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Improvements on Target Risk Portfolio in Strategic Asset Allocation

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Declaration

The work contained in this thesis is my own work unless otherwise stated.

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Abstract

Target risk funds are one of the fund of fund products which attempts to control the risk by managing the amount invested in debt and equity, to meet the demands of long-term investment plans of pension funds or individuals. This paper focus on the construction of target risk portfolios and exploring possible improvement methods. It first illustrates the advantage of risk based approach. Then it discusses risk diversification concept and shows the possible association between diversifying risk and improving return. Based on that, it explores the patterns of different concentration indices, and finds the best asset allocation distribution to construct a target risk FoF, in Chinese markets, involving the choices of different risk measures and lengths of sample data .

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Chapter 1

Introduction

For a long-term investment strategy, some investors may have some requirements to control the portfolio risk under an expected risk level. One type of fund product satisfying target risk requirement is called Target Risk Fund (TRF). This thesis is to explore some approaches for improving the portfolio performance of TRFs.

Markowitz Mean-Variance optimization is the most-known and the earliest theory for portfolio optimization, while it has some disadvantages which presents itself from being applied in the industries. Though it maximizes the *ex-ante* expected return of a portfolio under a predetermined volatility level, it does not perform desirably regarding of a *ex-post* performance measurement.

To find other approaches for improving portfolio performance, risk-based approach is to be focused on this project. First, risk budgeting model aims to find a portfolio which realizes a pre-assigned risk budgeting structure of the assets, which exhibits some superior aspects to mean-variance model. In particular, it has a steady weight allocation structure in the long run. The most widely used budget is the equal risk contribution. Second, in the view of risk diversification level, equal risk contribution (ERC) model can be regarded as the most risk-diversified model. Hence, based on the risk diversification concept of ERC model, alternative methods for diversifying risk are attempted. Regarding of three measurements for risk concentration, optimization problems are applied to minimize the risk concentration (or maximize risk diversification).

Under each of the target risks, portfolios are measured by several popular metrics such as annualized returns, annualized volatility, turnover,etc. In addition, portfolio risks can be measured in different ways, including volatility, VaR and ES. Apart from that, data of different window sizes is used for risk parameter estimation. Historical data in Chinese market are used to illustrate the performance under some different scenarios. Since the risk contribution portfolio is the most widely used and has great practical applications, it is widely used as a benchmark portfolio. The other three approaches for maximum diversification are explored and compared with ERC portfolios.

In particular, an improvement approach can be found by using the market data in Chinese markets. Portfolios with the best performance under different target risks are applied to invest in ETF fund products which are associated with the underlying indices. Through this way, based on risk diversification concepts, an alternative method to risk parity is applied and a fund of fund (FoF) product for particular target risk can be constructed for practical investment purposes.

Chapter 2 provides theoretical background and mathematical methods for the dissertation. Chapter 3 illustrates the procedure to explore target risk strategies step by step. Chapter 4 presents the results and some analysis. Chapter 5 provides a basic idea for applying the method into practive. Chapter 6 shows the conclusion of the thesis and Chapter 7 shows some discussion and further investigation.

Chapter 2

Background

2.1 Asset Allocation Strategy

In asset management practice, asset allocation strategy varies according to the time horizon of the allocation (or investment plans of investors). Generally, accoring to Roncalli [1], there exists three horizons: market timing (MT), tactical asset allocation (TAA) and strategic asset allocation (SAA).

- Market timing refers to a very short-term investment horizon, typically from 1 day to 1 month. For example, a daily strategy consisting in playing the mean-reverting property of stock returns, or a delta hedging strategy of a vanilla option.
- Tactical asset allocation is a short to medium term investment horizon, typically from 3 months to 3 years. These investment decisions are related to business cycles and medium-term market sentiments.
- Strategic asset allocation is a long-term investment horizon, usually ranging from 3 years to 50 years. Whereas TAA assumes that the risk premium of assets is time-varying, SAA is based on the stationary steady-state of the economy.

To manage wealth in a medium or long time horizons and to meet the demands of long-term investment plans of pension funds or individuals, the ways to allocate asset strategically will be the focus in this dissertation. Instead of arbitraging in high frequency trading or earning amount of money under great risk in market timing strategy, a portfolio with a *relatively low volatility and low trading frequency* is to be explored.

As one of SAA funds, target risk funds (TRFs) are one of the latest fund of fund (FoF) products which attempts to control the risk by managing the amount invested in debt and equity [2]. TRFs are frequently used in retirement plans and has an increasing use as an option in 401(k) plans in United States. In 2016, Elton, Gruber and de Souza [2] summarizes that in asset management industries, the compositions of categories of assets include equities, bonds, commodities and real state. The percentage of holdings for each categories varies in three classifications of TRFs: aggressive, moderate and conservative, which are based on different risk levels of portfolios strategies.

Inspired by TRFs, asset allocation strategies for different risk settings are attempted to be explored in Chinese markets.

2.2 Modern Portfolio Theory

To allocate weights of available assets in portfolio selection process, 'expected returns – variance of returns' rule (or mean-variance theory) was firstly proposed by Markowitz [3], which specifies a rule that the investor should diversify the risky assets and maximize expected returns.

To formulate the idea, the mathematical expression of (daily) **simple return** is introduced. [4].

$$R_t := \frac{S_t - S_{t-1}}{S_{t-1}}$$

Consider a universe of n assets, let $x = (x_1, ..., x_n)$ be the vector of weights in the portfolio. Denote $R = (R_1, ..., R_n)$ as the vector of asset returns where R_i is the return of asset i. The **return of the portfolio** is equal to

$$R(x) = \sum_{i=1}^{n} x_i R_i$$

Let $\mu = \mathbb{E}[R]$ and $\Sigma = \mathbb{E}[(R-\mu)(R-\mu)^T]$ be the vector of expected returns and the covariance matrix of asset returns. The **expected return of the portfolio** is:

$$\mu(x) = \mathbb{E}[R(x)] = \mathbb{E}[x^T R] = x^T \mu$$

The **variance of the portfolio** is

$$\sigma^2(x) = x^T \Sigma x$$

We can then formulate the investor's financial problems as follows ([1]):

1. Maximizing the expected return of the portfolio under a volatility constraint:

$$\max\ \mu(x)\ \text{u.c.}\ \sigma(x) \leq \sigma^*$$

2. Or minimizing the volatility of the portfolio under a return constraint:

$$\min\ \sigma(x)\ \text{u.c.}\ \mu(x) \geq \mu^*$$

The idea can be rewritten as follows:

$$\begin{aligned} x^* &= \arg\ \min x^T \Sigma x \\ u.c.\ 1^T x &= 1 \\ \mu(x) &\geq \mu^* \end{aligned} \tag{2.2.1}$$

Given a predetermined expected return of the portfolio, the solution x^* of 2.2.1 gives the weights which minimize the portfolio variance. Since 2.2.1 is a standard quadratic programming (QP) problem, available tools can be found to solve it easily (see A.1). Given different expected return levels μ^*, corresponding x^* and portfolio volatility σ^* can be found. The curve of μ^* and σ^* is called **efficient frontier**. Figure 2.1 shows an efficient frontier curve of three assets, under the quadratic programming 2.2.1.

To obtain portfolio weights in 2.2.1, estimation of expected return $u(x)$ and covariance matrix Σ of asset universe is required. In reality, estimation error of the parameters often outweights the diversification benefits of mean-variance optimization rendering an ex ante efficient portfolio

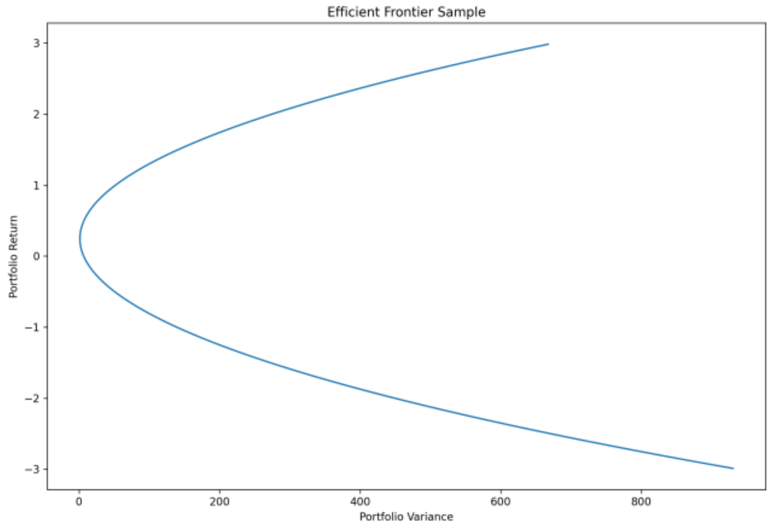

Figure 2.1: Efficient frontier sample of 2.2.1

rather an inefficient ex post portfolio [5]. In addition, Scherer [6] shows that mean variance picks up risk based pricing anomalies. And investment decision based on MV portfolio is hard to make in absence of the persistence of the anomalies. Roncalli [1] also demonstrates that MV strategies are relatively sensitive to input parameters. A practical example of rolling-sample based mean variance strategy is also illustrated in 4.1. It can be seen that parameter estimation is rather significant in mean variance portfolio. In a long-term investment, the low frequency trading and uncertainty of the possible anomalies in financial markets require a steady portfolio strategy, while the potential great change of asset allocation weights is not desirable.

2.3 Risk-Budgeting Approach

Risk-based asset allocation methodologies are of central importance since the introduction of the Basle Committee's Capital Accord in 1988. [7] Risk management is crucial in investment activities, since it can help to reduce costs of external financing and reduce variability. [4] To illustrate risk-budgeting portfolio, the concepts of three risk measures are to be introduced below. Firstly, **the loss of the portfolio** is $L(x) = -R(x)$, where $R(x)$ is the return of the portfolio.

- **Volatility of the loss**

  The volatility of the loss is the portfolio's volatility, which is given by

$$\sigma(L(x)) = \sigma(x)$$

- **Value-at-risk**

The value-at-risk at confidence level α is the α-quantile of the loss distribution, which is given by

$$\mathrm{VaR}_\alpha(x) = \inf\{l : \Pr\{L(x) \le l\} \ge \alpha\}$$

- **Expected shortfall**

  Let $\alpha \in (0,1)$. The ES of loss L at confidence level α is given by

$$\mathrm{ES}_\alpha(L) = \frac{1}{1-\alpha}\int_\alpha^1 q_u(L)du$$

To decompose the risk of the portfolio, two theorems are introduced as follows.

Theorem 2.3.1. ***Linearly Homogeneous***. *f is (positively) linearly homogeneous iff $f(\lambda x) = \lambda f(x)$ for all $\lambda > 0$ and $x >> 0_N$.*

Theorem 2.3.2. ***Euler's First Theorem***. *If f is linearly homogeneous and once continuously differentiable, then its first order partial derivative functions, $f_i(x)$ for $i = 1, 2, ..., N$, are homogeneous of degree zero and*

$$f(x) = \sum_{i=1}^{N} x_i f_i(x)$$

Since volatility, VaR and ES are linearly homogeneous, and are once continuously differentiable in general cases, which are focused in this project, they can be decomposed into the form in theorem 2.3.2, that is, let $\mathbb{R}(x)$ be the risk of the portfolio with weights $(x_1, ..., x_N)$,

$$\mathbb{R}(x) = \sum_{i=1}^{N} x_i \frac{\partial \mathbb{R}(x)}{\partial x_i}$$

According to Roncalli [1], the risk contribution for the i-th asset of a risk measure can always be defined as:

$$\mathbb{RC}_i = x_i \frac{\partial \mathbb{R}(x)}{\partial x_i}$$

Hence, we can have the following equation:

$$\mathbb{R}(x) = \sum_{i=1}^{N} x_i \frac{\partial \mathbb{R}(x)}{\partial x_i} = \sum_{i=1}^{N} x_i \mathbb{RC}_i$$

In Chinese markets, shorting is not allowed in bonds and equities, we generally prefer to obtain a long-only portfolio, indicating that all weights $(x_1, ..., x_N)$ are positive. Let b_i be the risk budget of the i-th asset, a proper risk budgeting portfolio is defined as follows:

$$\begin{cases} RC_i(x) = b_i R(x) \\ b_i \ge 0 \\ x_i \ge 0 \\ \sum_{i=1}^{N} b_i = 1 \\ \sum_{i=1}^{N} x_i = 1 \end{cases} \tag{2.3.1}$$

Since it is difficult to be find an analytical solution, the non-linear system 2.3.1 is transformed into an optimization problem:

$$\begin{aligned} x^* = \arg\min \textstyle\sum_{i=1}^{N}(x_i \partial_{x_i} RC(x) - b_i RC(x))^2 \\ u.c.\ 1^T x = 1 \\ 0 \leq x \leq 1 \end{aligned} \tag{2.3.2}$$

The Risk Parity (also called Equal Risk Contribution) portfolio is characterized by the requirements of having equal total risk contribution from each asset:

$$b_i = b_j, \forall i, j$$

Cesarone, Scozzari and Tardella [8] points out the conditions of the uniqueness of a Risk Parity Portfolio. In particular, for risk measure volatility, a unique solution of 2.3.2 exists. The theorem is present as follows:

Theorem 2.3.3. *For a continuously differentiable risk measure $\rho : \mathbb{R}_+^n \longrightarrow \mathbb{R}$, we have that (a) if ρ is positive and positively homogeneous of degree $\tau > 0$, then there exists a Risk Parity portfolio. (b) if ρ is convex, then there exist at most one Risk Parity portfolio.*

2.4 Target Risk Strategy

Though ERC strategy brings great performances, which is explained in previous section and will be shown in 4.1, it fails to satisfy an expected portfolio risk. A target risk strategy is constructed based on risk budgeting approach. The main concept is to maximize the risk diversification of a portfolio under a pre-setting risk. In risk parity portfolio, the risk is assigned equally to each asset. Based on risk budgeting model, to construct a target risk portfolio, we set a constant target risk level RC^*. Let $(x_1, ..., x_N)$ be the weights of N assets of a portfolio, $RC(x)$ be the portfolio risk. In a risk budgeting programming 2.3.1, a constraint $RC(x) = RC^*$ is imposed. In reality, to achieve a better return, many risk parity funds target a volatility greater than 8% by leveraging the portfolio, indicating that the sum of weights is possibly greater than 1 [1]. Generally, the long-only target risk strategy can be formulated as follows.

$$\begin{aligned} x^* = &\ \arg\min \textstyle\sum_{i=1}^{N}(x_i \partial_{x_i} RC(x) - b_i RC^*)^2 \\ \text{u.c.} &\ x \geq 0 \\ &\ RC(x) = RC^* \end{aligned} \tag{2.4.1}$$

If risk measurement is volatility, 2.4.1 becomes a convex optimization programming, since objective function and its constraints can be regarded as convex function. According to [9], a **convex optimization problem** is one of the form:

$$\begin{aligned} \text{minimize} &\quad f_0(x) \\ \text{subject to} &\quad f_i(x) \leq 0, i = 1, ..., M \\ &\quad a_i^T x = b_i, i = 1, ..., p \end{aligned} \tag{2.4.2}$$

Convex optimization problem can be solved efficiently and easily by available tools. In this dissertation, *optimize* of package *SciPy* in Python is used for these optimization problems. A.2 proves that 2.4.1 is a convex optimization problem.

In risk budgeting portfolio, the expected risk diversification is already obtained, according to b_i. And the realized diversification can be measured in the following three ways. Denote the risk contribution of i-th asset as π_i. Let $\pi \in \mathbb{R}_+^n$ such that $\mathbf{1}^T \pi = 1$. π is then a probability distribution. The probability distribution π^+ is perfectly concentrated if there exists one observation i_0 such that $\pi_{i_0}^+$ and $\pi_i^+ = 0$ if $i \neq i_0$. On the opposite, the probability distribution π^- such that $\pi_i^- = 1/n$ for all $i = 1, ..., n$ has no concentration.

- **Herfindahl index**

  The Herfindahl index is defined as follows:

$$\boldsymbol{H}(\pi) = \sum_{i=1}^{n} \pi_i^2$$

  This index takes the value 1 for a probability distribution π^+ and $1/n$ for a distribution with uniform probabilities.

- **Gini index**

  The Gini index for a discrete probability distribution π is

$$\boldsymbol{G}(\pi) = \frac{2\sum_{i=1}^{n} i\pi_{i:n}}{n\sum_{i=1}^{n} \pi_{i:n}} - \frac{n+1}{n}$$

  with $\{\pi_{1:n}, ..., \pi_{n:n}\}$ the ascending ordered statistics of $\{\pi_i, ..., \pi_n\}$. This index takes the value $\frac{n-1}{n}$ for a probability distribution π^+ and 0 for a distribution with uniform probabilities.

- **Shannon entropy**

  The Shannon entropy is defined as follows,

$$\boldsymbol{I}(\pi) = -\sum_{i=1}^{n} \pi_i ln \pi_i$$

  The diversity index corresponds to the statistic $\boldsymbol{I}^*(\pi) = -exp(\boldsymbol{I}(\pi))$. This index has $\boldsymbol{I}^*(\pi^-) = -n$ and $\boldsymbol{I}^*(\pi^+) = -1$

For a risk parity optimization portfolio, the expected risk contribution for each asset is equal, that is, $RC_i = \frac{1}{N} RC^*$ for $i = 1, ..., N$, indicating that concentration indices are expected to be the lowest, for all of the three measurements. Hence, from a view of risk diversification, a risk parity approach can be regarded as aiming to obtain a portfolio which is well diversified. However, an expected risk diversification may not be obtained. Especially in Chinese markets, leveraged portfolio is not applicable in long-term asset investment. If an additional constraint $\mathbf{1}^T x \leq 1$ is imposed to the programming problem, a diversification result may not be more difficult to obtain. When a relatively high target risk is imposed without a leverage constraint, a leveraged strategy outcome may be obtained. Risk budgeting portfolio is obtained by minimizing sum of squares of the difference between pre-assigned risk budgets and realized risk contribution. Based on the risk diversification concept of risk budgeting portfolio, alternative measurement for diversifying risks are applied. In order to find a maximum diversification portfolio, apart from using least squares objective function in 2.4.1, objective functions of concentration indices can also be considered.

Let $\boldsymbol{C}(x)$ represents the concentration indices (Herfindahl index, Gini index and Shannon index) or least squares of difference between realized risk contribution and equal risk budgets (ERC) of a $N-$asset portfolio $(x_1, ..., x_N)$ with risk contribution $RC_1, ..., RC_N$. $\boldsymbol{C}(x)$ is chosen to be $\boldsymbol{C}(x) = \sum_{i=1}^{N} RC_i^2$, $\boldsymbol{C}(x) = \frac{2\sum_{i=1}^{N} iRC_{i:N}}{N\sum_{i=1}^{N} RC_{i:N}} - \frac{N+1}{N}$ or $\boldsymbol{C}(x) = -exp(-\sum_{i=1}^{N} RC_i lnRC_i)$ or $\boldsymbol{C} = \sum_{i=1}^{N}(x_i\partial_{x_i}RC(x) - \frac{1}{N}RC^*)^2$. The *maximum diversification risk target* problem can be formulated as follows:

$$\begin{aligned} x^* &= \arg\min \boldsymbol{C}(x) \\ u.c.\ \mathbf{1}^T x &\leq 1 \\ x &\geq 0 \\ RC(x) &= RC^* \end{aligned} \tag{2.4.3}$$

Herfindahl index and Gini index can be proved to be convex, while the convexity of Shannon index cannot be determined.The convexity of Gini index is already proved by [10]. The proofs of convexity of Herfindahl index and Shannon index are shown in A.3. A technical explanation for Shannon index is missing in this dissertation and further investigation can be conducted on this part. The optimization problems are solved in Python and we basically obtain local optimization results using *SciPy*.

The sum of weights is not constrained to be strictly equal to 1, indicating that the portfolio is not to invest all the money into risky assets, which is in accordance with the practical use for target risk portfolio for a long-term investment.

Parameter estimation for volatility risk measure is obtained by calculating covariance matrix once each strategy. For VaR and ES, parameter estimation is largely based on the sample, and the asset returns are used for each iteration (shown in 2.5). If a strictly equality is imposed to target risk, available solutions are limited, especially, if a small sample is used for parameter estimation. A modification is the target risk which is relaxed to a small range. Denote $Error$ as the percentage of target risk we allow, optimization problem 2.4.3 is modified as follows:

$$\begin{aligned} x^* &= \arg\min \boldsymbol{C}(x) \\ u.c.\ \mathbf{1}^T x &\leq 1 \\ x &\geq 0 \\ |RC(x) - RC^*| &\leq Error \times RC^* \end{aligned} \tag{2.4.4}$$

That is, the realized risk contribution of the portfolio is in the range $[(1 - Error)RC^*, (1 + Error)RC^*]$. In VaR or ES settings, this constraint is affline. Optimization problem 2.4.4 is a convex optimization problem.

2.5 Parameter Estimation

For a **volatility risk measure**, the covariance matrix is computed based on **sample estimation** using daily returns on a rolling window, which can be formulated as follows,

$$\hat{\sigma_{i,j}} = \frac{1}{T}\sum_{t=1}^{T}(R_{it} - \mu_i)(R_{jt} - \mu_j)$$

where μ_i indicates the sample mean return of the $i-$th asset, $\mu_i = \frac{1}{T}\sum_{t=1}^{T} R_{it}$ The vector of marginal volatilities is:

$$\frac{\partial\sigma(x)}{\partial x} = \frac{1}{2}(x^T\Sigma x)^{-1}(2\Sigma x) = \frac{\sigma x}{\sqrt{x^T\Sigma x}}$$

The risk contribution of the i^{th} asset is then:

$$RC^{vol}(x_i) = x_i\frac{(\Sigma x)_i}{\sqrt{x^T\Sigma x}}$$

For **VaR and ES**, the risk contribution of $i-$th asset with confidence level α is given by [11]

$$RC_\alpha^{VaR}(x_i) = -x_i\mathbb{E}[R_i|R(x) = -VaR_\alpha(x)]$$

$$RC_\alpha^{ES}(x_i) = -x_i\mathbb{E}[R_i|R(x) \leq -VaR_\alpha(x)]$$

historial simulation is used for estimation. Suppose $R^{(1)}(x) \leq R^{(2)}(x) \leq ... \leq R^{(T)}(x)$ indicate the sorted outcomes of portfolio returns in a $T-$length estimation rolling window, the estimation of VaR and ES can be formulated as follows [12]:

$$RC_\alpha^{VaR}(x) = VaR_\alpha(x) = -R^{\lfloor\alpha T\rfloor}(x)$$

$$RC_\alpha^{ES}(x) = ES_\alpha(x) = -\frac{1}{\lfloor\alpha T\rfloor}\sum_{k=1}^{\lfloor\alpha T\rfloor} R^{(k)}(x)$$

where $\lfloor x\rfloor$ is floor function which gives the greatest integer which is less or equal than the input number x The risk contribution of $i-$th asset using VaR and ES is estimated as follows,

$$RC_\alpha^{VaR}(x_i) = -x_i\frac{1}{S}\sum_{k=1}^{S} R_i^{(k)}$$

$$RC_\alpha^{ES}(x_i) = -x_i\frac{1}{\lfloor\alpha T\rfloor}\sum_{k=1}^{\lfloor\alpha T\rfloor} R_i^{(k)}$$

where S indicates the number of asset return lying on the 5-th quantile, which can be inferred that $S = 1$ most of the time.

2.6 Performance Measurement of Portfolio

To compare the performances of different strategies, for each portfolio to be constructed, the analysis of portfolio performance relies on a **'rolling-sample' approach**. Inspired by DeMiguel [13] and De BONDT [14], given a $T-$month-long dataset of asset returns, an estimation window of length P is chosen. Starting from $t = P+1$, the data in the previous P months is used to estimate the parameters needed to implement a particular strategy, to determine the weights of the portfolio. Then we use the weights to calculate the return in the next $Q-$month-long period. Then the weights of the strategy is recalculated at $t = P+Q$. The weights for implementing strategies are calculated each Q months. The number of outcomes of this rolling-window approach is $\lfloor\frac{T-P}{Q}\rfloor$, where $\lfloor x\rfloor$ is floor function which gives the greatest integer which is less or equal than the input number x. Given the time series of *out-of-sample* returns generated by each portfolio strategy, the *out-of-sample volatility, Sharpe ratio, maximum drawdown* and *turnover* are computed as performance measurement.

To evaluate a strategy regarding of both risk premium and volatility, measurement **Ex Post Sharpe Ratio** is introduced as follows. [15] Let R_t be the return on the asset in period t, R_{Bt} the return on the benchmark portfolio or security in period t, D_t the differential return in period t: $D_t = R_t - R_{Bt}$. Let $\bar{D}$, σ_D be the sample mean and sample deviation of D_t over the period. The ex post, or historical **Sharpe Ratio** (S_h) is:

$$S_h = \frac{\bar{D}}{\sigma_D}$$

Sharpe Ratio can be used for either measuring the performance of one portfolio compared to its benchmark, or the ratio of risk premium of one asset compared to its volatility, if R_{Bt} is risk-free rate.

Another measurement **maximum drawdown** measures the maximum decline percentage from a historical peak. Suppose $M(\tau) = max_{t\in(0,\tau)}X(t)$ is the maximum return between time 0 and time t, maximum drawdown can be formulated as follows:

$$MDD(T) = max_{\tau\in(0,T)}[\frac{M(\tau) - X(\tau)}{M(\tau)}]$$

To evaluate the amount of trading required to implement each portfolio, regarding of rebalancing costs, **turnover** is introduced to measure the average sum of the absolute value of the trades across the N available assets, and $M = \lfloor\frac{T-P}{Q}\rfloor$, times of position tradings, which can be formulated as follows [13]:

$$Turnover = \frac{1}{M}\sum_{t=1}^{M}\sum_{j=1}^{N}(|\hat{w}_{j,t+1} - \hat{w}_{j,t^+}|),$$

in which $\hat{w}_{j,t}$ is the portfolio weight in asset j at time t, $\hat{w}_{j,t^+}$ is the portfolio weight before rebalancing at $t+1$, and $\hat{w}_{j,t+1}$ is the desired portfolio weight at time $t+1$, after rebalancing.

Chapter 3

Experiment

3.1 Data

CSI represents China Securities Index Company Limited, which is a leading index provider in China [16]. It publishes several indices about equities, bonds and commodities, which are used in this dissertation. *Nanhua* represents Nanhua Futures Company Limited. It publishes indices of commodities which are widely used in China [17]. *SWS* represents SWS Research Company Limited, which provides a wide range of indices in Asian market [18].

SSE Composite Index or *SSE Index* represents all the stocks in Shanghai Stock Exchange, which reflects the trends of the Chinese market.

CSI 300 Index selects 300 stocks which have the most market capitalization and liquidity in Shanghai Stock Exchange and Shenzhen Stock Exchange.

CSI Aggregate Bond Index selects treasury bonds, governmental bonds, corporate bonds and financial bonds, which have a rating greater than BBB and a time to maturity longer than 1 year in Shenzhen Exchange, Shanghai Exchange and the inter-bank market. It reflects trends of the whole bond market.

CSI Corporate Bond Index selects credit bonds which have a rating greater than BBB and a time to maturity longer than 1 year in Shenzhen Exchange, Shanghai Exchange and the inter-bank market.

Nanhua Commodities Index selects the future contracts of various types including agricultural products, metal etc. which have great liquidity, and the most close delivery date in Dalian Commodity Exchange, Zhengzhou Commodity Exchange and Shanghai Futures Exchange. It reflects trends of the commodity market in China.

Nanhua Precious Metal Index selects the future contracts of silver and gold in Dalian Commodity Exchange, Zhengzhou Commodity Exchange and Shanghai Futures Exchange. It reflects trends of the price of precious metal in China.

Exchange-traded Fund (ETF) refers to an investment fund that tracks a market index and that is itself traded in the same way as a stock [19]. In Chinese market, ETFs includng *SSE Index* ETF, *CSI 300 Index* ETF, *Gold Index* ETF etc, and have a similar performance to the corresponding indices. One great benefit of ETF is that it providing intraday liquidity for investors.

Listed Open-Ended Fund (LOF) refers to the mutual funds that can be listed and traded on the

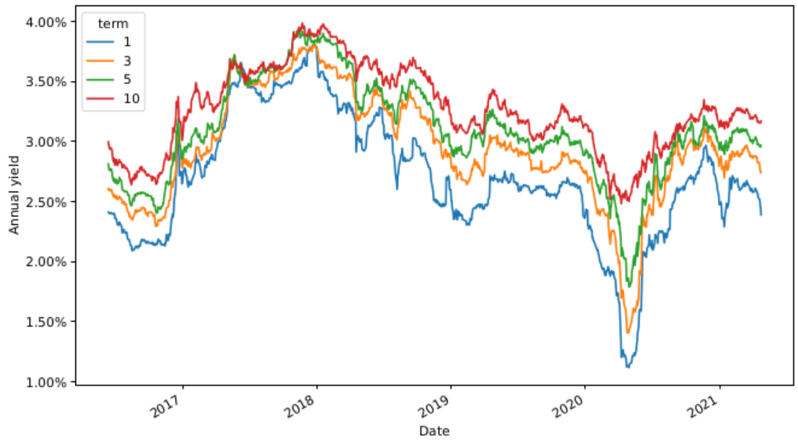

Figure 3.1: Annual rate of governmental bond ranging from 13/06/2016 to 23/04/2021. Maturities = 1 year, 3 years, 5 years and 10 years.

stock exchange, which adding liquidity of exchange trading.

The risk-free rate is chosen based on the Chinese 10-year treasury bond. Figure 3.1 shows the rate of governmental bonds in Chinese markets. Yield to maturity of 10-year treasury fluctuates from 2.50% to 4.00% and does not show a particular pattern. In this dissertation, the risk-free rate is chosen to be the mean of 10-year treasury bond rate, 0.03267.

3.2 Backtesting Methodology

3.2.1 Experiment Set Up

To backtest a portfolio, a portfolio with 1 unit net asset value is initialized and calculate a cumulative asset value for each asset. For the $t-$th trading, calculate the $i-$th asset value before reweighting V_{i,t^-} and the asset value after reweighting $V_{i,t}$, then the i-th asset value becomes $V_{i,t} - |V_{i,t} - V_{i,t^-}| \times TransactionCost$.

3.2.2 Parameters

The transaction cost of *Cash* is set to be 0. The risk-free rate of *Cash* is 0.0128% per day (3.267% per annual). The transaction cost of risky asset is set to be 0.4%.

3.3 A Comparison of Mean Variance Portfolio and Risk Based Portfolio

To illustrate the performance of mean variance portfolio and risk budgeting portfolio, and justify the reason for using risk based portfolio in the following exploration. Historical data of *CSI 300 Index, CSI Aggregate Bond, CSI Corporate Bond* and *Nanhua Commodity Index* ranging from 04/01/2017 to 28/06/2021 is used.

Portfolio settings of sample-based mean-variance and risk budgeting strategies are as follows:

- 1. Risk bugdeting portfolio: Under optimization programming setting 2.3.1, assign risk budgeting 0.4, 0.1, 0.2, 0.3 to *CSI 300 Index, CSI Aggregate Bond, CSI Corporate Bond* and *Nanhua Commodity Index* respectively.

- 2. Mean-Variance portfolio: Under setting 2.2.1, use the variance of the above risk-budgeting portfolio as target variance, maximize the expected return.

For risk budgeting portfolio, volatility of the last 6 months is used to measure risk. For mean variance portfolio, cumulative return of previous 6 months is used for expected return. That is, we set $P = 6$ in the rolling-based approach. Position is adjusted every 6 months and strategy performance is measured every 6 months ($Q = 6$).

3.4 Maximum Diversification (MD) Portfolio Optimization

3.4.1 Asset Selection

Generally, equities and commodities have high risk premium and low correlation, indicating that these two assets can diversify risk and have potential high return within a portfolio. Bonds have a negative relationship with equities and commodities. Though bonds have a relatively small risk premium, they play an important role to hedge risk of equities and commodities in asset allocation of a portfolio strategy.

Historical data of *SSE Index, CSI 300 Index, Small CAP Index, CSI Aggregate Bond, CSI Treasury Bond, CSI Corporate Bond, Nanhua Commodity Index* and *Nanhua Precious Metal Index* ranging from 04/01/2017 to 29/06/2021 are selected as asset universe. Table 3.1 shows the covariance matrix of the eight indices. As expected, equities have a high volatility (variance), and assets within equity category highly are correlated with each other. Bonds are negatively correlated with equities and have a small volatility, hence they will be assign a high weight when steady volatility and low risk are required in a long-term strategy. Besides, commodities have a relatively high volatility. One point worth noticing is that the correlation of *Nanhua Precious Metal Index* is not closely correlated with *Nanhua Commodity Index*. Since commodities are not correlated closely with other assets, they also are expected to be included in the portfolio for diversifying risk. Sharpe ratios of the eight assets are shown in Table 3.3. *CSI 300 Index* has the highest Sharpe ratio among equities (0.47 v.s. 0.088, -0.14). *Corporate Bond Index has* the highest Sharpe ratio among bonds (1.64 v.s. 0.23, 0.87). Sharpe ratio of *Nanhua Commodity Index* is higher than *Nanhua Precious Metal Index* (0.48 v.s. 0.10). Indices are chosen from each of the asset categories and the top three indices with the highest Sharpe ratios are selected for a *three-asset portfolio* construction.

	SSE	CSI 300	Small CAP	Aggre. Bond	Trea. Bond	Corp. Bond	Commodity	Precious Metal
SSE	297.07	323.91	338.82	-3.95	-6.30	-1.87	99.02	35.59
CSI 300	323.91	376.84	347.40	-4.19	-6.60	-2.01	103.47	38.38
Small CAP	338.82	347.40	510.93	-3.96	-6.43	-1.67	107.93	42.92
Aggre. Bond	-3.95	-4.19	-3.96	1.30	1.71	0.82	-1.76	2.18
Trea. Bond	-6.30	-6.60	-6.43	1.71	2.63	0.86	-3.46	3.31
Corp. Bond	-1.87	-2.01	-1.67	0.82	0.86	0.81	-0.43	1.23
Commodity	99.02	103.47	107.93	-1.76	-3.46	-0.47	216.18	64.31
Precious Metal	35.59	38.38	42.92	2.18	3.31	1.23	64.32	274.83

Table 3.1: Covariance matrix (annualized) of *SSE Index, CSI 300 Index, Small CAP Index, CSI Aggregate Bond, CSI Treasury Bond, CSI Corporate Bond, Nanhua Commodity Index* and *Nanhua Precious Metal Index* ranging from 04/01/2017 to 29/06/2021.

	SSE	CSI 300	Small CAP	Aggre. Bond	Trea. Bond	Corp. Bond	Commodity	Precious Metal
SSE	1.00	0.98	0.86	-0.20	-0.23	-0.12	0.34	0.11
CSI 300	0.98	1.00	0.85	-0.19	-0.21	-0.12	0.34	0.11
Small CAP	0.86	0.85	1.00	-0.15	-0.18	-0.08	0.28	0.09
Aggre. Bond	-0.20	-0.19	-0.15	1.00	0.93	0.80	-0.10	0.12
Trea. Bond	-0.23	-0.21	-0.18	0.93	1.00	0.59	-0.15	0.12
Corp. Bond	-0.12	-0.12	-0.08	0.80	0.59	1.00	-0.03	0.08
Commodity	0.34	0.34	0.28	-0.10	-0.15	-0.03	1.00	0.36
Precious Metal	0.11	0.11	0.09	0.12	0.12	0.08	0.36	1.00

Table 3.2: Correlation matrix of *SSE Index, CSI 300 Index, Small CAP Index, CSI Aggregate Bond, CSI Treasury Bond, CSI Corporate Bond, Nanhua Commodity Index* and *Nanhua Precious Metal Index* ranging from 04/01/2017 to 29/06/2021.

SSE	CSI 300	Small CAP	Aggre. Bond	Trea. Bond	Corp. Bond	Commodity	Precious Metal
0.088	0.47	-0.14	0.87	0.23	1.64	0.48	0.10

Table 3.3: Sharpe ratio of *SSE Index, CSI 300 Index, Small CAP Index, CSI Aggregate Bond, CSI Treasury Bond, CSI Corporate Bond, Nanhua Commodity Index* and *Nanhua Precious Metal Index* ranging from 04/01/2017 to 29/06/2021.

3.4.2 Portfolio Risk Diversification Strategy

Since leveraged portfolio is not applicable in Chinese markets, a task to assign risk budgets to diversify the portfolio is quite challenging. Illustrations of the performances of portfolios with and without leverage constraints are made first. Also, objective functions of equal risk contribution (ERC) and concentration minimization (maximum diversification) are compared under the same target risk. In addition, we adding more assets to test the improvement. And risk measures are changed for comparison.

Table 3.4 shows all the strategies in this part.

Number	Setting
strategy A	ERC + Vol + No leverage (Benchmark)
strategy B	ERC + Vol + Leverage
strategy C	Shannon + Vol + No leverage
strategy D	Henfindahl + Vol + No leverage
strategy E	GINI + Vol + No leverage
strategy F	ERC + VaR + No leverage
strategy G	Shannon + VaR + No leverage
strategy H	Henfindahl + VaR + No leverage
strategy I	GINI + VaR + No leverage
strategy J	ERC + ES + No leverage
strategy K	Shannon + ES + No leverage
strategy L	Henfindahl + ES + No leverage
strategy M	GINI + ES + No leverage

Table 3.4: Strategy settings in MD strategy optimization

Three-Asset Portfolio

CSI 300 Index, CSI Corporate Bond and *Nanhua Commodity Index* represent asset category *equities, bonds* and *commodities* respectively. In particular, futures are risky assets and have a low risk premium. It is set to be less than or equal to 20% of the total wealth. The weight of each risky asset should be greater than 2%. Volatility is used as risk measure. The period length of data used for risk estimation is 1 month, 6 months and 12 months ($P = 1, 3, 6, 12$) and trading is made every month ($Q = 1$). The portfolio settings are designed as follows:

- **Strategy A = ERC + Vol + No leverage:** Target risks 3%, 6%, 9%, 15% are assigned $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ to *CSI 300 Index, CSI Corporate Bond* and *Nanhua Commodity Index* respectively. The sum of the weights should be less than or equal than 100%.
- **Strategy B = ERC + Vol + Leverage:** Target risks 3%, 6%, 9%, 15% are assigned $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ to *CSI 300 Index, CSI Corporate Bond* and *Nanhua Commodity Index* respectively. No constraint is imposed to the sum of the weights.
- **Strategy C/D/E = Shannon/Herfindahl/Gini + Vol + No leverage:** Under a target risk 3%, 6%, 9%, 15%, find out strategies with minimum Herfindahl index, Gini index and Shannon index of risk contributions respectively. The sum of the weights is less than or equal 100%.

Then, risk measure is changed for portfolio construction. According to the results in the above portfolio, average value of $\text{VaR}_{0.95}$ and $\text{ES}_{0.95}$ for portfolios under the same risk (unconstrained ERC portfolio is excluded) are calculated. Since the estimation of VaR and ES largely depend on the sample size of data, $P = 1$ (less than 30 trading days) seems to be too small for estimation. Table 3.5 shows the corresponding value of volatility, $\text{VaR}_{0.95}$ and $\text{ES}_{0.95}$.

The portfolio settings are as follows.

- **Strategy F/G/H/I = ERC/Shannon/Herfindahl/Gini + VaR + No leverage:** Using $\text{VaR}_{0.95}$ as risk measurement, target risks 0.31, 0.64, 0.95 and 1.50 are assigned to conservative, moderate, aggressive and very aggressive portfolios respectively. Minimization concentration problems involving target risk error 2.4.4 is used with $Error = 0.05$. Rolling

Target Vol (%)	$\text{VaR}_{0.95}$ (%)	$\text{ES}_{0.95}$(%)
3	0.31	0.54
6	0.64	1.03
9	0.95	1.51
15	1.50	2.34

Table 3.5: $\text{VaR}_{0.95}$ and $\text{ES}_{0.95}$ estimation for different target risk settings

window is 6 months ($P = 6$). The weight of *Nanhua Commodity Index* should be less than or equal to 20%. The weight of each risky asset should be greater than 2%.

- **Strategy J/K/L/M = ERC/Shannon/Herfindahl/Gini + ES + No leverage:** Using $\text{ES}_{0.95}$ as risk measure, target risks 0.54, 1.03, 1.51 and 2.34 are assigned to moderate, aggressive and very aggressive portfolios respectively. Other settings are the same as $\text{VaR}_{0.95}$.

Four-Asset Portfolio

To improve the performance of target risk portfolios, four-asset portfolio is attempted. Based on asset correlation (see Table 3.2), *Nanhua Precious Metal Index* is chosen for a risk diversification. Though *Nanhua Commodities Index* is the same asset category as *Nanhua Precious Metal Index* and it already reflects trends of commodities including precious metal, it has a low correlation with other assets, which may improve the diversification of the portfolio. A maximum weight 10% is imposed to this index. In previous section, $P = 6$ and $P = 1$ presents a steady portfolio and a high-return portfolio respectively, hence $P = 1, 6$ is set in this section.

The portfolio setting is as follows,

- **Strategy A = ERC + Vol + No leverage:** Under target risks 3%, 6%, 9%, 15%, risk budgets $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$ are assigned to *CSI 300 Index, CSI Corporate Bond*, *Nanhua Commodity Index* and *Nanhua Precious Metal Index* respectively. The weight of each risky asset should be greater than 2%. The weight of *Nanhua Commodity Index* and *Nanhua Precious Metal Index* should be less than 20% and 10%. The sum of the weights should be less than or equal to 100%.
- **Strategy C/D/E = Shannon/Herfindahl/Gini + Vol + No leverage:** Under a target risk 3%, 6%, 9%, 15%, find out strategies with minimum Herfindahl index, Gini index and Shannon index of risk contributions respectively. Weight of each risky asset should be greater than 2%. The weight of *Nanhua Commodity Index* and *Nanhua Precious Metal Index* should be less than 20% and 10%. The sum of the weights is less than or equal to 100%.

Chapter 4

Results And Analysis

4.1 Comparison between MV Portfolio and RB Portfolio

The weight distribution can be shown directly from Figure 4.1 and Figure 4.2. Weight distribution of MV portfolio has a great change each time, while weights of RB portfolio change slightly, under the condition that, the two portfolios have the same expected risk (volatility), indicating a stable property of risk-budgeting approach. A possible lower rebalancing costs of risk budgeting strategy makes it more reasonable and applicable in tactical and strategic asset allocation portfolios.

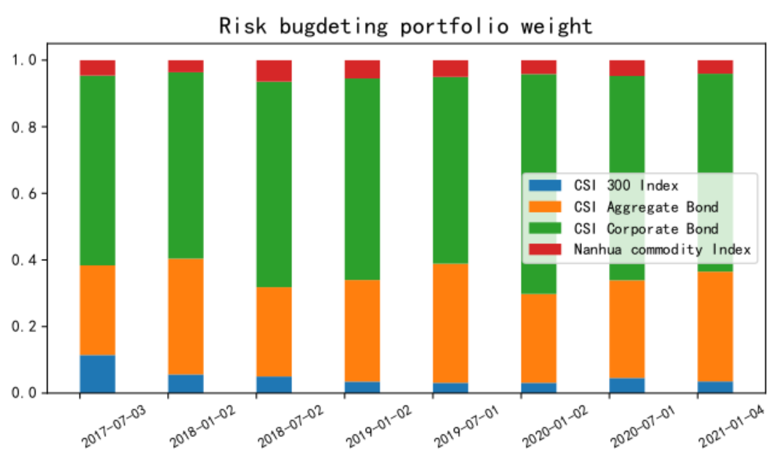

Figure 4.1: Weight Allocation of Risk-budgeting Portfolio

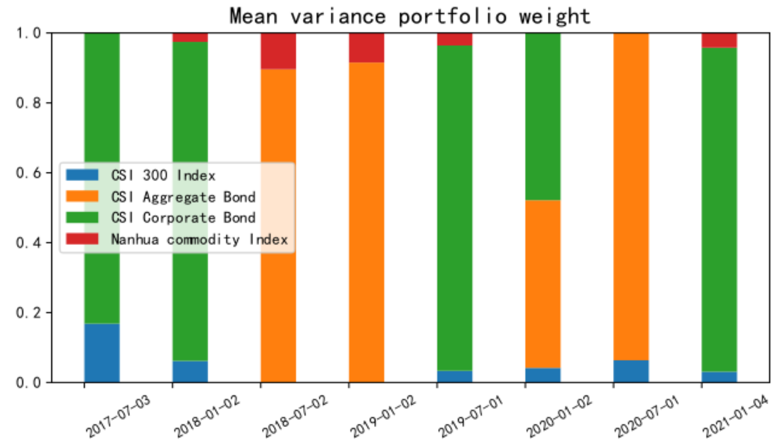

Figure 4.2: Weight Allocation of Mean-variance Portfolio

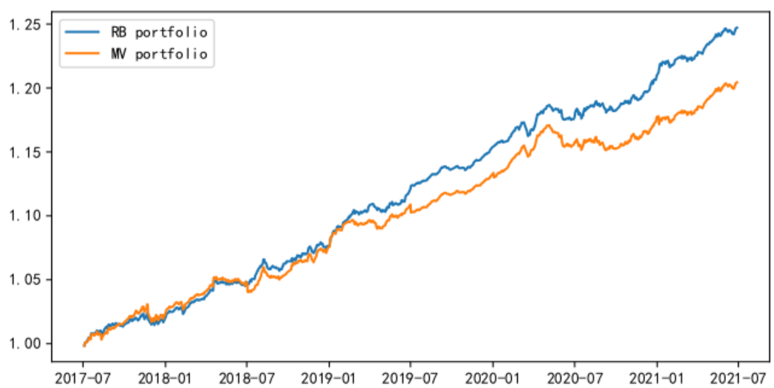

Figure 4.3: Backtesting outcomes of RB and MV strategies. Transaction cost = 0.4%

The two strategies are backtested with a 0.4% transaction cost for each trading. Figure 4.3 shows the time series of the two strategies ranging from 03/07/2017 to 28/06/2021. Two strategies have similar outcomes, while risk budgeting portfolio gives a higher return after 07/2020. Table 4.1 shows the performance measurement of the two strategies. Under the condition that these two portfolios have the same volatility, risk-budgeting portfolio performs better than mean-variance portfolio. In particular, RB strategy has a much lower turnover (12.99 % v.s. 114.43%). The illustration of these two strategies shows that, to construct a long-term, steady-risk portfolio, risk-budgeting strategy has great advantages, compared with mean-variance strategy. Especially, it eliminates the sensitivity of the sample based estimation of mean-variance strategy. A steady weight allocation during a long period is more reasonable in TAA and SAA strategies.

	Return p.a. (%)	Volatility p.a. (%)	SR	MDD (%)	Turnover (%)
RB strategy	5.80	1.50	1.72	0.98	12.99
MV stratey	4.89	1.61	1.04	1.66	114.43

Table 4.1: Out-of-sample performance of RB strategy and MV strategy

4.2 Maximum Diversification Portfolio Optimization

4.2.1 Three-Asset Portfolio

For $P = 1, 3, 6, 12$, average weight allocation of assets, average value of concentration indices and out-of-sample performance measurement are presented respectively. (see B)

Comparison of ERC between leverage and no leverage, P=1, Q=1

To illustrate the general patterns of target risk portfolios with leverage and no leverage, outcomes of constrained ERC (no leverage) and unconstrained ERC (leverage) are presented, using 1-month-length estimation window. Table 4.2 shows the average weights of portfolios over time. The results show that we need to invest more in risky assets as we set a higher risk target. The allocation distribution which spares a proportion of cash is applicable to practical investment, since investors are not going to invest all into risky assets and conservative investors are willing to hold more cash than aggressive investors. For portfolios with $P = 1$, a target risk 6%, 9%, 15%, 3%, 23% and 50% leverage ratio are required if leverage is allowed.

The realized diversification results are shown in Table 4.3. To achieve equal risk contribution, the Herfindahl index, Gini index and Shannon index of the portfolio is expected to be 0.33, 0.00, -3.00 respectively. The lower the concentration value is, the more diversified the portfolio will be. It can be seen that if leverage is allowed, portfolio achieves a better diversification, regardless of in-sample and out-of-sample. Under each target risk, leveraged portfolio has a lower concentration values (better risk diversification) and better out-of-sample performance, which may have a *implication that a more risk diversified portfolio may bring a better return.* Table 4.4 shows the out-of-sample performance. Transaction cost is set to 0.4%. The table shows that target risk strategy is able to constrain the volatility to the target level, and the monthly turnover rate is relatively low, which are of practical meaning for target risk strategy. If leverage is allowed, for a conservative (3%) or a moderate (6%) target risk, constrained ERC and unconstrained ERC seems to have similar performances, apart from that unconstrained ERC has a higher turnover rate. For a high target risk (9% or 15%), unconstrained ERC achieves a better performance, regarding of return, Sharpe ratio and maximum drawdown, though the turnover rate is relatively high. A implication may be that *if there is no leverage constraint, achieved risk contribution of a portfolio is more close to the expected level, which leads to a more desirable performance*, in terms of return and Sharpe ratio, especially when an aggressive target risk is set.

	Target	CSI 300 Index	CSI Aggregate Bond	Gold Index	Cash
Cons. ERC	3%	12.0	47.0	13.0	29.0
Uncons. ERC	3%	11.0	62.0	13.0	13.0
Cons. ERC	6%	27.0	28.0	19.0	25.0
Uncons. ERC	6%	27.0	57.0	19.0	-3.0
Cons. ERC	9%	44.0	16.0	20.0	20.0
Uncons. ERC	9%	46.0	57.0	20.0	-23.0
Cons. ERC	15%	69.0	6.0	17.0	8.0
Uncons. ERC	15%	71.0	56.0	23.0	-50.0

Table 4.2: Average weights of risk parity strategies ranging from 04/01/2017 to 28/08/2021. $P =$ 1 month. $Q = 1$ month.

	Target	Shannon Index (-3.00)		Herfindahl Index (0.33)		Gini Index (0.00)	
		In	Out	In	Out	In	Out
Cons. ERC	3%	-2.22	-2.07	0.48	0.54	0.31	0.40
Uncons. ERC	3%	-2.31	-2.15	0.46	0.53	0.29	0.39
Cons. ERC	6%	-1.87	-1.83	0.59	0.60	0.43	0.45
Uncons. ERC	6%	-1.96	-1.89	0.57	0.59	0.42	0.45
Cons. ERC	9%	-1.65	-1.64	0.68	0.69	0.51	0.52
Uncons. ERC	9%	-1.72	-1.67	0.67	0.69	0.50	0.52
Cons. ERC	15%	-1.42	-1.41	0.80	0.81	0.57	0.58
Uncons. ERC	15%	-1.50	-1.49	0.77	0.79	0.56	0.57

Table 4.3: Average value of concentration indices from 04/01/2017 to 28/06/2021. In: in-sample value. Out: out-of-sample value. Expected value of Herfindahl Index, Gini Index, Shannnon Index is **0.33, 0.00, -3.00** respectively. $P = 1$ month. $Q = 1$ month.

	Target	Return p.a. (%)	Volatility p.a.(%)	SR	MDD (%)	Turnover (%)
Cons. ERC	3%	6.30	3.78	0.81	4.49	38.38
Uncons. ERC	3%	6.42	3.98	0.81	4.08	47.19
Cons. ERC	6%	9.04	6.89	0.85	10.04	36.00
Uncons. ERC	6%	9.09	6.99	0.84	8.80	50.63
Cons. ERC	9%	11.74	10.09	0.85	17.44	30.68
Uncons. ERC	9%	12.27	10.29	0.88	16.12	54.83
Cons. ERC	15%	15.42	15.10	0.81	30.05	33.40
Uncons. ERC	15%	17.95	15.63	0.94	27.97	64.04

Table 4.4: Out of sample performance measurement. $P = 1$ month. $Q = 1$ month. Transaction cost = 2%.

Comparison of ERC and MD, P=12, Q=1

For portfolios with $P = 12$ months (see B.1, B.2, B.3), unconstrained ERC portolios always outperforms the other four portfolios. As target risks increases, annualized return increases from 6% to 8%, Sharpe ratio decreases from 1.09 to 0.30, maximum drawdown increases from 4% to

32%. The monthly turnover rate remains a relatively low level (less than 10%). In particular, for a conservative (3%) and a moderate risk target (6%), *Herfindahl* strategy improves the performance compared with ERC, regarding of return (6.87% v.s. 6.16%, 7.23% v.s. 6.96%), Sharpe raio (1.09 v.s. 0.87, 0.60 v.s. 0.56). Under aggressive risks (9% and 15%), *Herfindahl and Gini* strategy do not have outstanding results. *Shannon* strategy seems to perform poorly for each scenario.

Comparison of ERC and MD, P=6, Q=1

For portfolios with $P = 6$ months (see B.4, B.5, B.6), similarly, unconstrained ERC always outperforms the other four. Annualized return ranging from 7% to 11% and Sharpe ratio decreases from 1.11 to 0.51. The monthly turnover rate fluctuates from 8% to 17%. To be specific, *Herfindahl* portfolio performs better than ERC portfolio under target risk 3%, 6%, and 9%, regarding of return, Sharpe ratio and turnover. For target risk 15%, *Herfindahl* performs the same as ERC portfolio, with a slightly lower turnover. *Shannon* strategy performs the worst under 3%, 6% and 9% and has a normal performance under 15%.

Comparison of ERC and MD, P=3, Q=1

Using $P = 3$ rolling window (see B.7, B.8, B.9), compared with the previous two window lengths, annualized return has a greater increment (from 6% to 14%). Shape ratios are more stable, from 0.89 to 0.71. However, the monthly turnover rate are higher, fluctuating from 19% to 27%. For a conservative strategy, *Gini and Herfindahl* can improve the performance compared with ERC portfolio slightly. For the other targets, the four portfolios perform similarly. Still, *Shannon strategy* performs the worst under each risk target.

Comparison of ERC and MD, P=1, Q=1

The results with $P = 1$ month are shown in B.10, B.11, B.12. Portfolio return increases from 6.73% to 17.27 %, which is an great improvement compared with previous results and the Sharpe ratios are higher, ranging from 0.80 to 0.92. Portfolios with $P = 1$ have the highest turnover (from 31% to 39%). In particular, *Shannon* strategy no long performs badly compared with the previous outcomes. It brings a highest return for 6%, 9% and 15% and performs best for a very aggressive target (15%), achieving an annualized return 17.27% and Sharpe ratio 0.90. *Herfindahl* strategy is the best for 3%, regarding of return (6.76% v.s. 6.30%), Sharpe ratio (0.92 v.s. 0.81).

Comparison of Different Window Lengths, P=1, 3, 6, 12

Figure 4.4, Figure 4.5, Figure 4.6 and Figure 4.7 show backtesting results of portfolios under each of the target risks and each of the estimation window lengths. The same y-axis range is set to make the figures more comparable. Since Gini strategy performs rather similarly to Herfindahl strategy but slightly worse than Herfindahl, it is not presented in the figures to make the performances of other portfolios shown more clearly. A shorter window estimate seems to bring a more desirable return, though as mentioned before, it brings a higher turnover. In particular, for a conservative target risk, *Herfindahl* strategy outperforms other strategies for every estimation rolling window. Its dominant performance is more obvious for a longer estimation rolling window. As target risk increases, the advantage of Herfindahl strategy disappears and it performs similarly to ERC strategy, which implies *a stable property* of Herfindahl Strategy. In contrast to Herfindahl strategy, *Shannon* strategy exhibits *an unstable property*. For a low target risk and short estimation period, Shannon strategy shows a poor performance than other strategies. However, for a high target risk and short estimation period, Shannon strategy outperforms other strategies greatly. In particular, backtesting results with $P = 1$ show a great performance of Shannon strategy (Figure 4.7.

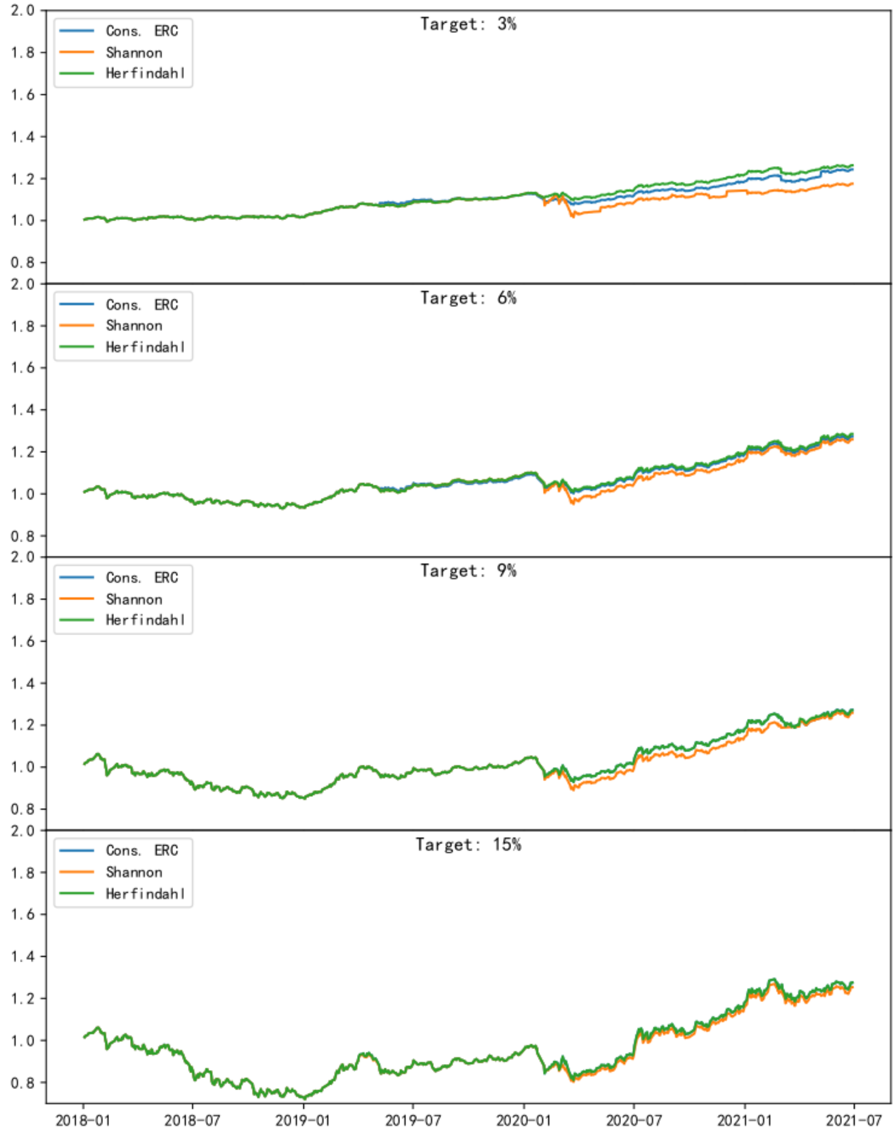

Figure 4.4: Backtesting outcomes of target risk portfolios from 05/01/2018 to 28/06/2021. $P = 12$ months. $Q = 1$ month.

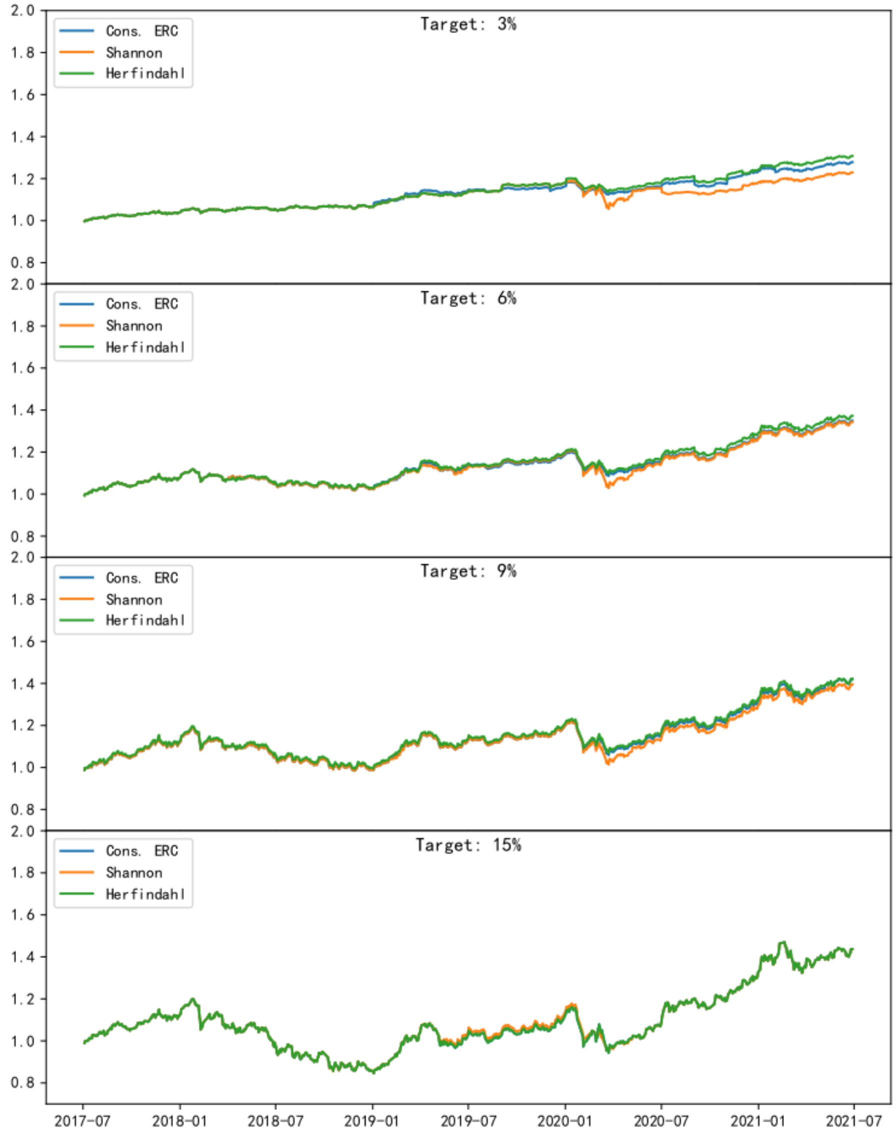

Figure 4.5: Backtesting outcomes of target risk portfolios from 03/07/2018 to 28/06/2021. $P = 6$ months. $Q = 1$ month.

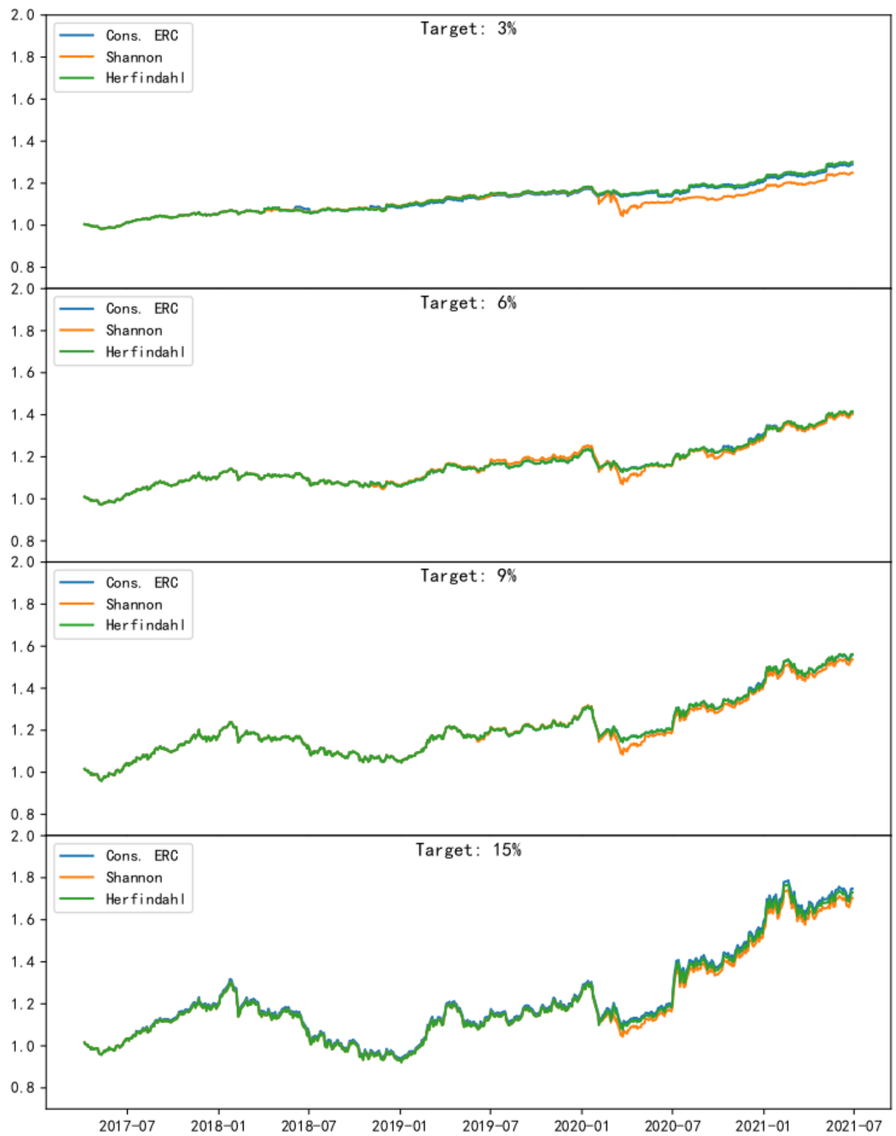

Figure 4.6: Backtesting outcomes of target risk portfolios from 05/04/2018 to 28/06/2021. $P = 3$ months. $Q = 1$ month.

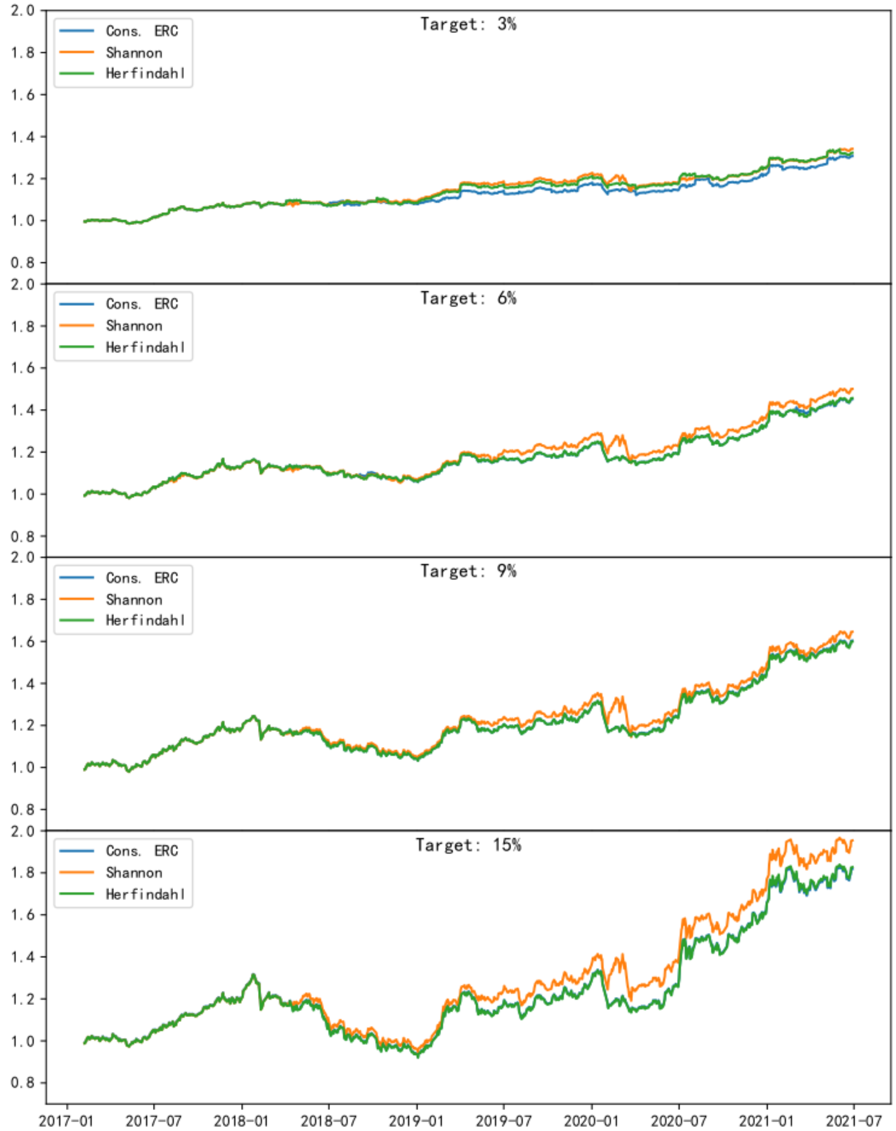

Figure 4.7: Backtesting outcomes of target risk portfolios from 03/02/2018 to 28/06/2021. $P = 1$ months. $Q = 1$ month.

Portfolios with $P = 12$ perform the worst, regarding of return, Sharpe ratio, maximum drawdown, but they bring a relatively low turnover (less than 10%). Portfolios with $P = 1$ performs the best, regarding the highest annualized return and a relatively high Sharpe ratio, however, the monthly turnover rate is greater than 30%. Based on a practical point of view, if a conservative or a moderate target risk is set, portfolios with $P = 6$ are more applicable, with a relatively high return, Sharpe ratio and low monthly turnover rate (less than 15%). If an aggressive target risk

	Risk	Return p.a.(%)	Volatility p.a.(%)	SR	MDD (%)	Turnover (%)	VaR(0.05)	ES(0.05)
Conservative	Volatility	6.50	3.46	0.95	5.21	15.05	0.29	0.50
	$VaR_{0.95}$	7.14	3.79	1.04	3.75	73.22	0.32	0.52
	$ES_{0.95}$	6.83	4.55	0.79	7.09	58.83	0.39	0.67
Moderate	Volatility	8.15	6.64	0.74	9.78	13.73	0.64	1.00
	$VaR_{0.95}$	10.10	7.46	0.92	11.40	56.02	0.66	1.13
	$ES_{0.95}$	9.62	8.45	0.76	12.32	44.12	0.79	1.27
Aggressive	Volatility	9.80	9.95	0.66	17.20	12.75	0.95	1.52
	$VaR_{0.95}$	10.12	11.27	0.61	18.64	42.33	1.09	1.73
	$ES_{0.95}$	10.24	11.99	0.59	21.92	31.32	1.14	1.84
Very Aggressive	Volatility	10.82	15.42	0.49	29.59	8.90	1.52	2.38
	$VaR_{0.95}$	10.37	15.72	0.45	27.85	24.86	1.54	2.44
	$ES_{0.95}$	11.85	16.93	0.51	30.56	14.09	1.66	2.58

Table 4.5: Out of sample performance of ERC portfolios under different risk measures

is set, portfolios with $P = 1$ are more applicable, since it has a high return, high Sharpe ratio but as well as high monthly turnover.

Comparison of different risk measures, P=6, Q=1

The out-of-sample results are shown in Table B.13 and Table B.14. First, to see the differences between different risk measures, a comparison of ERC portfolios is made, under the same sample length ($P = 6$). The results are shown in Table 4.5. Out-of-sample results show that realized $VaR_{0.95}$ of VaR portfolios are close to the target setting, but ES portfolios fail to constrain the $ES_{0.95}$ and the realized $ES_{0.95}$ is larger than target. Though VaR brings an improvement in return for a low risk target, the turnover rate is rather high (greater than 50%). ES portfolios seem not to bring obvious improvements, except under a very aggressive target.

Secondly, according to Table B.13, concentration minimization portfolios does not bring any improvement. Besides, VaR portfolios bring a rather high monthly turnover rate, ranging from 25% to 73% (mostly greater than 40%). Shannon strategy shows the unstable property since it fails to control risk under a conservative target and performs poorly in other scenarios. According to Table B.14, *Gini and Herfindahl* strategy perform similar to ERC. *Shannon* strategy improves the performance for a high target risk. ES portfolios also bring a relatively high turnover rate, ranging from 59% to 14%.

Summary

In summary, to obtain a better diversification and higher return, leverage is always a possible method. However, under a leverage constraint, the improvement for target risk portfolios is not easy to achieve. Generally, *Herfindahl strategy has a steady property, and has performance which is of the same level or slighter greater level compared with ERC strategy, regarding of annualized return and monthly turnover rate.* Specifically, under a conservative target risk, Herfindahl is able to bring great improvements. However, under other risk targets, the improvement is not obvious. *Gini strategy has the similar properties to Herfindahl but performs worse than it. Shannon strategy brings portfolios which has unsteady performances, but has some patterns and is able to bring a desirable return for particular scenarios.* Specifically, it improves the performance greatly compared with ERC strategy if a short estimation window is used and an aggressive target is to be achieved.

In addition, a long estimation-rolling window ($P = 12$) performs poorly, except that it brings a rather low monthly turnover rate. One-month rolling-window seems to be desirable while it brings a more than 30% monthly turnover rate. When investment strategy is applied in practice, a trade-off should be made between turnover rate and annualized return towards to the selection of length of data.

Regarding of different risk measures, VaR ERC portfolio can improve annualized return and Sharpe ratio for risk 3% and 6%, but it has a high monthly turnover rate (73% and 56%). For ES risk measures, though ES portfolios fail to control out-of-sample risk, *Shannon* ES strategy *does improve* return and Sharpe ratio compared to ERC portfolio, under 6%, 9% and 15%, which can be regarded as an improvement method when a long length of estimation window is applied ($P = 6$). In the following application, ES strategy is not applied since volatility strategy with $P = 1$ has a more desirable performance for a high target risk.

	Target	P	Return p.a. (%)	Volatility p.a.(%)	SR	MDD (%)	Turnover (%)
Cons. ERC	3%	6	6.50	3.46	0.95	5.21	15.05
Herfindahl	3%	6	7.10	3.51	1.11	5.48	12.08
Cons. ERC	3%	1	6.30	3.78	0.81	4.49	38.38
Herfindahl	3%	1	6.76	3.87	0.92	4.60	38.65
Cons. ERC	6%	6	8.15	6.64	0.74	9.78	13.73
Herfindahl	6%	6	8.60	6.67	0.81	9.78	11.74
Cons. ERC	6%	1	9.04	6.89	0.85	10.04	36.00
Herfindahl	6%	1	9.12	6.95	0.85	9.93	35.90
Cons. ERC	9%	6	9.80	9.95	0.66	17.20	12.75
Herfindahl	9%	6	9.92	9.93	0.68	16.96	9.98
Cons. ERC	9%	1	11.74	10.09	0.85	17.44	30.68
Shannon	9%	1	12.49	11.12	0.83	16.50	31.99
Cons. ERC	15%	6	10.82	15.42	0.49	29.59	8.90
Herfindahl	15%	6	10.82	15.41	0.49	29.59	8.38
Cons. ERC	15%	1	15.42	15.10	0.81	30.05	33.40
Shannon	15%	1	17.27	15.68	0.90	28.11	34.62

Table 4.6: Out of sample performance measurement of target risk portfolios. $P = 1$, 6 months. $Q = 1$ month.

Table 4.6 summarizes the outcomes of three-asset portfolios and backtesting results. Under the same volatility level and rather close maximum drawdown of different rolling-window lengths, a trade-off is made between return and turnover rate. The best strategy alternative to ERC strategy is marked bold in Table 4.6. For a low target risk (3% and 6%), Herfinhadl strategy is the choice. Regarding of turnover rate, a longer estimation window is chosen ($P = 6$). For a high target risk (9% and 15%) with $P = 1$, Shannon strategy has an outstanding performance.

4.2.2 Four-Asset Portfolio

Comparison of ERC and MD, $P = 6, 1$

Figure 4.8 and Figure 4.9 show the backtesting results. The patterns are rather similar to thos in Section 4.2.1. *Shannon* strategy has a poor performance for $P = 6$, especially for a low target risk, but has a dominant advantage for $P = 1$, especially for high target risks. *Gini and Herfindahl* strategy give a steady performance, and bring a high return for a low target risk.

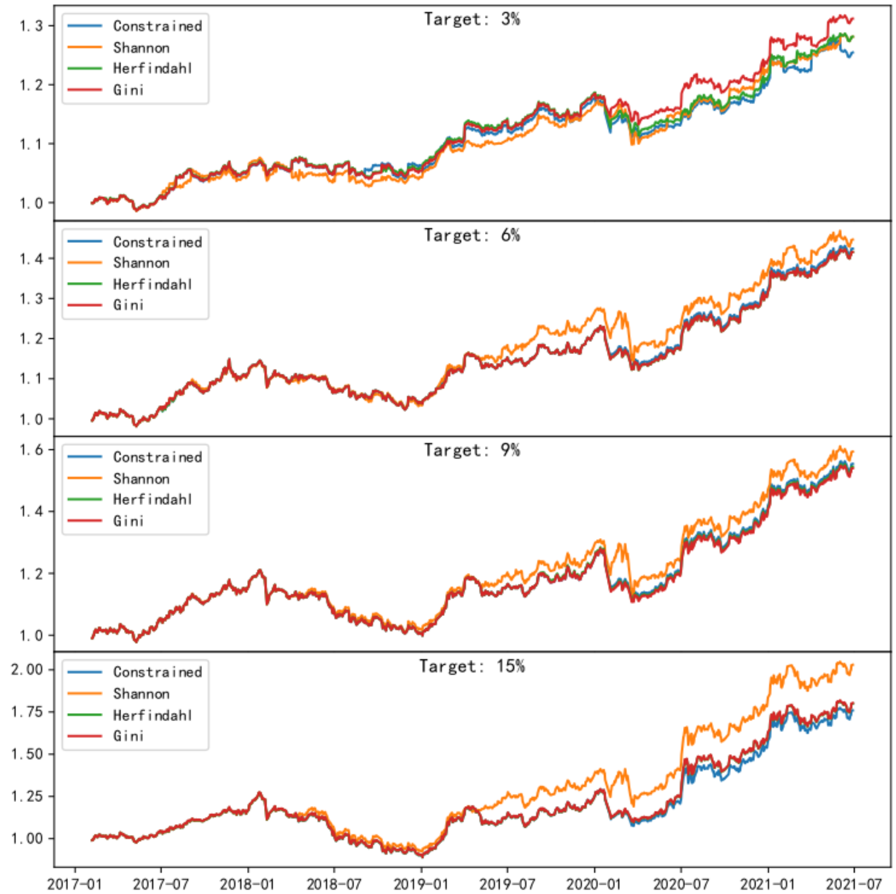

Figure 4.8: Backtesting outcomes of target risk *four-asset* portfolios from 03/07/2018 to 28/06/2021. $P = 1$ months. $Q = 1$ month.

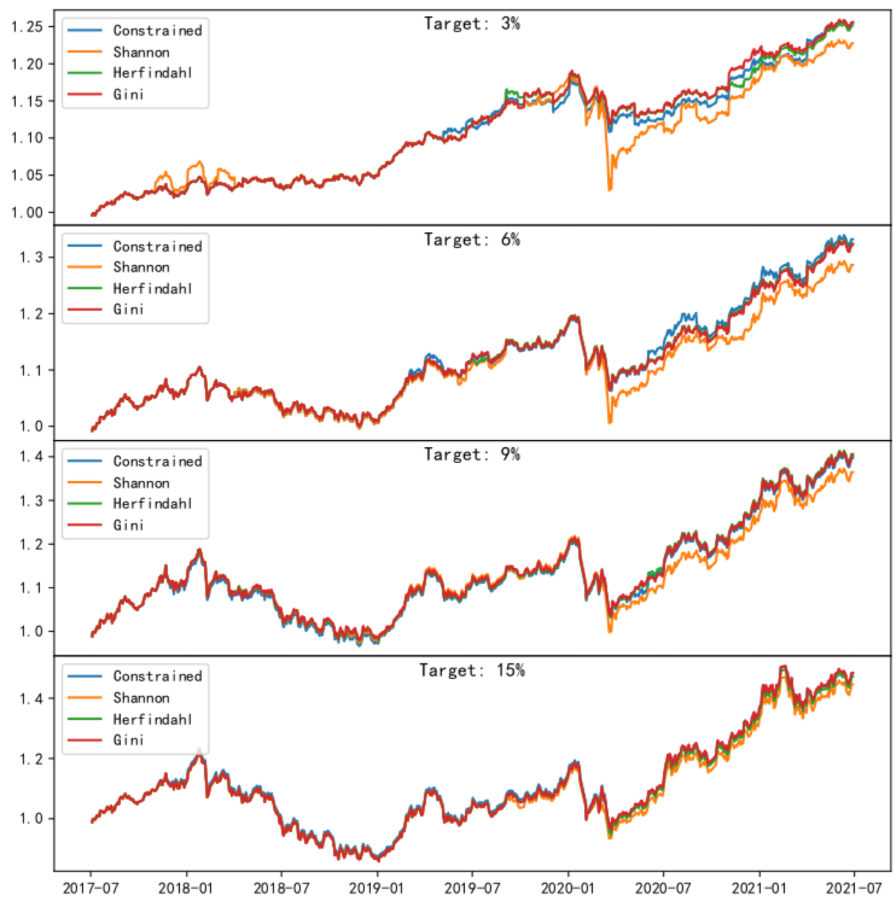

Figure 4.9: Backtesting outcomes of target risk *four-asset* portfolios from 04/02/2018 to 28/06/2021. $P = 6$ months. $Q = 1$ month.

Comparison of Three-Asset Portfolio and Four-Asset Portfolio

If a comparison is made between three-asset portfolios and four-asset portfolios, it can be seen that adding a diversified asset seems not to improve the performance, and gives a worse out-of-sample backtesting results, though the most uncorrelated asset is chosen. Results are shown in Table 4.7. One exception is that, *Shannon* four-asset strategy gives a better result than three-asset strategy. Though an improvement can be achieved by applying concentration minimization strategy, the performance of the portfolios closely depends on the selection of the asset universe.

	Target	No. Assets	Return p.a.(%)	Volatility p.a.(%)	SR	MDD (%)	Turnover (%)
ERC	3%	3	6.50	3.46	0.95	5.21	15.05
Herfindahl	3%	3	7.10	3.51	1.11	5.48	12.08
Constrained	3%	4	5.86	3.42	0.77	6.18	16.39
Gini	3%	4	6.09	3.41	0.84	6.20	12.00
ERC	6%	3	8.15	6.64	0.74	9.78	13.73
Herfindahl	3%	4	8.60	6.67	0.81	9.78	11.74
Constrained	6%	4	7.72	6.73	0.67	11.07	15.00
Gini	6%	4	7.52	6.65	0.65	11.07	10.89
ERC	9%	3	11.74	10.09	0.85	17.44	30.68
Shannon	9%	3	12.49	11.12	0.83	16.50	31.99
Constrained	9%	4	10.98	9.76	0.80	17.80	32.39
Shannon	9%	4	11.70	10.40	0.82	16.60	35.52
ERC	15%	3	15.42	15.10	0.81	30.05	33.40
Shannon	15%	3	17.27	15.68	0.90	28.11	34.62
Constrained	15%	4	14.67	15.10	0.76	30.18	39.68
Shannon	15%	4	18.10	15.37	0.97	27.47	46.18

Table 4.7: Comparisons of three-asset portfolios and four-asset portfolios

Chapter 5

Application in Portfolio Optimization

5.1 ETF Construction

Tthree-asset portfolios in Table 4.6 is used to construct FoF product. In addition, four-asset portfolios in Table 4.7 are also applied to FoF construction. In Chinese mutual fund markets, there are index fund products which not only track index but also make improvement based on index (Enhanced Index Fund, EIF), especially equity index, which generally have a better results than ETFs, according to their historical performances. For this part, ETFs are applied for illustration, and a better result may be possbly achieved if EIF is used.

For equities, two CSI 300 Index ETFs which have the highest Sharpe ratio before 01/01/2017 are chosen (ticker = 159919, 110020) and equal 1/2 weight are assigned. For bonds, there is no CSI Corporate Bond ETF in market that was established before 01/01/2017 and is being running at the current time. Hence, eight mutual LOF products of corporate bonds, which has an asset size greater than 300 million CNY (approximately 33 million GBP) are chosen. They have a rather low correlation (less than 0.2) and a 1/8 weight is assigned to each of the fund product. For commodities, one Commodity Index ETF and one gold ETF are used. (ticker = 510170, 159934). For illustration, a 3% ERC product has an average weight as shown in 5.1.

Asset Category	Asset Weight(%)	Fund Product Ticker	Product Weight(%)
Equity	10.0	159919	5.0
		110020	5.0
Bond	44.0	161115	5.5
		161119	5.5
		160618	5.5
		161716	5.5
		161216	5.5
		164210	5.5
		164509	5.5
		164703	5.5
Commodity	13.0	510170	13.0
Cash	34.0	NA	NA

Table 5.1: Average asset weights in 3% ERC FoF construction

Target	Strategy	Return p.a.(%)	Volatility p.a.(%)	SR	MDD (%)	Turnover (%)
	ERC	6.11	6.16	0.47	6.35	1.99
Conservative	ERC - 6 mons	6.50	6.33	0.52	6.01	15.28
	Herfindahl - 6 mons	7.42	6.98	0.60	8.45	12.35
Moderate	ERC - 6 mons	9.03	9.48	0.61	15.03	13.88
	Herfindahl - 6 mons	10.03	9.67	0.70	15.11	11.91
Aggressive	ERC - 1 mons	12.77	12.64	0.76	23.23	30.61
	Shannon - 1 mons	13.62	13.56	0.77	23.42	31.72
Very Aggresive	ERC - 1 mons	15.44	16.98	0.72	32.96	33.26
	Shannon - 1 mons	17.19	17.38	0.80	32.33	34.30

Table 5.2: Out-of-sample performance of three-asset FoF

Figure 5.1: Backtesting results of three-asset FoF.

5.2 ETF Performance

ERC without target risk constraint, which is of the form in 2.3.1 is the most widely used strategy. Though it cannot control portfolio risk, it is can be regarded as a benchmark and the performances of target risk strategies can be compared wit it. The out-of-sample performance are shown in B.17. Though the portfolios can control the volatility level when indices data is used, it brings a higher volatility level in practical products. In particular, realized volatility of conservative portfolio is greater than 6% and that of moderate portfolio is greater than 9%. It seems that more strategy should be investigation for risk control. The desirable point is that concentration portfolios *do improve* the return and give higher Sharpe ratios. Backtesting results for each target risk are shown in Figure B.1.

Figure 5.1 and Figure 5.2 shows the FoF strategies under each of the target risk. FoFs for satisfying different risk targets can be constructed. Compared with a naive ERC portfolio, 6%, 9% and 15% FoFs are more volatile, and more sensitive to the market, but they can achieve a higher return, which are able to satisfy expectations of investors.

Hence, risk-based approach is a feasible method for FoF construction. Second, through minimization concentration approaches, it is possible to obtain a better product than ERC approach. Through this way, *a basic but applicable and relatively desirable FoF product can be constructed* according to different risk expectations.

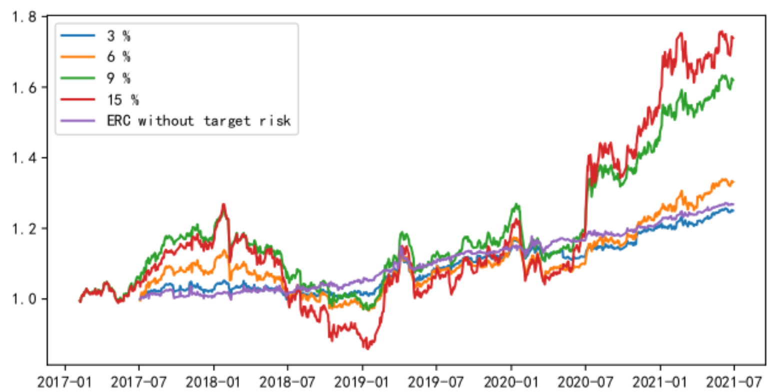

Figure 5.2: Backtesting results of four-asset FoF.

Chapter 6

Conclusion

In this thesis, a construction of target risk fund and the improvement on it are explored.

First, by comparing mean variance strategy and risk budgeting strategy, we find the several desirable properties of risk budgeting strategy: a steady weight allocation over time and a great performance comparable with mean variance portfolio in long-term investment, which justifies the following research based on risk-allocation approaches.

Second, based on investors' requirement for constant risk control in investment, we use a strategy in which a target risk constraint is imposed. It solves two problems in realistic: firstly, it realizes a portfolio with a steady risk level as expected over time; second, it gives a proper weight allocation to cash.

Third, from the point of view that higher risk diversification is associated with a better yield, four different objective functions (ERC and concentration index) to minimize risk diversification level are applied. The results show that the choice of objective functions should be related to target risk level and window length of sample data. *Herfindahl and Gini* strategies show a *steady property*, and bring 3% and 6% portfolios with desirable results. *Shannon* strategy shows an *unstable property*. It brings a great return to 9% and 15% portfolios, if just a few latest sample data (approximately 22 trading days in this dissertation) is used for risk estimation. By applying concentration indices as objective functions, improvement can be obtained if we use as ERC a benchmark portfolio. Their general patterns can be used for further investment in different assets.

Fourth, to illustrate that using indices for asset allocation is applicable in practical investment, ETF and LOF products which tracks indices or reflect the performance of indices are used to construct target risk FoF. Since the results are as expected, we regard it as a feasible way for index FoF construction.

Chapter 7

Discussion

According to section 4.2.2, the choice of assets plays an importance role. It is closely associated with the performance of strategy. Selecting it manually may not be able to get a desirable result. [8] proposed the Optimization + Risk Parity diversification strategy, which solves the problem between the selection of asset universe and maximum expected return. This can be formulated as follows,

$$\begin{aligned} x^* = \quad & \arg\max \mu(x) \\ \text{u.c.} \quad & x_i \frac{\partial \rho(x)}{\partial x_i} = \alpha y_i \\ & x_i \leq y_i \\ & y \in 0, 1^n \\ & \alpha \in \mathbb{R} \\ & i = 1, \ldots, n \end{aligned} \tag{7.0.1}$$

where $\mu(x)$ is utility function which aims to maximize an expected return, among all the solutions for satisfying risk parity. Inspired by this optimization problem, I was attempting to modify this for concentration minimization, which can be formulated as follows,

$$\begin{aligned} x^* = \quad & \arg\min \boldsymbol{C}(x) \\ \text{u.c.} \quad & \textstyle\sum_{i=1}^{n} R_i(x_i y_i) = RC^* \\ & x_i \leq y_i \\ & y \in 0, 1^n \\ i = 1, \ldots, n & \end{aligned} \tag{7.0.2}$$

It needs an efficient algorithm and I fail to realize it. Further investigation may be possible conducted based on that.

In addition, when FoF is constructed based on practical product, FoF fails to control the risk well and the realized volatility is higher than expected. That may be due to that ETFs fails to track indices closely as well as the price volatility in the secondary market. Hence, possibly adjustment can be made to risky assets for practical investment. Beside, further investigation can involve the construction of more asset categories including ABS and overseas assets to see if risk diversification can be improved and better returns can be gained.

Appendix A

Technical Proofs

A.1 Standard Quadratic Programming

According to [20], a general quadratic programming is defined as follows,

$$\begin{array}{ll} x^* = & \arg\min \frac{1}{2} x^T G x + x^T c \\ \text{u.c.} & a_i^T x = b_i, i \in \boldsymbol{\epsilon} \\ & a_i^T x \geq b_i, i \in \boldsymbol{I} \end{array} \tag{A.1.1}$$

where G is a symmetrix $n \times n$ matrix, $\boldsymbol{\epsilon}$ and $\boldsymbol{I}$ are finite sets of indices, and c, x, and a_i, $i \in \boldsymbol{\epsilon} \cup \boldsymbol{I}$, are vectors in R^n.

Especially, we set $c = \mathbf{0}$. For $i \in \boldsymbol{\epsilon}$, there is one constraint that $a_i = \mathbf{1}$, $b_i = 1$, a_i, and for $i \in \boldsymbol{I}$, let $a_i = (\mu_1, ..., \mu_N)$ which is the expected return of N assets, and $b_i = \mu^*$, which is the minimum return we want to achieve. The optimization problem is then of the same form of 2.2.1. If G is positive semidefinite, it is an *convex* QP. Since G is assigned to be the covariance matrix of asset returns, which is positive semidefinite, hence 2.2.1 is an *convex* QP.

A.2 Target risk approach in convex optimization problem form

If volatility is used for measuring risk, and let target risk be σ^*, variance-covariance matrix be Σ, 2.4.1 can be reformulated as follows:

$$\begin{array}{ll} x^* = & \arg\min \sum_{i=1}^{N} (x_i \frac{(\Sigma x)_i}{\sigma^*} - b_i \sigma^*)^2 \\ \text{u.c.} & \boldsymbol{x} \geq 0 \\ & \sqrt{x^T \Sigma x} = \sigma^* \end{array} \tag{A.2.1}$$

The objective function is equivalent to minimize the following formula

$$\sum_{i=1}^{N} (x_i (\Sigma x)_i - b_i {\sigma^*}^2)^2$$

It can be written in matrix notation:

$$||x \cdot (\Sigma x) - \sigma^{*2}\boldsymbol{b}||_2^2$$

According to [9], every norm on R^n is convex. And since Σ is positive semidefinite matrix, $x^T \Sigma x$ is convex. Hence, the target programming problem can be rewritten as

$$\begin{aligned} x^* = \quad & \arg\min ||x \cdot (\Sigma x) - \sigma^{*2}\boldsymbol{b}||_2^2 \\ \text{u.c.} \quad & \boldsymbol{x} \geq 0 \\ & x^T \Sigma x = \sigma^{*2} \end{aligned} \tag{A.2.2}$$

which is a convex optimization programming as the form of 2.4.2.

A.3 Convexity of Herfindahl Index and Shannon Index

Herfindahl index is expressed as follows,

$$\boldsymbol{H}(\pi) = \sum_{i=1}^{n} \pi_i^2$$

The convexity of $f_i(\pi_i) = \pi_i^2$ is easily obtained by taking the second order derivative, that is, $f_i^{''}(\pi_i) = 2$, which is greater than 0. According to [9], the weighted sum of convex functions is convex. Hence, $\boldsymbol{H} = \sum_{i=1}^{n} f_i(\pi_i)$ is convex.

The Shannon entropy is defined as follows,

$$\boldsymbol{I}(\pi) = -\sum_{i=1}^{n} \pi_i ln \pi_i$$

The diversity index corresponds to the statistic

$$\boldsymbol{I}^*(\pi) = exp(\boldsymbol{I}(\pi))$$

. Similarly, the concavity of $f_i(\pi_i) = -\pi_i ln \pi_i$ can be obtained by taking the second order derivative. Since $f_i^{''}(\pi_i) = -1 < 0$, $f_i(\pi_i)$ is concave, and the sum of concave function is concave. $g(x) = -exp(x)$ is a *concave and non-increasing* function. According to [9], since $\boldsymbol{I}(\pi)$ is convex, only if g is *concave and non-decreasing*, $g(\boldsymbol{I}(\pi))$ is concave. Neither the concavity of Shannon Index nor the convexity of it can be determined.

Appendix B

Full Results

B.1 Three-Asset

B.1.1 Volatility

	Target	CSI 300 Index	CSI Aggregate Bond	Gold Index	Cash
Cons. ERC	3%	9.0	34.0	13.0	44.0
Uncons. ERC	3%	9.0	44.0	13.0	34.0
Shannon	3%	10.0	66.0	13.0	11.0
Herfindahl	3%	9.0	54.0	13.0	24.0
Gini	3%	9.0	54.0	13.0	24.0
Cons. ERC	6%	21.0	24.0	20.0	34.0
Uncons. ERC	6%	21.0	44.0	20.0	15.0
Shannon	6%	22.0	33.0	21.0	24.0
Herfindahl	6%	21.0	29.0	20.0	30.0
Gini	6%	21.0	26.0	20.0	33.0
Cons. ERC	9%	37.0	16.0	20.0	27.0
Uncons. ERC	9%	37.0	44.0	20.0	-1.0
Shannon	9%	36.0	18.0	20.0	26.0
Herfindahl	9%	37.0	15.0	20.0	28.0
Gini	9%	37.0	16.0	20.0	27.0
Cons. ERC	15%	65.0	2.0	18.0	15.0
Uncons. ERC	15%	66.0	35.0	20.0	-21.0
Shannon	15%	63.0	4.0	19.0	14.0
Herfindahl	15%	65.0	2.0	18.0	15.0
Gini	15%	65.0	2.0	18.0	15.0

Table B.1: Average weights (%) of target risk portfolios from 02/01/2018 to 28/06/2021. $P = 12$ months. $Q = 1$ month.

	Target	Shannon Index (-3.00)		Herfindahl Index (0.33)		Gini Index (0.00)	
		In	Out	In	Out	In	Out
Cons. ERC	3%	-2.03	-1.97	0.53	0.55	0.40	0.42
Uncons. ERC	3%	-2.07	-2.00	0.52	0.55	0.39	0.41
Shannon	3%	-2.02	-2.00	0.57	0.58	0.42	0.43
Herfindahl	3%	-2.08	-2.04	0.53	0.56	0.39	0.42
Gini	3%	-2.08	-2.04	0.53	0.56	0.39	0.42
Cons. ERC	6%	-1.82	-1.82	0.61	0.61	0.47	0.47
Uncons. ERC	6%	-1.87	-1.85	0.60	0.61	0.46	0.47
Shannon	6%	-1.82	-1.83	0.61	0.61	0.47	0.47
Herfindahl	6%	-1.83	-1.82	0.61	0.61	0.47	0.47
Gini	6%	-1.82	-1.83	0.61	0.61	0.47	0.47
Cons. ERC	9%	-1.61	-1.61	0.70	0.71	0.53	0.53
Uncons. ERC	9%	-1.67	-1.65	0.70	0.71	0.53	0.53
Shannon	9%	-1.58	-1.60	0.71	0.71	0.53	0.53
Herfindahl	9%	-1.61	-1.61	0.70	0.71	0.53	0.53
Gini	9%	-1.61	-1.61	0.70	0.71	0.53	0.53
Cons. ERC	15%	-1.37	-1.36	0.82	0.83	0.60	0.60
Uncons. ERC	15%	-1.42	-1.40	0.82	0.82	0.59	0.60
Shannon	15%	-1.36	-1.38	0.81	0.82	0.59	0.59
Herfindahl	15%	-1.37	-1.36	0.82	0.83	0.60	0.60
Gini	15%	-1.37	-1.36	0.82	0.83	0.60	0.60

Table B.2: Average value of concentration indices of target risk portfolios from 04/01/2017 to 28/06/2021. In: in-sample value. Out: out-of-sample value. Expected value of Herfindahl Index, Gini Index, Shannnon Index is 0.33, 0.00, -3.00 respectively. $P = 12$ months. $Q = 1$ month.

	Target	Return p.a. (%)	Volatility p.a. (%)	SR	MDD (%)	Turnover (%)
Cons. ERC	3%	6.16	3.38	0.87	4.72	7.11
Uncons. ERC	3%	6.70	3.41	1.02	4.72	8.52
Shannon	3%	4.77	4.92	0.32	10.33	18.64
Herfindahl	3%	6.87	3.35	1.09	3.81	5.29
Gini	3%	6.80	3.29	1.09	3.64	5.74
Cons. ERC	6%	6.96	6.70	0.56	10.13	6.63
Uncons. ERC	6%	8.05	6.68	0.72	8.39	9.42
Shannon	6%	6.68	7.57	0.46	13.15	13.07
Herfindahl	6%	7.23	6.71	0.60	10.13	6.66
Gini	6%	7.06	6.76	0.57	10.13	9.48
Cons. ERC	9%	7.09	10.15	0.38	20.02	6.96
Uncons. ERC	9%	8.67	10.08	0.54	16.44	10.62
Shannon	9%	6.78	10.25	0.35	20.02	14.03
Herfindahl	9%	7.05	10.17	0.38	20.02	6.98
Gini	9%	7.09	10.15	0.38	20.02	6.96
Cons. ERC	15%	8.06	16.35	0.30	32.25	5.38
Uncons. ERC	15%	9.67	16.71	0.39	30.31	21.14
Shannon	15%	7.47	15.90	0.27	32.22	9.59
Herfindahl	15%	8.06	16.34	0.30	32.22	4.99
Gini	15%	8.06	16.34	0.30	32.22	4.99

Table B.3: Out of sample performance measurement of target risk portfolios from 02/01/2018 to 28/06/2021. $P = 12$ months. $Q = 1$ month.

	Target	CSI 300 Index	CSI Aggregate Bond	Gold Index	Cash
Cons. ERC	3%	10.0	44.0	13.0	34.0
Uncons. ERC	3%	10.0	57.0	13.0	21.0
Shannon	3%	11.0	67.0	13.0	9.0
Herfindahl	3%	10.0	58.0	13.0	19.0
Gini	3%	10.0	58.0	13.0	19.0
Cons. ERC	6%	23.0	29.0	20.0	28.0
Uncons. ERC	6%	23.0	53.0	20.0	4.0
Shannon	6%	24.0	37.0	21.0	18.0
Herfindahl	6%	23.0	34.0	20.0	23.0
Gini	6%	23.0	32.0	20.0	25.0
Cons. ERC	9%	41.0	15.0	20.0	24.0
Uncons. ERC	9%	40.0	47.0	20.0	-7.0
Shannon	9%	41.0	17.0	21.0	22.0
Herfindahl	9%	41.0	18.0	20.0	22.0
Gini	9%	40.0	16.0	20.0	24.0
Cons. ERC	15%	64.0	3.0	20.0	13.0
Uncons. ERC	15%	69.0	42.0	22.0	-33.0
Shannon	15%	62.0	5.0	21.0	12.0
Herfindahl	15%	64.0	2.0	20.0	14.0
Gini	15%	64.0	2.0	20.0	14.0

Table B.4: Average weights (%) of target risk portfolios from 03/07/2017 to 28/06/2021. $P = 6$ months. $Q = 1$ month.

	Target	Shannon Index (-3.00)		Herfindahl Index (0.33)		Gini Index (0.00)	
		In	Out	In	Out	In	Out
Cons. ERC	3%	-2.10	-2.03	0.51	0.53	0.37	0.40
Uncons. ERC	3%	-2.16	-2.09	0.50	0.52	0.35	0.38
Shannon	3%	-2.05	-2.01	0.55	0.57	0.40	0.41
Herfindahl	3%	-2.15	-2.08	0.51	0.53	0.37	0.39
Gini	3%	-2.16	-2.10	0.51	0.53	0.37	0.39
Cons. ERC	6%	-1.87	-1.83	0.59	0.60	0.45	0.47
Uncons. ERC	6%	-1.93	-1.86	0.58	0.60	0.44	0.46
Shannon	6%	-1.87	-1.85	0.59	0.60	0.45	0.47
Herfindahl	6%	-1.87	-1.84	0.59	0.60	0.45	0.47
Gini	6%	-1.87	-1.84	0.59	0.60	0.45	0.46
Cons. ERC	9%	-1.63	-1.61	0.70	0.70	0.53	0.53
Uncons. ERC	9%	-1.69	-1.66	0.69	0.70	0.52	0.52
Shannon	9%	-1.62	-1.62	0.70	0.70	0.52	0.52
Herfindahl	9%	-1.64	-1.62	0.69	0.70	0.52	0.52
Gini	9%	-1.64	-1.63	0.69	0.69	0.52	0.52
Cons. ERC	15%	-1.44	-1.43	0.79	0.79	0.57	0.58
Uncons. ERC	15%	-1.48	-1.45	0.78	0.79	0.57	0.58
Shannon	15%	-1.45	-1.46	0.77	0.78	0.56	0.57
Herfindahl	15%	-1.44	-1.43	0.79	0.79	0.57	0.58
Gini	15%	-1.44	-1.43	0.79	0.79	0.57	0.58

Table B.5: Average value of concentration indices of target risk portfolios from 04/01/2017 to 28/06/2021. In: in-sample value. Out: out-of-sample value. Expected value of Herfindahl Index, Gini Index, Shannnon Index is 0.33, 0.00, -3.00 respectively. $P = 6$ months. $Q = 1$ month.

	Target	Return p.a. (%)	Volatility p.a. (%)	SR	MDD (%)	Turnover (%)
Cons. ERC	3%	6.50	3.46	0.95	5.21	15.05
Uncons. ERC	3%	6.85	3.55	1.02	5.19	18.66
Shannon	3%	5.55	4.92	0.47	11.20	14.99
Herfindahl	3%	7.10	3.51	1.11	5.48	12.08
Gini	3%	6.80	3.50	1.02	5.38	15.55
Cons. ERC	6%	8.15	6.64	0.74	9.78	13.73
Uncons. ERC	6%	8.66	6.67	0.82	9.78	20.16
Shannon	6%	8.09	7.37	0.66	14.57	16.48
Herfindahl	6%	8.60	6.67	0.81	9.78	11.74
Gini	6%	8.50	6.68	0.79	9.78	9.20
Cons. ERC	9%	9.80	9.95	0.66	17.20	12.75
Uncons. ERC	9%	10.66	9.94	0.75	14.60	19.68
Shannon	9%	9.36	10.14	0.61	17.20	17.14
Herfindahl	9%	9.92	9.93	0.68	16.96	9.98
Gini	9%	9.83	9.95	0.66	17.11	9.85
Cons. ERC	15%	10.82	15.42	0.49	29.59	8.90
Uncons. ERC	15%	13.20	16.11	0.62	27.20	24.91
Shannon	15%	10.73	14.79	0.51	29.52	14.78
Herfindahl	15%	10.82	15.41	0.49	29.59	8.38
Gini	15%	10.82	15.41	0.49	29.59	8.38

Table B.6: Out of sample performance measurement of target risk portfolios from 03/07/2017 to 28/06/2021. $P = 6$ months. $Q = 1$ month.

	Target	CSI 300 Index	CSI Aggregate Bond	Gold Index	Cash
Cons. ERC	3%	10.0	45.0	13.0	32.0
Uncons. ERC	3%	10.0	60.0	13.0	17.0
Shannon	3%	12.0	58.0	14.0	16.0
Herfindahl	3%	11.0	50.0	13.0	27.0
Gini	3%	11.0	53.0	13.0	24.0
Cons. ERC	6%	25.0	29.0	19.0	27.0
Uncons. ERC	6%	24.0	56.0	19.0	0.0
Shannon	6%	26.0	35.0	21.0	18.0
Herfindahl	6%	25.0	32.0	20.0	23.0
Gini	6%	25.0	31.0	20.0	24.0
Cons. ERC	9%	43.0	16.0	20.0	21.0
Uncons. ERC	9%	43.0	56.0	20.0	-19.0
Shannon	9%	43.0	19.0	21.0	17.0
Herfindahl	9%	43.0	17.0	20.0	20.0
Gini	9%	43.0	16.0	20.0	21.0
Cons. ERC	15%	66.0	5.0	19.0	10.0
Uncons. ERC	15%	71.0	56.0	22.0	-48.0
Shannon	15%	65.0	7.0	19.0	8.0
Herfindahl	15%	66.0	5.0	19.0	10.0
Gini	15%	66.0	5.0	19.0	10.0

Table B.7: Average weights (%) of target risk portfolios from 05/04/2017 to 28/06/2021. $P = 3$ month. $Q = 1$ month.

	Target	Shannon Index (-3.00)		Herfindahl Index (0.33)		Gini Index (0.00)	
		In	Out	In	Out	In	Out
Cons. ERC	3%	-2.15	-2.06	0.50	0.53	0.36	0.40
Uncons. ERC	3%	-2.24	-2.13	0.48	0.52	0.33	0.39
Shannon	3%	-2.19	-2.08	0.50	0.54	0.36	0.40
Herfindahl	3%	-2.17	-2.07	0.50	0.53	0.35	0.40
Gini	3%	-2.18	-2.10	0.49	0.53	0.35	0.39
Cons. ERC	6%	-1.86	-1.83	0.59	0.61	0.45	0.47
Uncons. ERC	6%	-1.94	-1.89	0.58	0.60	0.43	0.46
Shannon	6%	-1.86	-1.83	0.60	0.61	0.45	0.47
Herfindahl	6%	-1.87	-1.83	0.59	0.61	0.45	0.47
Gini	6%	-1.87	-1.82	0.59	0.61	0.45	0.47
Cons. ERC	9%	-1.65	-1.63	0.69	0.70	0.52	0.53
Uncons. ERC	9%	-1.69	-1.67	0.69	0.70	0.52	0.53
Shannon	9%	-1.64	-1.64	0.69	0.70	0.52	0.52
Herfindahl	9%	-1.65	-1.63	0.69	0.70	0.52	0.53
Gini	9%	-1.64	-1.63	0.69	0.70	0.52	0.53
Cons. ERC	15%	-1.44	-1.43	0.79	0.79	0.57	0.57
Uncons. ERC	15%	-1.50	-1.48	0.78	0.79	0.57	0.58
Shannon	15%	-1.43	-1.45	0.78	0.78	0.57	0.57
Herfindahl	15%	-1.43	-1.43	0.79	0.79	0.57	0.57
Gini	15%	-1.43	-1.43	0.79	0.79	0.57	0.57

Table B.8: Average value of concentration indices of target risk portfolios from 04/01/2017 to 28/06/2021. In: in-sample value. Out: out-of-sample. Expected value of Herfindahl Index, Gini Index, Shannnon Index is 0.33, 0.00, -3.00 respectively. $P = 3$ month. $Q = 1$ month.

	Target	Return p.a. (%)	Volatility p.a.(%)	SR	MDD (%)	Turnover (%)
Cons. ERC	3%	5.95	3.43	0.80	4.18	23.24
Uncons. ERC	3%	6.11	3.51	0.83	3.50	28.82
Shannon	3%	5.24	4.96	0.41	11.75	21.50
Herfindahl	3%	6.17	3.44	0.86	4.39	20.70
Gini	3%	6.27	3.42	0.89	4.31	20.67
Cons. ERC	6%	8.22	6.61	0.76	9.31	22.22
Uncons. ERC	6%	8.50	6.61	0.80	8.24	34.34
Shannon	6%	8.18	7.57	0.66	14.85	26.67
Herfindahl	6%	8.33	6.61	0.77	9.57	20.34
Gini	6%	8.35	6.62	0.78	9.45	22.51
Cons. ERC	9%	10.85	9.99	0.76	15.92	20.46
Uncons. ERC	9%	11.13	9.95	0.79	13.51	37.03
Shannon	9%	10.55	10.39	0.71	17.78	24.28
Herfindahl	9%	10.90	10.00	0.77	15.92	20.51
Gini	9%	10.83	9.98	0.76	16.16	20.48
Cons. ERC	15%	14.47	15.48	0.73	29.53	20.84
Uncons. ERC	15%	14.85	15.81	0.74	26.22	40.38
Shannon	15%	13.82	15.32	0.69	29.45	21.60
Herfindahl	15%	14.22	15.47	0.71	29.45	18.97
Gini	15%	14.22	15.47	0.71	29.48	18.75

Table B.9: Out of sample performance measurement of target risk portfolios from 05/04/2017 to 28/06/2021. $P = 3$ month. $Q = 1$ month.

	Target	CSI 300 Index	CSI Aggregate Bond	Gold Index	Cash
Cons. ERC	3%	12.0	47.0	13.0	29.0
Uncons. ERC	3%	11.0	62.0	13.0	13.0
Shannon	3%	14.0	58.0	15.0	12.0
Herfindahl	3%	12.0	56.0	13.0	19.0
Gini	3%	12.0	51.0	13.0	24.0
Cons. ERC	6%	27.0	28.0	19.0	25.0
Uncons. ERC	6%	27.0	57.0	19.0	-3.0
Shannon	6%	29.0	36.0	21.0	14.0
Herfindahl	6%	28.0	30.0	19.0	23.0
Gini	6%	28.0	30.0	19.0	23.0
Cons. ERC	9%	44.0	16.0	20.0	20.0
Uncons. ERC	9%	46.0	57.0	20.0	-23.0
Shannon	9%	45.0	21.0	22.0	12.0
Herfindahl	9%	44.0	16.0	20.0	20.0
Gini	9%	44.0	15.0	20.0	20.0
Cons. ERC	15%	69.0	6.0	17.0	8.0
Uncons. ERC	15%	71.0	56.0	23.0	-50.0
Shannon	15%	67.0	9.0	18.0	6.0
Herfindahl	15%	69.0	6.0	17.0	8.0
Gini	15%	69.0	6.0	17.0	8.0

Table B.10: Average weights (%) of target risk portfolios from 03/02/2017 to 28/06/2021. $P = 1$ month. $Q = 1$ month.

	Target	Return p.a. (%)	Volatility p.a.(%)	SR	MDD (%)	Turnover (%)
Cons. ERC	3%	6.30	3.78	0.81	4.49	38.38
Uncons. ERC	3%	6.42	3.98	0.81	4.08	47.19
Shannon	3%	6.93	4.30	0.87	6.48	31.94
Herfindahl	3%	6.76	3.87	0.92	4.60	38.65
Gini	3%	6.52	3.84	0.86	4.57	39.36
Cons. ERC	6%	9.04	6.89	0.85	10.04	36.00
Uncons. ERC	6%	9.09	6.99	0.84	8.80	50.63
Shannon	6%	9.89	7.83	0.85	11.08	39.27
Herfindahl	6%	9.12	6.95	0.85	9.93	35.90
Gini	6%	9.15	6.93	0.86	9.93	35.98
Cons. ERC	9%	11.74	10.09	0.85	17.44	30.68
Uncons. ERC	9%	12.27	10.29	0.88	16.12	54.83
Shannon	9%	12.49	11.12	0.83	16.50	31.99
Herfindahl	9%	11.68	10.10	0.84	17.44	30.56
Gini	9%	11.73	10.09	0.84	17.47	30.63
Cons. ERC	15%	15.42	15.10	0.81	30.05	33.40
Uncons. ERC	15%	17.95	15.63	0.94	27.97	64.04
Shannon	15%	17.27	15.68	0.90	28.11	34.62
Herfindahl	15%	15.62	15.16	0.82	30.05	32.30
Gini	15%	15.62	15.16	0.82	30.05	32.30

Table B.12: Out of sample performance measurement of target risk portfolios from 03/02/2017 to 28/06/2021. $P = 1$ month. $Q = 1$ month.

	Target	Shannon Index (-3.00)		Herfindahl Index (0.33)		Gini Index (0.00)	
		In	Out	In	Out	In	Out
Cons. ERC	3%	-2.22	-2.07	0.48	0.54	0.31	0.40
Uncons. ERC	3%	-2.31	-2.15	0.46	0.53	0.29	0.39
Shannon	3%	-2.26	-2.12	0.48	0.53	0.32	0.39
Herfindahl	3%	-2.24	-2.13	0.48	0.53	0.30	0.39
Gini	3%	-2.24	-2.11	0.48	0.53	0.30	0.40
Cons. ERC	6%	-1.87	-1.83	0.59	0.60	0.43	0.45
Uncons. ERC	6%	-1.96	-1.89	0.57	0.59	0.42	0.45
Shannon	6%	-1.86	-1.84	0.59	0.60	0.44	0.46
Herfindahl	6%	-1.87	-1.84	0.59	0.60	0.43	0.45
Gini	6%	-1.87	-1.84	0.59	0.60	0.43	0.45
Cons. ERC	9%	-1.65	-1.64	0.68	0.69	0.51	0.52
Uncons. ERC	9%	-1.72	-1.67	0.67	0.69	0.50	0.52
Shannon	9%	-1.64	-1.65	0.68	0.68	0.51	0.52
Herfindahl	9%	-1.65	-1.64	0.68	0.69	0.51	0.52
Gini	9%	-1.65	-1.64	0.68	0.69	0.51	0.52
Cons. ERC	15%	-1.42	-1.41	0.80	0.81	0.57	0.58
Uncons. ERC	15%	-1.50	-1.49	0.77	0.79	0.56	0.57
Shannon	15%	-1.39	-1.43	0.79	0.79	0.57	0.57
Herfindahl	15%	-1.40	-1.40	0.80	0.81	0.58	0.58
Gini	15%	-1.40	-1.40	0.80	0.81	0.58	0.58

Table B.11: Average value of concentration indices of target risk portfolios from 04/01/2017 to 28/06/2021. In: in-sample value. Out: out-of-sample. Expected value of Herfindahl Index, Gini Index, Shannnon Index is 0.33, 0.00, -3.00 respectively. $P = 1$ month. $Q = 1$ month.

B.1.2 Using VaR and ES as Risk Measure

	Target(%)	Return p.a.(%)	Volatility p.a.(%)	SR	MDD (%)	Turnover (%)
ERC	0.31	7.14	3.79	1.04	3.75	73.22
Shannon	0.31	7.80	6.89	0.67	8.70	47.31
Herfindahl	0.31	6.99	5.30	0.71	7.45	58.97
Gini	0.31	7.16	4.68	0.84	6.57	66.99
ERC	0.64	10.10	7.46	0.92	11.40	56.02
Shannon	0.64	7.08	8.10	0.48	12.07	36.65
Herfindahl	0.64	9.50	8.03	0.78	11.88	50.34
Gini	0.64	9.40	8.10	0.76	11.88	59.82
ERC	0.95	10.12	11.27	0.61	18.64	42.33
Shannon	0.95	6.98	10.55	0.36	17.66	43.00
Herfindahl	0.95	10.10	11.60	0.59	20.08	48.24
Gini	0.95	8.87	11.47	0.49	20.39	48.38
ERC	1.5	10.37	15.72	0.45	27.85	24.86
Shannon	1.5	7.16	15.23	0.26	28.14	38.56
Herfindahl	1.5	10.91	15.95	0.48	28.08	29.24
Gini	1.5	10.53	16.03	0.46	29.07	24.61

Table B.13: Out of sample performance of target risk portfolios using $\text{VaR}_{0.95}$ from 04/07/2017 to 28/06/2021. $P = 6$. $Q = 1$

	Target	Return p.a.(%)	Volatility p.a.(%)	SR	MDD (%)	Turnover (%)
ERC	0.54	6.83	4.55	0.79	7.09	58.83
Shannon	0.54	9.84	8.29	0.80	10.56	36.74
Herfindahl	0.54	7.03	4.75	0.80	7.40	57.71
Gini	0.54	7.02	4.75	0.80	7.43	58.81
ERC	1.03	9.62	8.45	0.76	12.32	44.12
Shannon	1.03	11.92	9.28	0.94	12.21	38.57
Herfindahl	1.03	9.80	8.46	0.78	12.32	44.51
Gini	1.03	9.74	8.46	0.77	12.64	44.83
ERC	1.51	10.24	11.99	0.59	21.92	31.32
Shannon	1.51	11.77	10.56	0.81	16.49	30.16
Herfindahl	1.51	10.37	11.98	0.60	21.92	31.78
Gini	1.51	10.36	11.98	0.60	21.92	31.83
ERC	2.34	11.85	16.93	0.51	30.56	14.09
Shannon	2.34	12.03	12.58	0.70	18.96	17.92
Herfindahl	2.34	11.85	16.93	0.51	30.56	14.09
Gini	2.34	11.85	16.93	0.51	30.56	14.09

Table B.14: Out of sample performance of target risk portfolios using $\text{ES}_{0.95}$ from 04/07/2017 to 28/06/2021. $P = 6$. $Q = 1$

B.2 Four-Asset

	Target	Return p.a.(%)	Volatility p.a.(%)	SR	MDD (%)	Turnover (%)
ERC	3%	5.44	3.94	0.56	6.08	42.27
Shannon	3%	5.81	4.03	0.64	6.32	46.59
Herfindahl	3%	5.76	3.81	0.67	6.09	34.17
Gini	3%	6.31	3.72	0.83	4.42	41.77
ERC	6%	8.50	6.74	0.78	10.97	40.32
Shannon	6%	9.04	7.30	0.80	10.56	40.58
Herfindahl	6%	8.37	6.81	0.76	10.97	40.36
Gini	6%	8.36	6.80	0.76	10.97	40.33
ERC	9%	10.98	9.76	0.80	17.80	32.39
Shannon	9%	11.70	10.40	0.82	16.60	35.52
Herfindahl	9%	10.85	9.87	0.77	17.80	32.85
Gini	9%	10.77	9.89	0.76	17.80	32.36
ERC	15%	14.67	15.10	0.76	30.18	39.68
Shannon	15%	18.10	15.37	0.97	27.47	46.18
Herfindahl	15%	15.18	14.96	0.80	30.18	36.58
Gini	15%	15.23	14.96	0.80	30.18	36.31

Table B.15: Out of sample performance measurement of four-asset target risk portfoliio from 03/02/2017 to 28/06/2021. $P = 1$ month. $Q = 1$ month.

	Target	Return p.a.(%)	Volatility p.a.(%)	SR	MDD (%)	Turnover (%)
ERC	3%	5.86	3.42	0.77	6.18	16.39
Shannon	3%	5.40	5.00	0.44	12.83	21.63
Herfindahl	3%	6.00	3.37	0.83	6.20	8.62
Gini	3%	6.09	3.41	0.84	6.20	12.00
ERC	6%	7.72	6.73	0.67	11.07	15.00
Shannon	6%	6.85	7.24	0.50	15.75	15.25
Herfindahl	6%	7.56	6.64	0.66	11.07	13.02
Gini	6%	7.52	6.65	0.65	11.07	10.89
ERC	9%	9.36	10.07	0.61	18.08	14.49
Shannon	9%	8.77	10.08	0.55	18.08	15.76
Herfindahl	9%	9.52	10.03	0.63	17.97	11.55
Gini	9%	9.46	10.03	0.62	17.80	12.81
ERC	15%	11.68	15.51	0.55	29.91	10.91
Shannon	15%	10.96	14.90	0.52	29.90	15.24
Herfindahl	15%	11.50	15.49	0.53	29.92	9.57
Gini	15%	11.71	15.42	0.55	29.92	9.74

Table B.16: Out of sample performance measurement of four-asset target risk portfoliio from 04/07/2017 to 28/06/2021. $P = 6$ month. $Q = 1$ month.

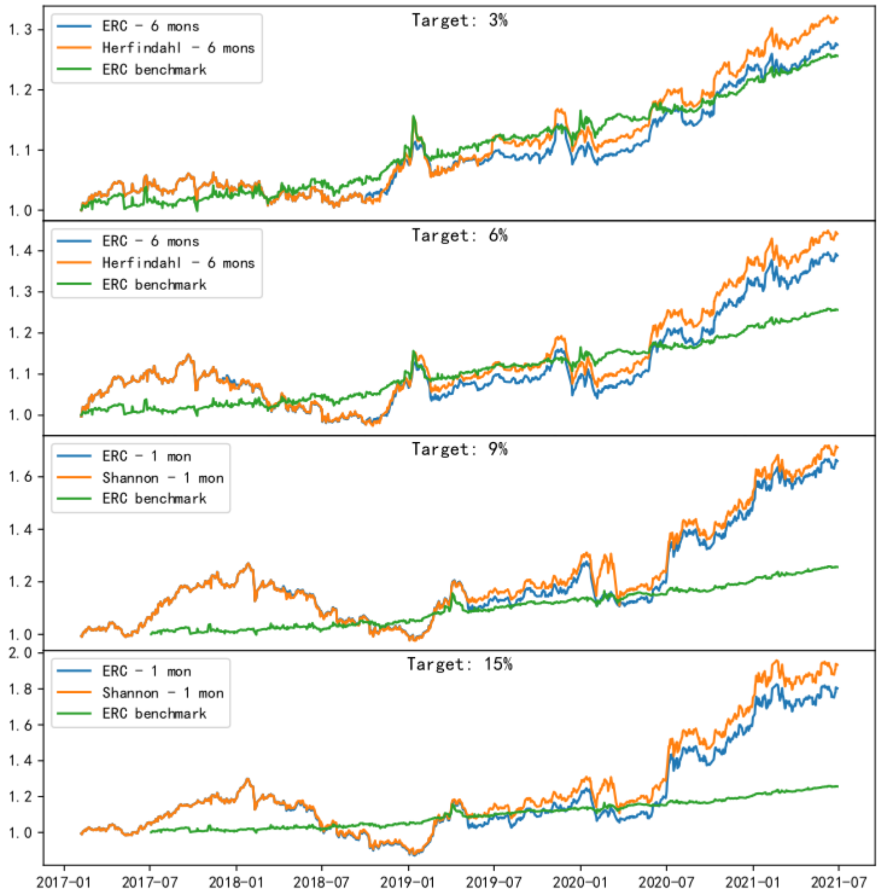

Figure B.1: Backtesting results of three-asset FoF

B.3 Fund of fund products

	Return p.a.(%)	Volatility p.a.(%)	SR	MDD (%)	Turnover (%)
ERC - 6 mons	6.50	6.33	0.52	6.01	15.28
Herfindahl - 6 mons	7.42	6.98	0.60	8.45	12.35
ERC - 6 mons	9.03	9.48	0.61	15.03	13.88
Herfindahl - 6 mons	10.03	9.67	0.70	15.11	11.91
ERC - 1 mons	12.77	12.64	0.76	23.23	30.61
Shannon - 1 mons	13.62	13.56	0.77	23.42	31.72
ERC - 1 mons	15.44	16.98	0.72	32.96	33.26
Shannon - 1 mons	17.19	17.38	0.80	32.33	34.30

Table B.17: Out of sample performance of three-asset FoF

	Return p.a.(%)	Volatility p.a.(%)	SR	MDD (%)	Turnover (%)
ERC - 6 mons	5.83	6.15	0.43	7.56	36.17
Gini - 6 mons	6.03	6.31	0.45	7.76	23.15
ERC - 6 mons	8.13	9.02	0.55	15.18	38.28
Gini - 6 mons	7.80	9.21	0.50	15.06	26.40
ERC - 1 mons	12.25	11.95	0.76	22.97	32.31
Shannon - 1 mons	12.13	12.05	0.74	22.97	32.78
ERC - 1 mons	14.28	16.60	0.67	32.39	39.61
Shannon - 1 mons	14.51	16.44	0.69	32.39	36.45

Table B.18: Out of sample performance of four-asset FoF

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FINAL GRADE

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GENERAL COMMENTS

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