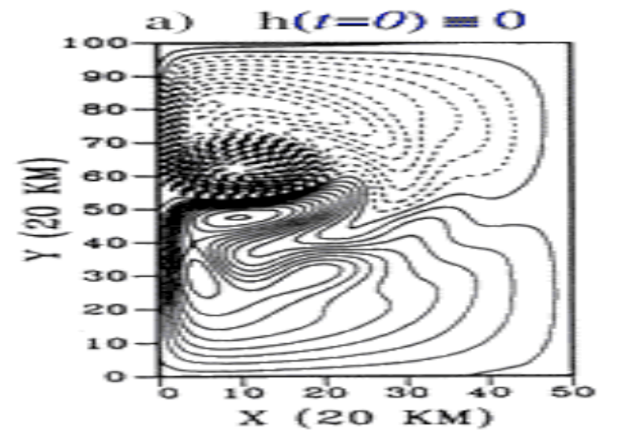
$$\begin{aligned}
\frac{\partial}{\partial t}(\nabla^2 h_1 \;&-\; \lambda_1^2(h_1 - h_2)) + \beta\frac{\partial h_1}{\partial x} = -\frac{g'}{f_0}J[h_1, \nabla^2 h_1 - F_1(h_1 - h_2)] \\
&+\; A_h\nabla^4 h_1 - C\nabla^2(h_1 - h_2) + \frac{f_0}{\rho_0 g' H_1}curl\vec{\tau} \\
\frac{\partial}{\partial t}(\nabla^2 h_2 \;&-\; \lambda_2^2(h_2 - h_1)) + \beta\frac{\partial h_2}{\partial x} = -\frac{g'}{f_0}J[h_2, \nabla^2 h_2 - F_2(h_2 - h_1)] \\
&+\; A_h\nabla^4 h_2 - C\nabla^2(h_2 - h_1) - R\nabla^2 h_2
\end{aligned}$$

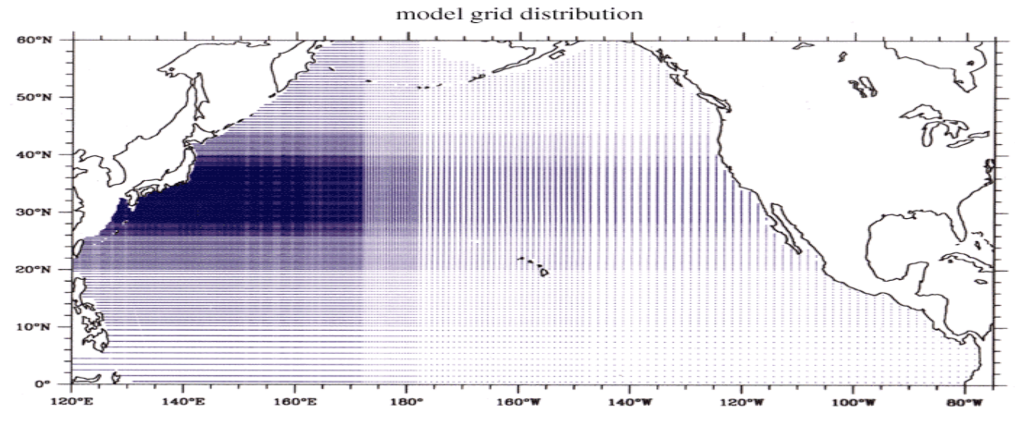
where

h_1, h_2:	height anomaly for upper and lower layer	**(stream functions)**
H_1, H_2:	mean height for upper and lower layer	
λ_1, λ_2:	Rossby radius of deformation	$\equiv \sqrt{h'H_1/f_0^2}, \sqrt{h'H_2/f_0^2}$
$\vec{\tau}$:	wind stress	
A_h:	viscosity coefficient	
C, R:	Rayleigh coefficient for interface and lower layer	
f_0, β:	Coriolis and beta parameters	
ρ_0, g':	mean density and reduced gravity	

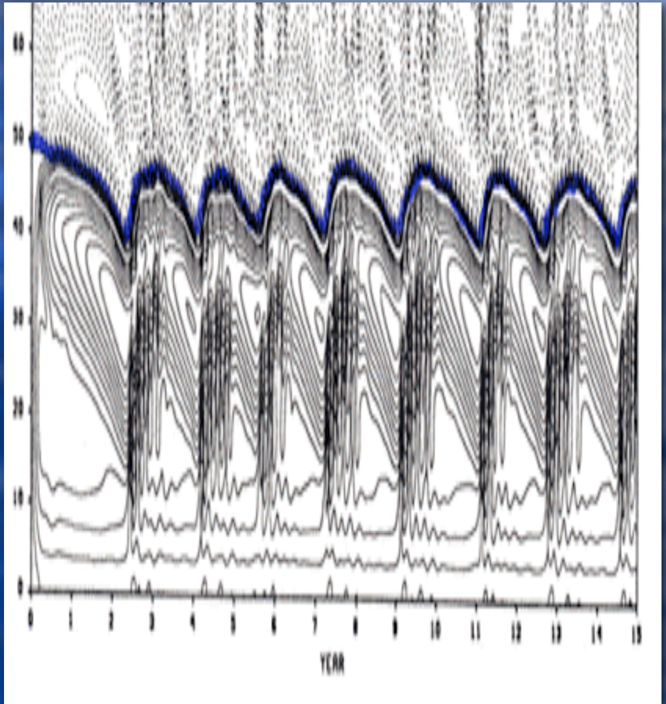
Bo Qiu, U. of Hawaii, pers. commun., 1997

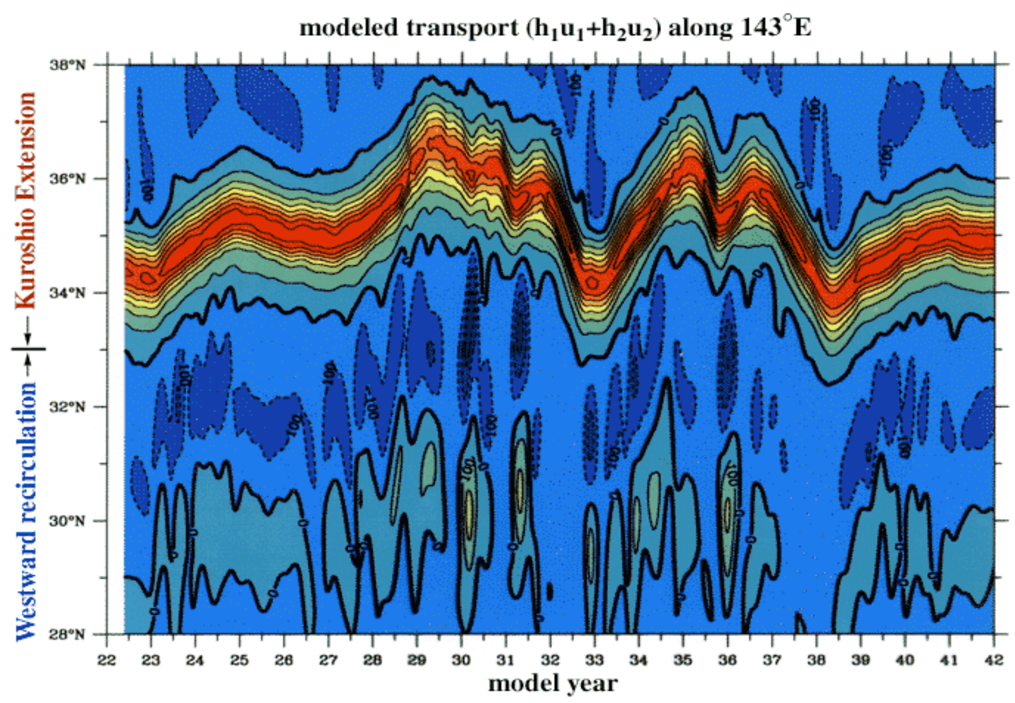
Model-to-model, qualitative comparison





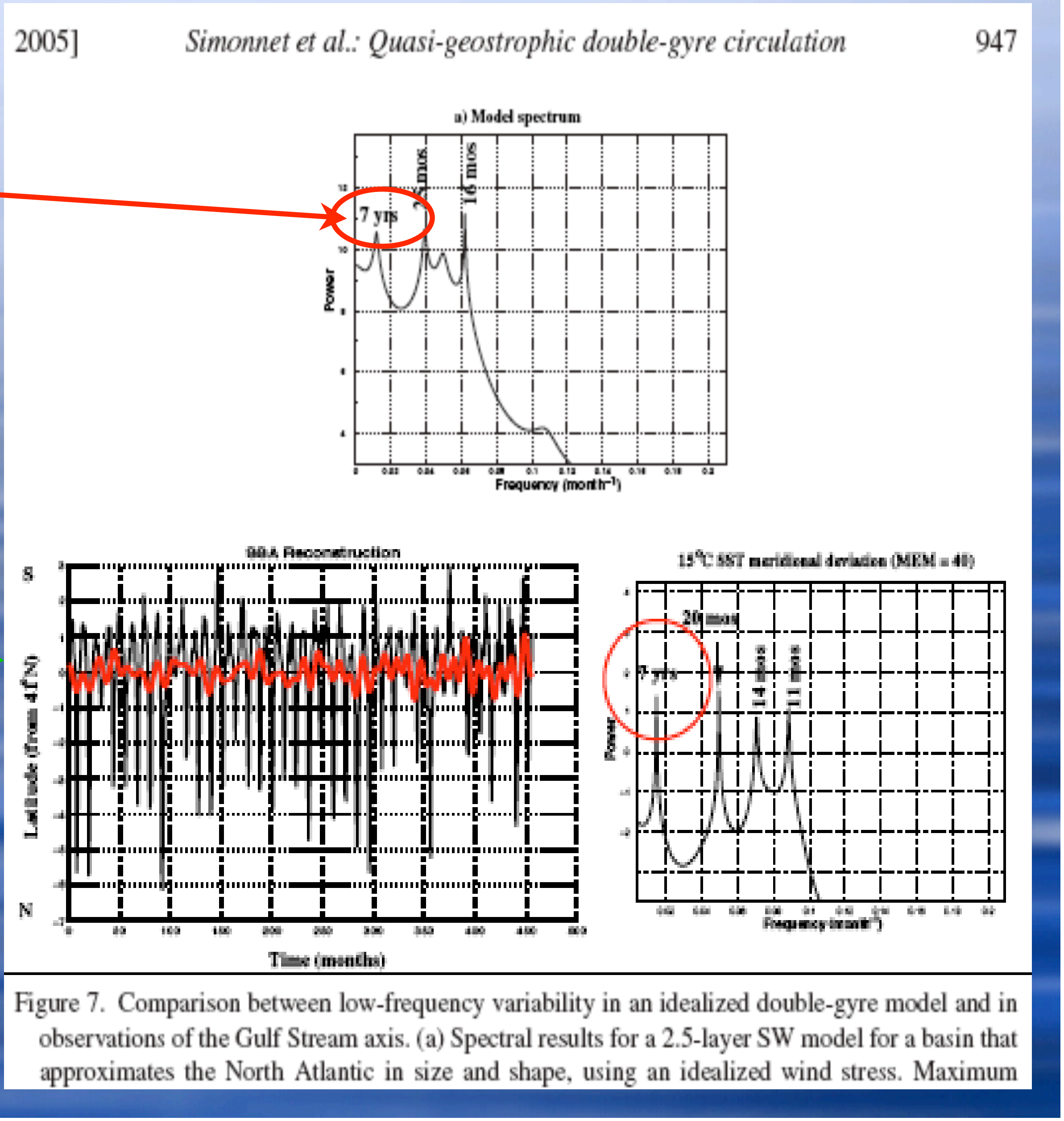
Bo Qui, 2.5 - layer QG model, 1997





Model-and-observations, quantitative comparison

Spectra of
(a) kinetic energy of
2.5-layer shallow-water
model in North-Atlantic–
shaped basin; and
(b) Cooperative Ocean-
Atmosphere Data Set
(COADS) Gulf-Stream
axis data



2005] *Simonnet et al.: Quasi-geostrophic double-gyre circulation* 947

Figure 7. Comparison between low-frequency variability in an idealized double-gyre model and in observations of the Gulf Stream axis. (a) Spectral results for a 2.5-layer SW model for a basin that approximates the North Atlantic in size and shape, using an idealized wind stress. Maximum

More spatio-temporal data

Multi-channel SSA analysis of the UK Met Office monthly mean SSTs for the century-long 1895–1994 interval

Marked similarity with the 7–8-year “gyre mode” of a full hierarchy of ocean models, on the one hand, and with the North Atlantic Oscillation (NAO), on the other: explanation?

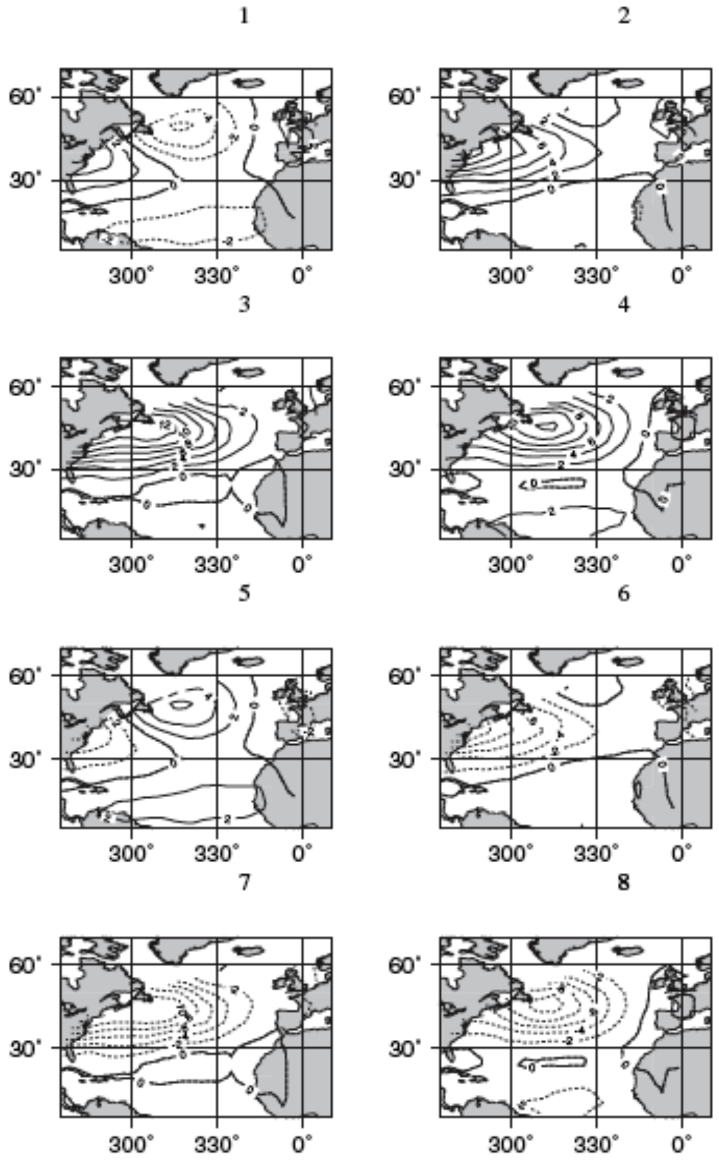


Figure 8. Phase composites of the reconstructed 7–8-year SST oscillation. The MSSA window length is 40 year and the contour interval is 0.02°C.

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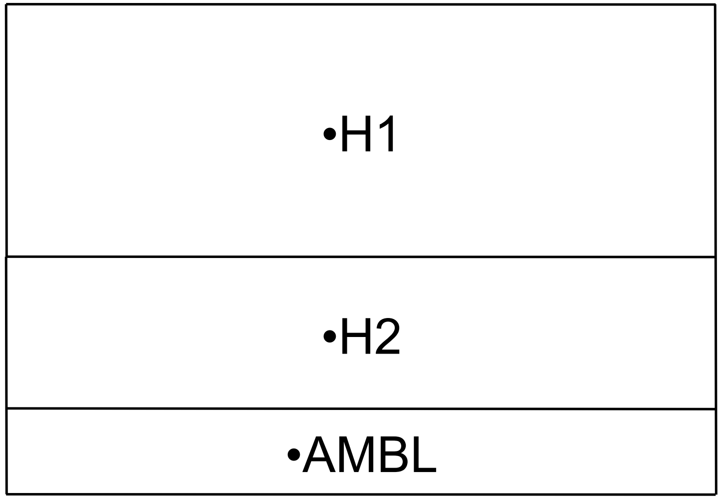
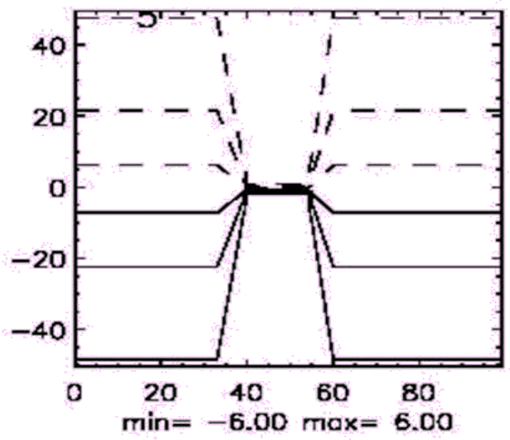
Atmospheric impact of mid-latitude SST anomalies: A highly contentious issue



•H1

•H2

•AMBL



- A quasi-geostrophic (QG) atmospheric model in a periodic β-channel, first barotropic (Feliks *et al.*, *JAS*, 2004; FGS'04), then baroclinic (FGS'07).
- Marine atmospheric boundary layer (ABL), analytical solution.
- Forcing by idealized oceanic SST front.

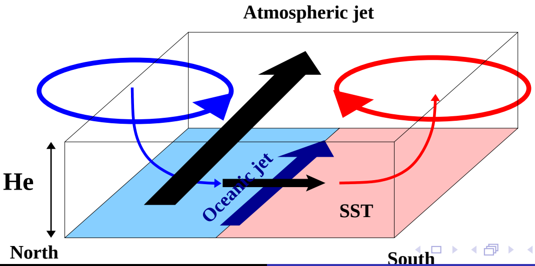
Ocean-atmosphere coupling mechanism (II)

Vertical velocity at the top of the marine ABL

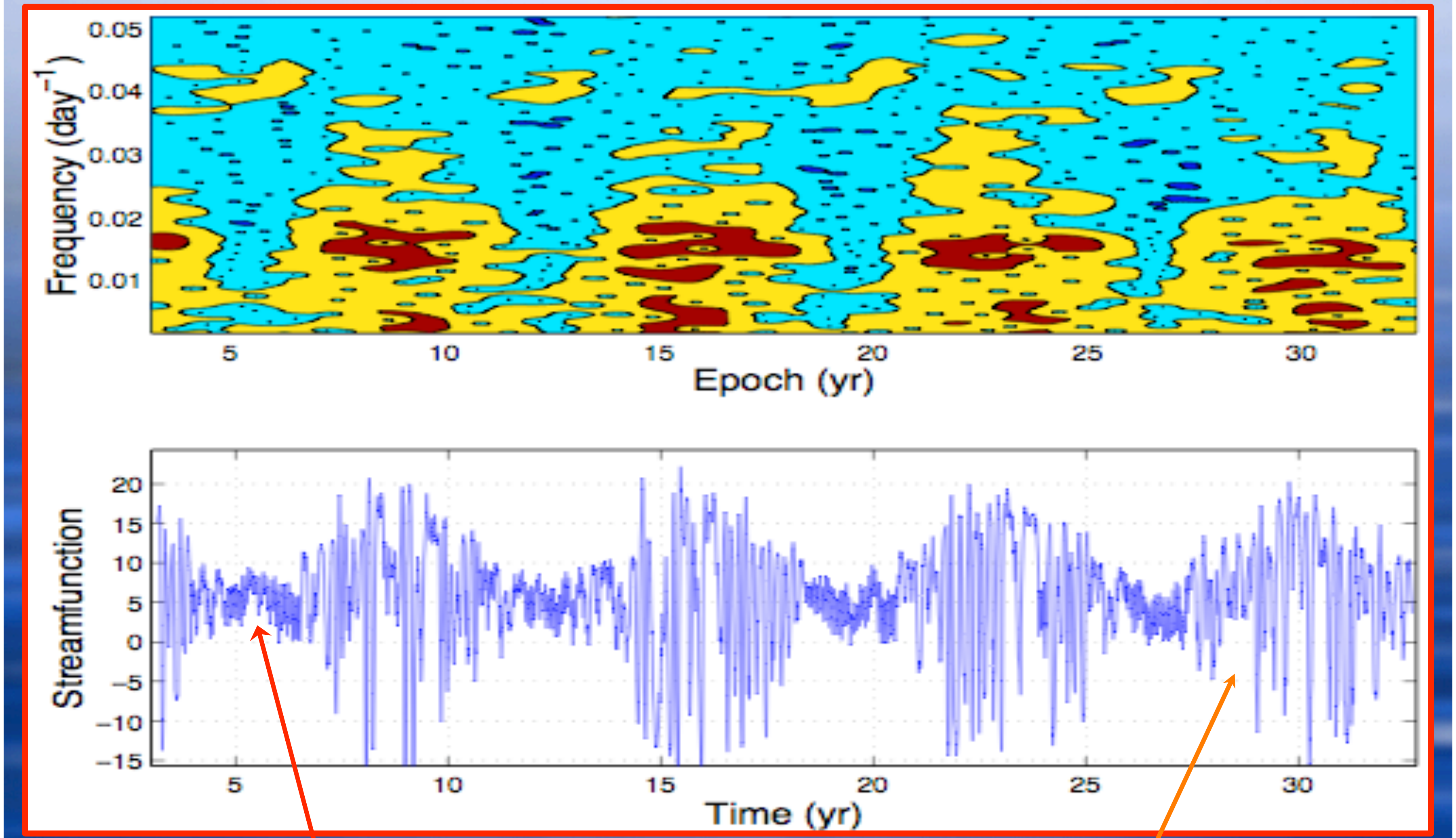
- The nondimensional $w(H_e)$ is given by

$$w(H_e) = \left[\gamma\zeta_g - \alpha\nabla^2 T\right],$$

  with $\gamma = c_1(f_0L/U)(H_e/H_a)$ and $\alpha = c_2(g/T_0U^2)(H_e^2/H_a)$, where H_a is the layer depth of the free atmosphere ($\sim$ 10 km), and ζ_g the atmospheric geostrophic vorticity.
- Two components: one **mechanical**, due to the geostrophic flow ζ_g above the marine ABL and one **thermal**, induced by the SST front.



Evolutive spectral analysis



30-day oscillation

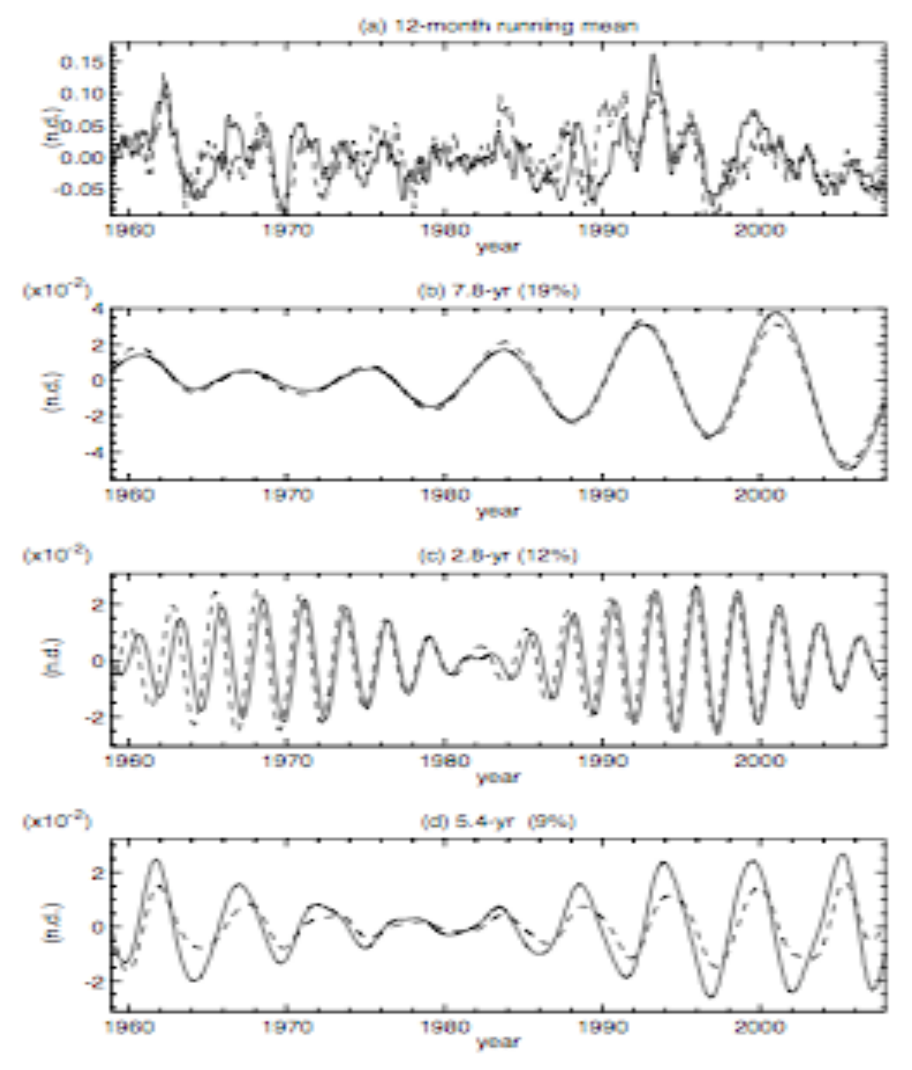
70-day oscillation

The 7–8-yr mode in atmospheric data

Likewise a contentious issue

Simulate atmospheric response to SODA data over the Gulf Stream region

- Use SST (–5 m) data from the SODA reanalysis (50 years)
- Use the FGS'07 QG model in periodic β-channel
  - – baroclinic + marine ABL
- Figure shows NAO index:
  - – simulated (solid)
  - – observed (dashed)

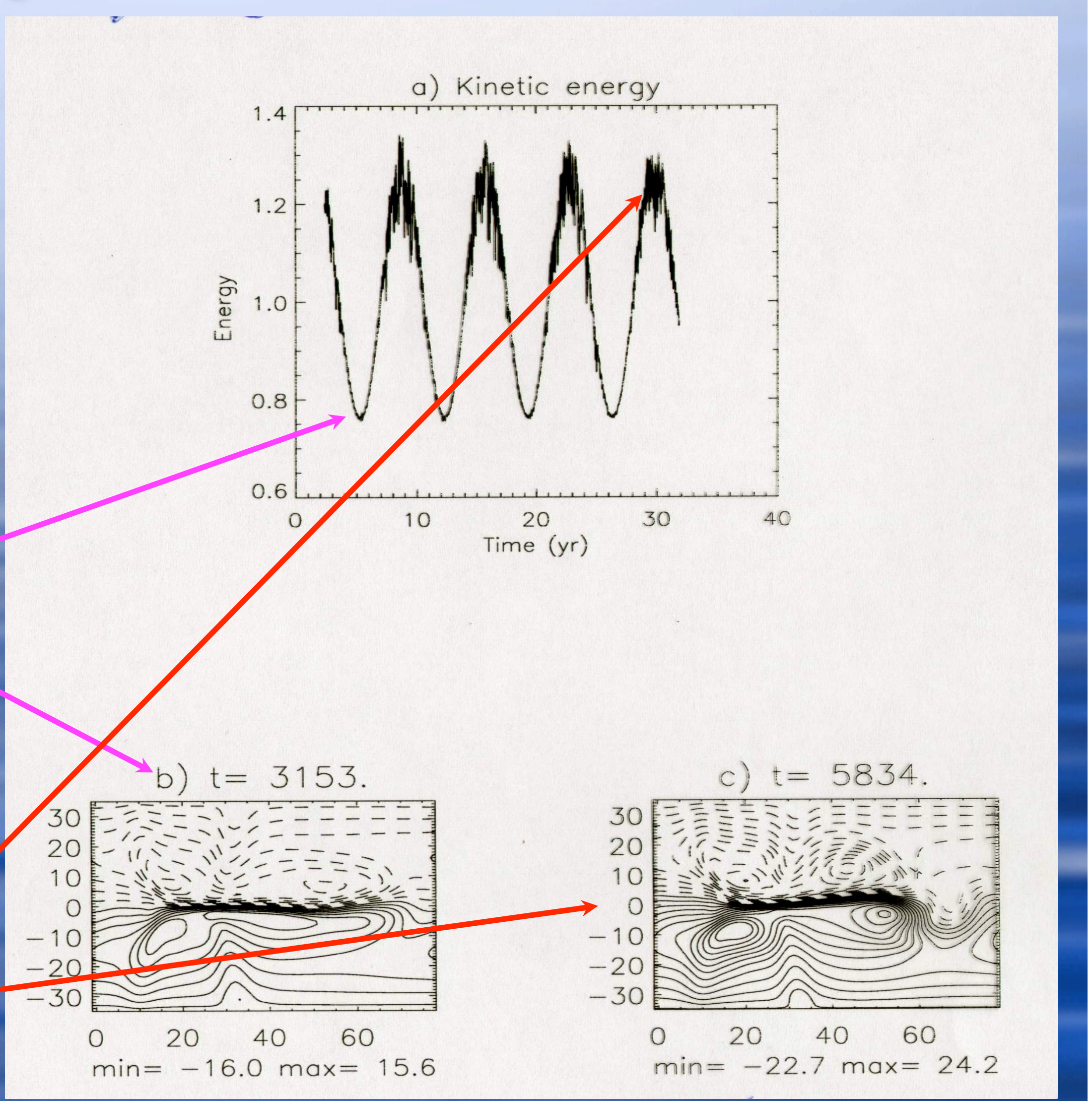


Concluding remarks, II

- Tipping points and bifurcations: do they really help?
  - Yes, if properly understood and carefully applied!
- Can we predict them?
  - Yes, depending on the problem and the data!

Forced 7-year cycle in the FGS'04 model

Slow amplitude modulation of 1 ^{0}C in the SST front



Low-energy phase

High-energy phase

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Waves vs. Particles: A Pathway to Prediction?

Is predicting as hard

as it is claimed to be?

No, it's actually quite easy:

Just flip a coin or roll a die!

What's difficult, though, is

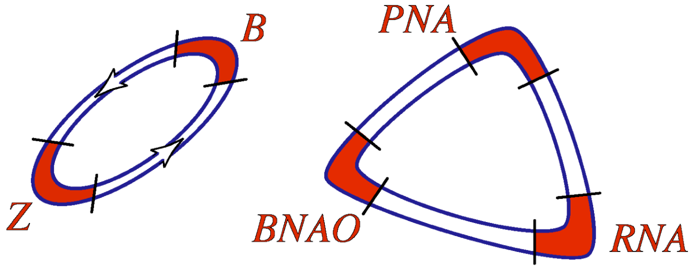
trusting the prediction

That's where a little understanding of what we're trying to predict helps!

Based on Ghil & Robertson (2002)

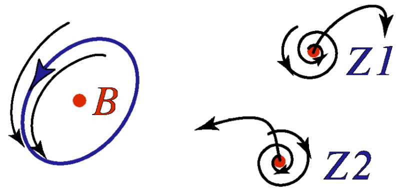
"Waves vs. Particles"
in Atmospheric Low-Frequency Variability

1. Are the regimes but slow phases of the oscillations?



Kimoto & Ghil (*JAS*, 1993a, b)

2. Are the oscillations but instabilities of particular equilibria?

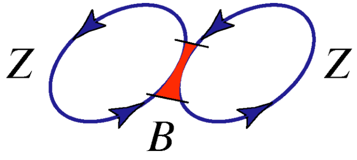


Legras & Ghil (*JAS*, 1985)

3. How about both: "chaotic itinerancy" (Itoh & Kimoto, *JAS*, 1999)

4. How about neither? Null hypotheses:

a) It's all due to interference of linear waves, *e.g.*, neutrally stable Rossby waves;



Lindzen *et al.* (*JAS*, 1982)

b) It's all due to red noise — Hasselmann (*Tellus*, 1976), Mitchell (*Quatern. Res.*, 1976), Penland & co. (Magorian, Sardeshmukh, 1990s).

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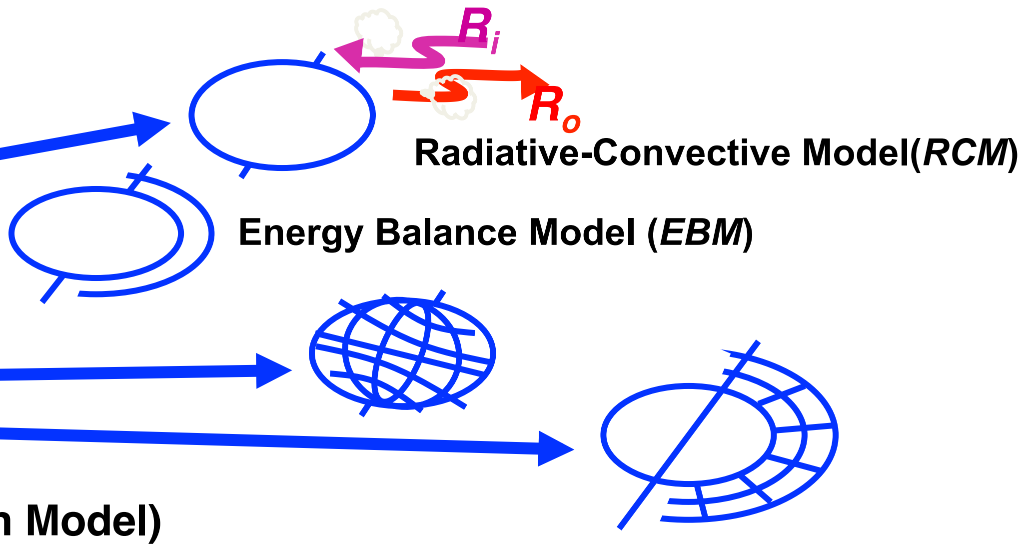
Reserve slides

Climate models (atmospheric & coupled) : A classification

Temporal

- stationary, (quasi-)equilibrium
- transient, climate variability

Space



- 0-D (dimension 0)
- 1-D
    - vertical
    - latitudinal
- 2-D
    - horizontal
    - meridional plane
- 3-D, GCMs (General Circulation Model)
- Simple and intermediate 2-D & 3-D models

Coupling

- Partial
    - unidirectional
    - asynchronous, hybrid
- Full

➔ ***Hierarchy*:** back-and-forth between the simplest and the most elaborate model, and between the models and the observational data

Spin-up of atmospheric jet

SST front:

L_{oc} = 600 km,

ΔT = 3.5 ^{0}C,

d = 50 km

Atmospheric jet

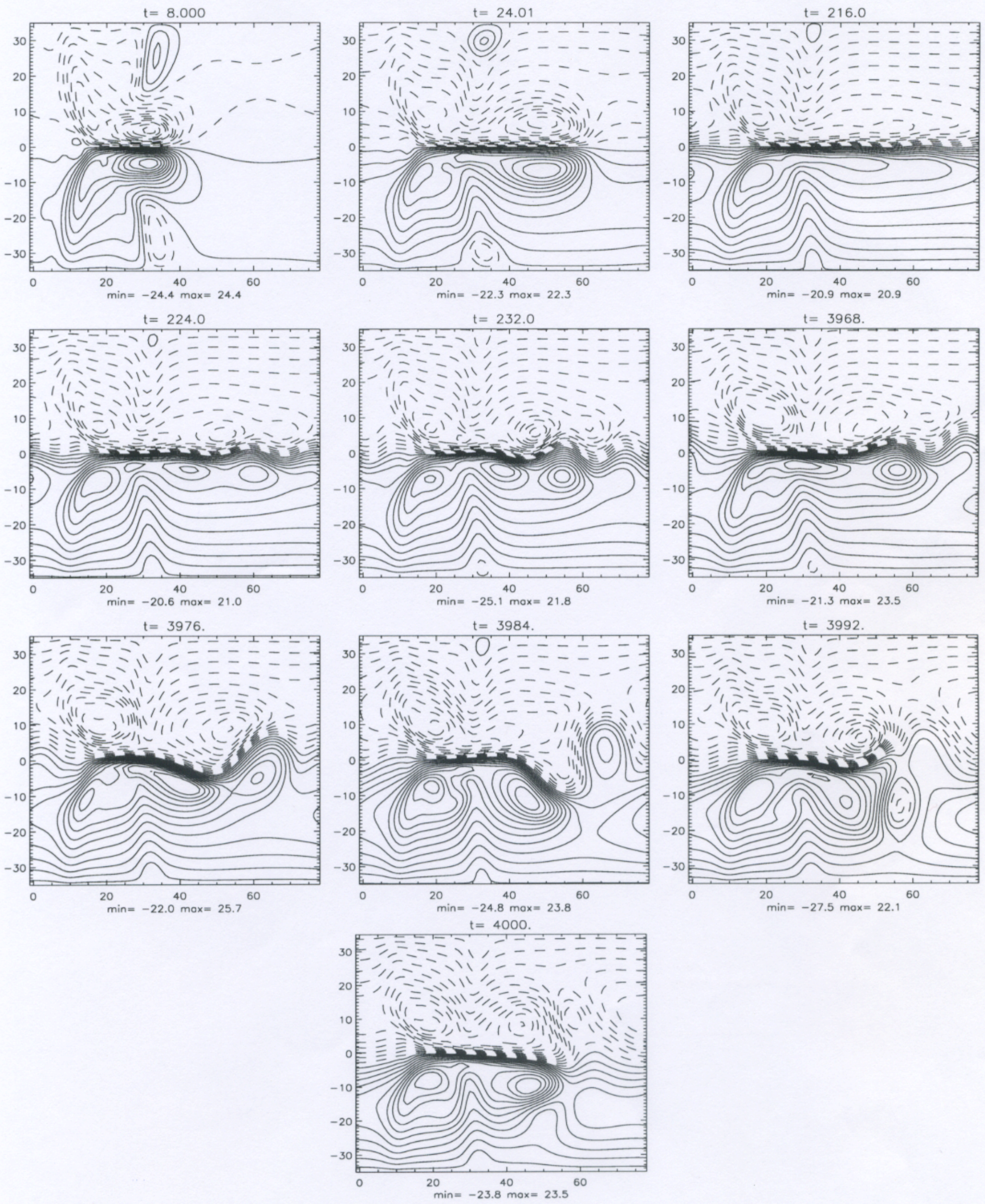
spins up from

L_a = 2000 km to

L_a = 4000 km, much

greater speed and

strong recirculation



Can we, nonlinear people, help?

The uncertainties might be ***intrinsic***, rather than mere "tuning problems"

If so, maybe ***stochastic structural stability*** *could help*!

Might fit in nicely with recent taste for "stochastic parameterizations"

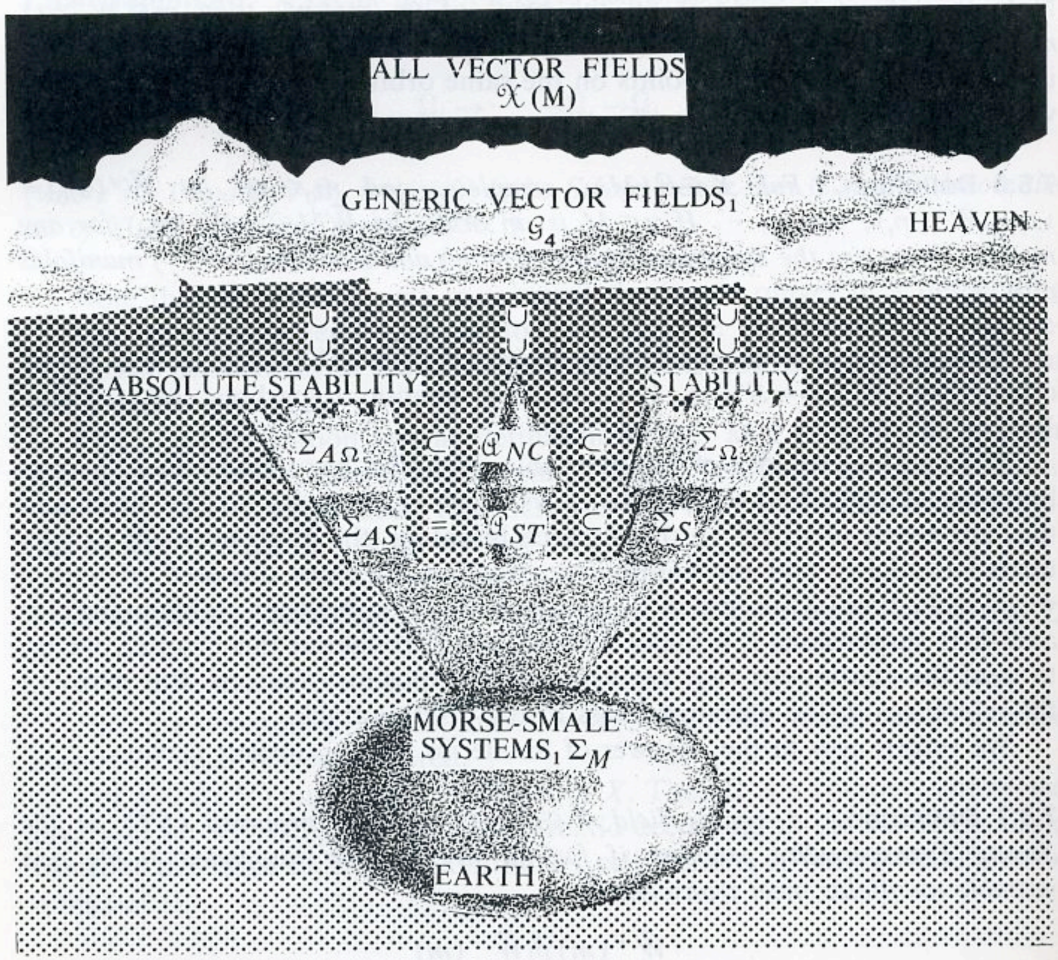


Figure 7.5-1. The three towers of differentiable dynamics.

The DDS dream of structural stability (from *Abraham & Marsden*, 1978)