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Bootstrapping Past Inflation Returns

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Declaration

The work contained in this thesis is my own work unless otherwise stated.

Abstract

With the resurgence of sustained inflation for the first time in many decades, investors are now faced with the question of how to allocate assets during economic scenarios having little to no recent data, and a scarcity of first-hand experience. A key question for investors is whether long term inflation over the next few years may be accompanied with by stagnation, or interest rate rises, and what these interactions will mean for the wider economy. While it is impossible to predict with high certainty what economic scenarios may arise, understanding what happened in past periods which had similar economic conditions can provide valuable insight into the actions we need to take today. In this thesis, we propose a simple framework for analyzing the past returns of certain assets under defined economic scenarios. We consider a number of methods for applying statistical inference to autocorrelated financial time series and apply these to a number of pressing inflation scenarios today.

Acknowledgements

I would like to thank the following people for the help they provided me with this thesis:

- George Lagarias, Chief Economist at Mazars, for devising the research topic and providing guidance with the financial and economic aspects of this thesis.
- Etienne Segalen, Quantitative Analyst at Mazars for his help in the technical aspects of this thesis, including implementation and statistical foundation.
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Introduction

The work carried out during this thesis was completed with the Wealth Management Team at Mazars UK with support and guidance from the Quantitative Solutions team. The Wealth Management Team at Mazars holds a portfolio for its clients, which is managed by an investment team that makes strategic and tactical asset allocation decisions every quarter. In the recent years, the team has been interested in introducing more advanced quantitative and data driven perspectives on top of their existing fundamental and econometric work, and this is what led to this project. A key question for the investment team this year has been how to allocate assets as inflation rose to historic levels during the past year, and determining what we could learn from past time periods when similar rates of inflation were observed.

In addition to simply calculating historical returns, it is important to the investment team to understand the statistical significance of results: if one asset class records positive returns during past inflationary periods, is this trustworthy? Is there enough data to make a strong judgement or is there too much uncertainty in the estimation of parameters?

Aside from obtaining the results themselves, this project was designed for one additional purpose: To determine whether having a quantitative analyst on the Mazars Wealth Management investment team to assist with investment decisions is viable. Ultimately the judgement has been that it is, and in part due to the work undertaken during this thesis, the Wealth Management team has for the first time created a full-time quantitative analyst position within its Wealth Management practice.

This thesis will be structured as follows: In Chapter 1, we will discuss our motivations for carrying out this type of research and our objectives, in Chapter 2 we will describe the statistical techniques we used, in Chapter 3 we will apply the methods of Chapter 2 to real data and look at a number of recently relevant scenarios relating to inflation, growth, and interest rate hikes, and in Chapter 4 we will present our conclusions.

Chapter 1

Learning From the Past

1.1 Motivation

In October 2020, one of the most renowned quantitative strategists and head of quantitative strategy at Bernstein, Inigo Frazer Jenkins, provocatively stated [1] that he was no longer a quant, arguing that the industry has often attached too little importance to the changing of market regimes. Indeed in a world where shocks like COVID-19 can happen, and in which large purchases by the United States Federal Reserve can have profound impacts on the economy , we must ask ourselves, if the validity of statistical techniques and backtests are at the mercy of changing regimes, what, if any, is the place that that quantitative techniques have within long term investment strategy?

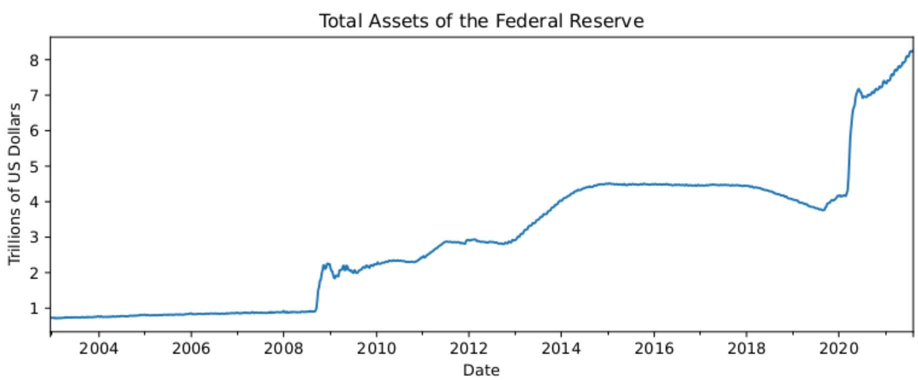


Figure 1.1: Between 17 March 2020 and 17 March 2021, the total number of assets held by the Federal Reserve, rose by $4.7 trillion due to actions taken by the Federal Reserve to combat the economic disruptions caused by the COVID-19 Pandemic [2]. This policy had a significant impact on the corporate bond market, increasing liquidity and reducing transaction costs. While clearly very impactful for the economy, it is clear that one cannot rely on statistical models to predict such regime changes (source: FRED).

There is never a guarantee that the future will closely resemble the past. Nevertheless, the intelligent asset allocation strategist must form their decisions based on a careful understanding of the world, and part of this requires an accurate understanding of past events. It is a common occurrence in investment committee meetings that different analysts will remember past events differently. Did stocks go up during times of high inflation? Did bonds do poorly in times of low growth? Were these events indicative of a pattern or simply a coincidence? These are questions that any investor must seek to know the answer to objectively.

Therefore, in this paper, we provide a framework for more objectively analyzing the past performance of different asset classes under various economic scenarios, with inflation being studied in the most detail. We examine a number of techniques for applying bootstrapping techniques to financial return data that violate common independent and identically distributed (iid) assumptions, and apply these techniques to a number of recently relevant economic scenarios relating to the historically high inflation rates that have been seen in recent times. Finally we will discuss the results obtained and how they may be used to make more informed investment decisions.

1.2 Methodology

Our approach will be to study the returns of a large number of economic, equity and bond indexes, outlined in Appendix A, which will be our interface to the performance of the corresponding asset classes under simple buy-and-hold strategies. In particular, we wish to estimate the average returns, historical sharpe ratio, and the level of uncertainty in the estimation of these quantities during these periods. More than simply calculating values, it is crucial that we must establish statistical significance for any conclusions we reach: any parameter we estimate must be accompanied with a confidence interval, showing the uncertainty in the estimate, and any comparison made must be done with attention paid to whether differences are a matter of coincidence or true deviations between two datasets.

The purpose of our analysis is to obtain a better understanding of the past. It is practically very difficult to define economic scenarios such that individual occurrences of the scenario are all comparable. Case in point: inflation has been affected by factors as diverse as wars, revolutions, and pandemics. Therefore, we cannot expect our results to directly hold out of sample. Nevertheless, such backward looking analysis can augment the understanding and experience of an investment analyst.

The statistical dynamics of stock returns are characterized by a number of stylized facts which ensure that one must be careful when modelling and estimating returns. Particularly salient to our purposes include the following:

- Returns do not follow a normal distribution
- Returns are often heavy tailed and asymmetric
- Returns are not independent: squared returns and absolute returns are serially correlated
- Volatility is clustered and highly persistent

In light of these properties of financial returns, we find it appropriate to use *bootstrap* techniques for the estimation of parameters and construction of confidence intervals for our purposes, and we shall outline some of the techniques which may be used to bootstrapping financial time series returns in the next section.

Chapter 2

Bootstrapping of Financial Time Series

2.1 The Standard Bootstrap

The bootstrap, introducted by Efron in [3] works by sampling with replacement. Given empirical return data:

$$r_1, r_2, r_3, ..., r_n \tag{2.1.1}$$

and a sample statistic $T(r_1, r_2, r_3, ..., r_n)$, one may calculate a bootstrap confidence interval for the statistic of level α as follows:

1. Let $r_1^i, r_2^i, r_3^i, ..., r_n^i$ be a sample from the discrete uniform distribution $\{r_1, r_2, r_3, ..., r_n\}$ independently and with replacement for each i in $1, \ldots, B$ where B is the number of bootstrap iterations. It is important that this sample is of the same size as the original sample. This is our *bootstrap resample*.

2. Calculate $T_i^* = T(r_1^i, r_2^i, r_3^i, ..., r_n^i)$ for i in range $1, ..., B$.

3. Calculate $\delta_i^* = T_i^* - T(r_1, r_2, ..., r_n)$ for each $i = 1, 2, ..., B$.

4. Calculate the $\alpha/2$ and $(1 - \alpha/2)$ critical values of $\{\delta_i^*\}_{i=1}^B$: $\delta_{\alpha/2}^*, \delta_{(1-\alpha/2)}^*$

5. Then, our $100(1 - \alpha)\%$ bootstrap confidence interval for T is given by

$$[T(r_1, r_2, ..., r_n) - \delta_{\alpha/2}^*, T(r_1, r_2, ..., r_n) - \delta_{(1-\alpha/2)}^*]. \tag{2.1.2}$$

Bootstrapping carries a number of advantages for our purposes. It is model free, and takes into account that our data may be non-normal or assymmetric. It also allows one to estimate complex statistics T in a simple way. This makes it applicable to a wide range of data.

In principle, bootstrapping should ensure that the empirical distribution of the resampled data $r_j^*, j = 1, ..., n$ should have the same distribution as the original data. However, for this to hold, it is important to note that bootstrapping assumes that the returns are independent and identically distributed. This is generally not the case for financial data returns; returns generally exhibit autocorrelation and volatility clustering. Figure 2.1 shows how bootstrapping can destroy the autocorrelation structure of returns.

2.2 Block Bootstrapping

The classical remedy to the problem of bootstrapping correlated data is to use *block bootstrapping*. In block bootstrapping, instead of individual observations, *blocks* of data are sampled with replacement from the raw data. Following this, the remaining procedure is the same as in Section 2.1

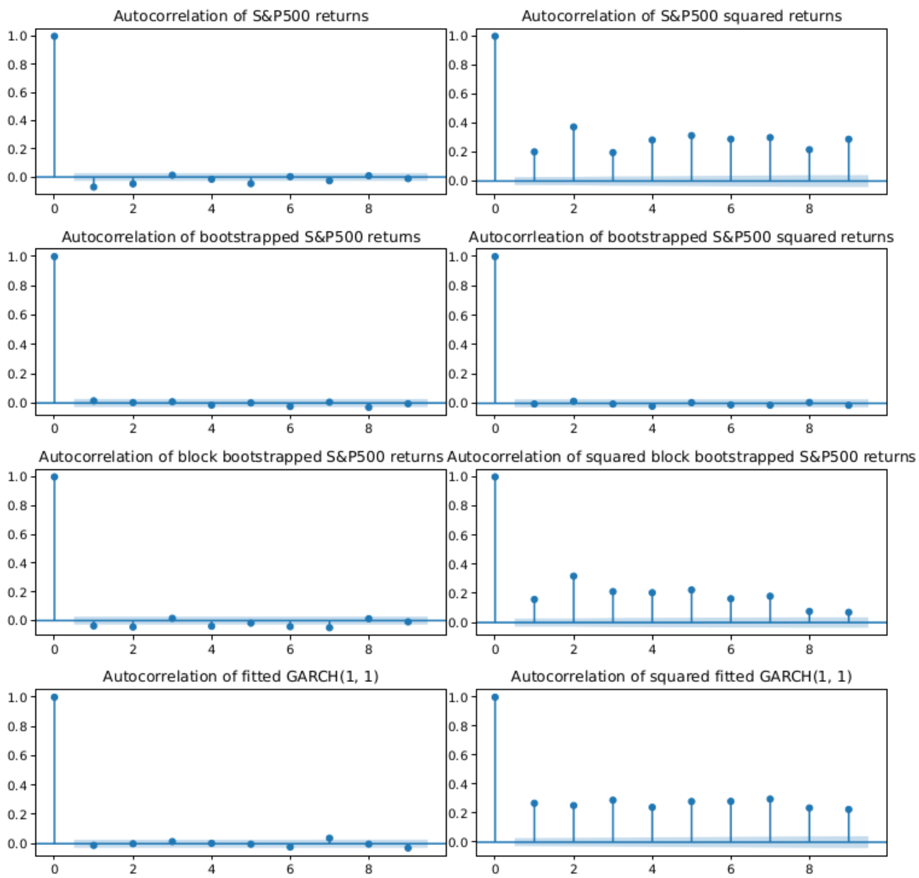


Figure 2.1: Left Column: Raw returns. Right Column: Squared Returns. While raw returns have low autocorrelation, squared returns show significant and persistent autocorrelation. This autocorrelation structure is destroyed by regular bootstrapping, while block bootstrapping and GARCH models can help to alleviate this problem.

A number of versions of block bootstrapping have been devised, involving variations in the way blocks are chosen and the size of the blocks. A number of these have been studied in [4], and we will summarize them below:

In the following sections we will use the following setup: Let our original sample be the sequence $\{r_i\}_{i=1}^n$. Then, define $r_i = r_{(i \bmod n)}$ for $i > n$ such that we periodically extend the series. Finally, denote by $\mathcal{B}(i,k) = (r_i, r_{i+1}, ..., r_{i+k-1})$, the block of size $k \geq 1$ starting from r_i

2.2.1 Moving Block Bootstrapping (MBB)

In moving block bootstrapping, one samples blocks with replacement from the collection of blocks

$$\{\mathcal{B}(i,l) : i = 1, \dots, n-l+1\} \tag{2.2.1}$$

where l is the fixed block size. To be precise, our final bootstrap resample will consist of

$$\mathcal{B}(I_1,l), \dots, \mathcal{B}(I_b,l) \tag{2.2.2}$$

where $b = \lfloor \frac{n}{l} \rfloor$ and $I_1, \dots I_b$ are discrete uniform random variables taking values $i = 1, \dots, n-l+1$

2.2.2 Nonoverlapping Block Bootstrapping (NBB)

In NBB, we disallow choosing overlapping blocks; we choose from the following without replacement

$$\{\mathcal{B}((i-1)l+1, l) : 1 \leq i \leq b\} \tag{2.2.3}$$

where our final bootstrap sample is once again given by (2.2.2)

2.2.3 Circular Block Bootstrapping (CBB)

In circular block bootstrapping, one samples blocks with replacement from the collection of blocks

$$\{\mathcal{B}(i,l) : 1 \leq i \leq n\}, \tag{2.2.4}$$

that is, we allow wrapping around of the start and the end of the data. The purpose of allowing this wrap around is to prevent systematic undersampling of data from the start and end of the time series, which occurs in MBB due to the fact that indexes less than l or greater than $n-l+1$ occur in fewer of the blocks the set (2.2.1)

2.2.4 Stationary Bootstrap (SB)

Stationary block bootstrapping differs from the methods previously mentioned because the block size is no longer fixed, but exponentially distributed. This method is notable for producing bootstrap samples which are statistically stationary, as proved by [5].

2.2.5 Block Size Selection

As noted in [6], the size of blocks has a crucially important role in determining the bias and variance of a block bootstrap estimator. Therefore it is of critical importance to choose appropriate block sizes (or mean block sizes in the case of stationary bootstrap). Fortunately, [7] provides an automatic block size selection algorithm for selecting the optimal block size for the Stationary and Circular Block bootstraps. The method is sensitive to the autocorrelation function of the time series and has been demonstrated to attain excellent performance in choosing the optimal block size in [8].

2.2.6 Comparison of Block Bootstrapping Methods

The block bootstrapping methods mentioned above are compared in [4]. It was found that all four methods have the same amount of bias asymtotically, but have different leading terms in expansions of their variance.

Ultimately the conclusion of [4] is that methods with overlapping blocks are to be preferred and that the Stationary Bootstrap typically leads to larger mean-squared errors. Thus, we find that circular block bootstrapping is the best method for our purposes as it has the added feature over moving block bootstrapping of not under-sampling data at the start and end of series.

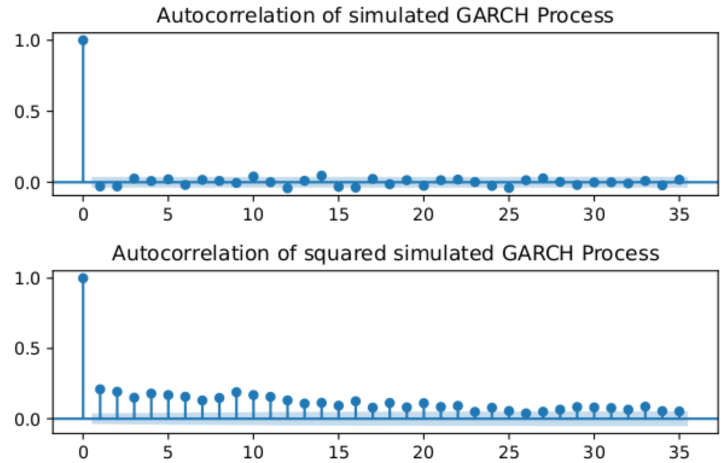


Figure 2.2: Autocorrelation of Simulated Garch Process using the `arch` package in Python

2.3 Parametric methods/Monte Carlo

Another approach which can be used to respect the correlation structure when estimating parameters from financial time series is to assume certain time series models. Models such as the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model were designed to be able to capture the properties of financial returns and thus we felt that such methods merited serious consideration. We ultimately found that these methods did not suit our purposes very well, however we will still outline the work that we carried out in this area in order to explain our reasoning for preferring the approaches in Section 2.2.

2.3.1 GARCH Models

The generalized autoregressive conditional heteroskedasticity (GARCH) model, introduced by [9] is generally considered to be one of the most important models for returns of financial returns. A process $(X_t)_{t\in\mathbb{Z}}$ is a GARCH(p, q) process if it is strictly stationary, and for a positive valued process $(\sigma_t)_{t\in\mathbb{Z}}$ satisfies

$$X_t = \sigma_t Z_t, \qquad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{2.3.1}$$

for $t \in \mathbb{Z}$ where $\alpha_0 > 0$, $\alpha_i \geq 0, i = 1, \ldots, p$ and $\beta_j \geq 0, j = 1, \ldots, q$, and $(Z_t)_{t\in\mathbb{Z}}$ being a series of iid square-integrable random variables with mean 0 and variance 1 [10]. This may be further extended [11] to a Constant Mean Garch Model, $(R_t)_{t\in\mathbb{Z}}$,

$$R_t = \mu + X_t \tag{2.3.2}$$

which allows for the process to have a constant mean. In practice, lower order GARCH models are the most used, with GARCH$(1, 1)$ perhaps being the most popular, and thought to be a good fit to logarithmic return data. The standard t distribution is also commonly used for the distribution of Z_t as it has heavier tails than the normal distribution. Thus, we will focus on GARCH$(1, 1)$ with standard t errors in the forthcoming discussion.

An important feature of GARCH models is that they are able to capture the autocorrelation structure and volatility clustering properties of returns. In Figure 2.2 a GARCH(1, 1) model is simulated, showing significant autocorrelation in the squared series, but essentially negligible autocorrelation in the original series.

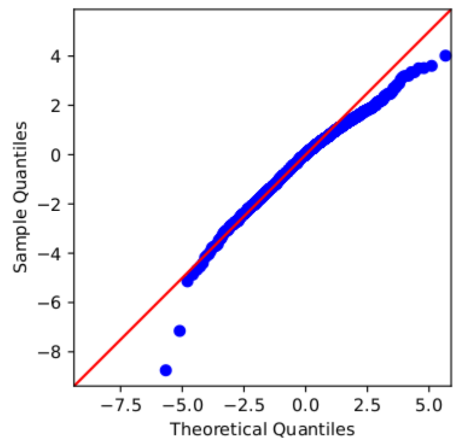


Figure 2.3: Quantile-Quantile plot of standardized residuals of GARCH model fitted S&P500 returns with Student's t errors. One can see significant deviations in the distribution particularly at the extremes

If we believe that a GARCH model is a good fit to our data, the GARCH model allows for statistical inference to be carried out in a straightforward way. Maximum likelihood estimators, along with asymptotic results for the distribution of these estimators have been derived [10] and calculated standard errors are readily available for parameters such as μ through software packages. Alternatively, we may take a Monte Carlo approach, by simulating the data using the estimated parameters and assumed distribution for Z_t in order to obtain Monte Carlo confidence intervals.

We gave serious consideration to using these techniques over the block bootstrapping techniques outlined in section 2.2. Ultimately, we decided against this for practical reasons. Primarily, our concern was with ensuring that these models are a good fit to a large number of indexes, as the quality of results is largely dependent on how good of a fit the model is. Our intention is to apply these methods to around 50-60 indexes, where it would be impractical to assess the goodness of fit for each model. In addition, this constraint means that it is also impractical to consider the best GARCH variants for each index (for example different values of p and q, different variants of GARCH, different distributions for standard errors, and other variants of GARCH such as IGARCH, GJR-GARCH, etc).

It is also our intention in the implementation side of things to create a program which can recreate our analysis for arbitrary new scenarios and new indicies at a moment's notice, and the need to assess fits each time the components of the model are modified makes this process considerably less efficient.

Chapter 3

US Inflation Case Study

3.1 Motivation

The question of how to invest in inflationary times is a question that investment managers have not needed to consider in decades. Following the sharp rise in inflation during the COVID-19 pandemic, and economic shocks caused by restrictions and lockdowns, investors are faced with the prospect of a number of uncertain long term economic scenarios over the next few years. A number of scenarios of particular interest to Mazars are outlined in Table 3.2

Scenario	Description
Stagflation	High inflation coupled with low economic growth
Inflation + Growth	High inflation coupled with higher economic growth rates
High Inflation, Interest Rate Hikes	Federal Reserve increasing the federal funds rate to combat inflation
High Inflation, No Interest Rate Hikes	High Inflation, but the Federal Reserve takes no action

Table 3.2: Inflation Scenarios

It is of course extremely difficult to perfectly anticipate any of these scenarios before they happen, and even harder to perfectly predict how they will affect markets. Hence we instead aim to obtain a good understanding of what happened when each of the scenarios occurred in the past. Such knowledge can then be used to inform and guide our decisions as it becomes more clear which scenarios are more likely. In this section, we will start by defining our scenarios clearly, and then explain our methodology for analyzing the past performance.

3.2 Defining Scenarios

3.2.1 Inflation

It is difficult to precisely quantify inflation. More than one measure exists for measuring inflation: Personal consumption expenditure, Consumer Price Index, GDP deflator, to name a few, each having their own limitations and drawbacks, which have been hotly debated by economists for decades.

Moreover, two fundamental issues with inflation measures are highlighted by [12]: The deflationary forces of technology, and the fact that the baskets of goods relied on by many inflation indexes, can often be non-representative for a given investor, or even somewhat arbitrary.

In the end, we must accept that there is no perfect measure of inflation. For our purposes, we follow the decision of [12] which, in a similar analysis, use the United States Consumer Price Index (CPI) as reported by the Bureau of Labor Statistics as their measure of inflation. As with any index of inflation, this has its flaws [13]: it is a lagging indicator, and the fixed basket of goods can often be non-representative. Nevertheless, there are two advantages to using this measure. Firstly, the CPI has a large amount of historical data, going as far back as 1913, and secondly, it is the most popularly used measure of inflation, meaning that is the measure that market participants ultimately react to.

As inflation is defined by a rise in prices, we consider the year on year percentage rise of CPI and set the cutoff between high and low inflation at 3%. Figure 3.1 shows the full year on year CPI rate time series along with the periods when this was above 3%.

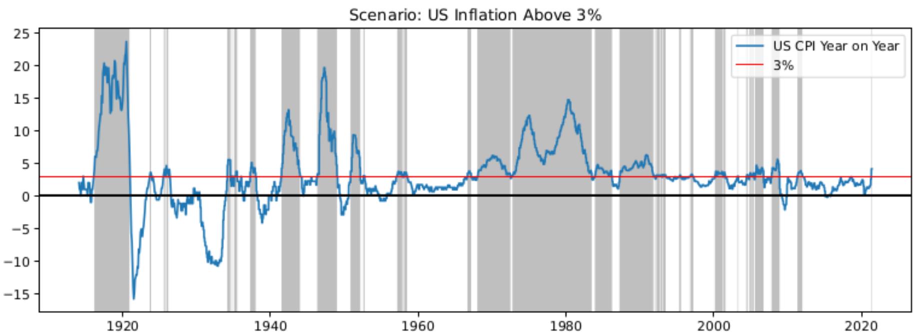


Figure 3.1: US CPI Year on Year rate
Shaded regions indicate high inflation ($> 3\%$) and non-shaded regions indicate low inflation ($< 3\%$)

3.2.2 Growth

The most common measure of economic growth is by far the Gross Domestic Product (GDP). Similarly to the Consumer Price index, the GDP has been highly scrutinized and criticized by economists as a measure of economic growth. Nevertheless, it is still considered an extremely important measure [14], which is capable of capturing important economic events such as recessions.

As with the consumer price index, the GDP has the benefits of having a large amount of historical data, and being a measure which is widely used and reacted to by market participants.

3.2.3 Combined Inflation and Growth Scenarios

Inflationary periods are often characterized by the type of growth during that period. For example, high Inflation coupled with low growth (stagnation) is commonly referred to as "stagflation". In a stagflationary environment, prices rise, and money loses value while economic growth stagnates, leading to wages decreases and unemployment, creating misery for everyday people on two fronts.

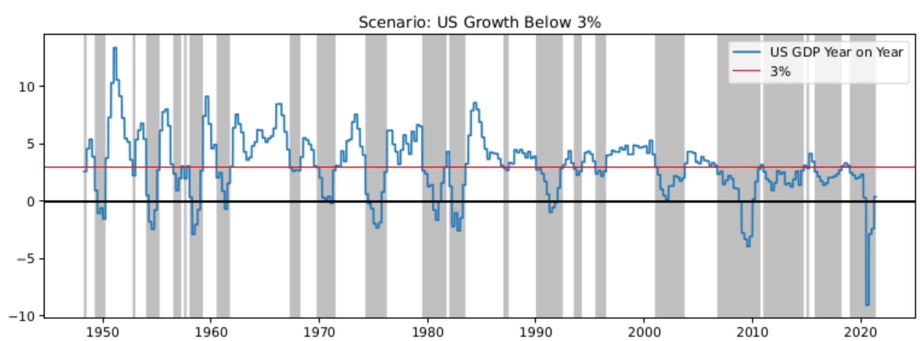


Figure 3.2: Low Growth

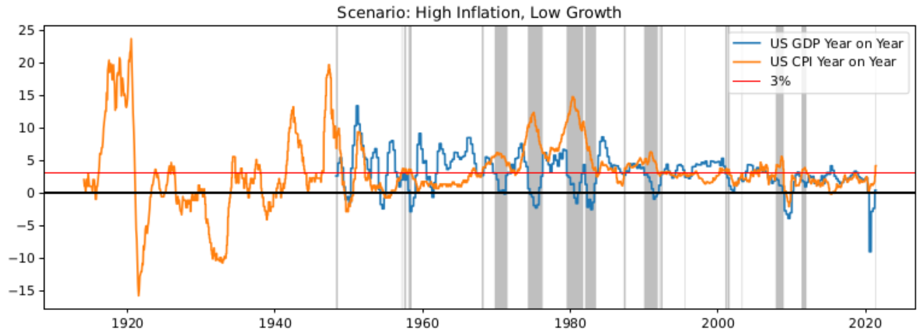


Figure 3.3: High Inflation, Low Growth (Stagflation)

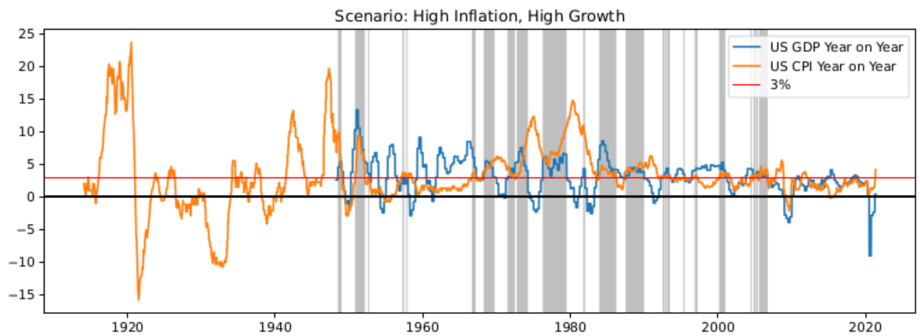


Figure 3.4: High Inflation, High Growth (Managed Growth)

Such a scenario has a vastly different effect on the overall economy compared to one where inflation is coupled with high growth. We define the High Inflation, Low Growth Scenario to be when inflation is above 3% and Growth is below 3%, and the High Inflation, High Growth Scenario to be when inflation is above 3% and growth is above 3% and illustrate these scenarios in Figures 3.3 and 3.4

3.2.4 Interest Rate Hikes

A key question for markets is whether the Federal Reserve will raise interest rates in order to combat higher inflation. We create two additional scenarios based around this question. For the High Inflation and Interest Rate Hikes scenario, we take all time periods when inflation rose above 3%, and restricted the scenario to only the periods where the Federal Funds Target Rate was raised by at least 0.5% at one point during that period.

Conversely, the High Inflation and No Interest Rate Hikes scenario consists of inflationary periods where the Federal Funds Target Rate was never raised by more than 0.5% at any point during the period. We illustrate these scenarios in Figures 3.5 and 3.6.

3.3 Data Preprocessing

Before using our data we undertake the following preprocessing steps

- **Converting Daily Data to weekly data**: We use weekly data to calculate returns. That is, the price level on each Friday. Our justification is the following: Global markets tend to normalize at the end of the week. That is to say that, news which broke during the week tends to be fully priced in by the end of the week in all geographical markets. For a long

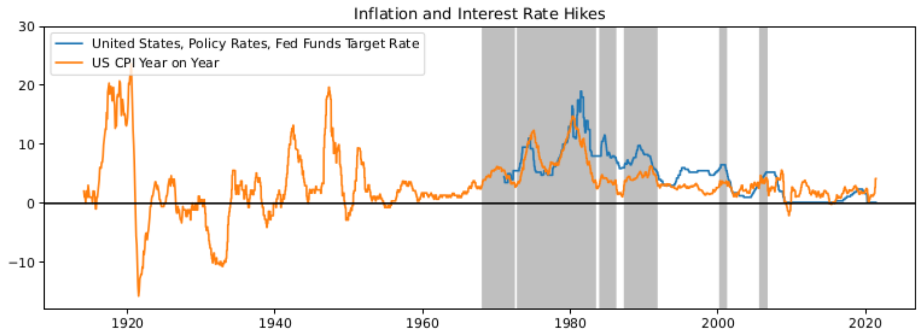


Figure 3.5: High Inflation, Interest Rate Hikes

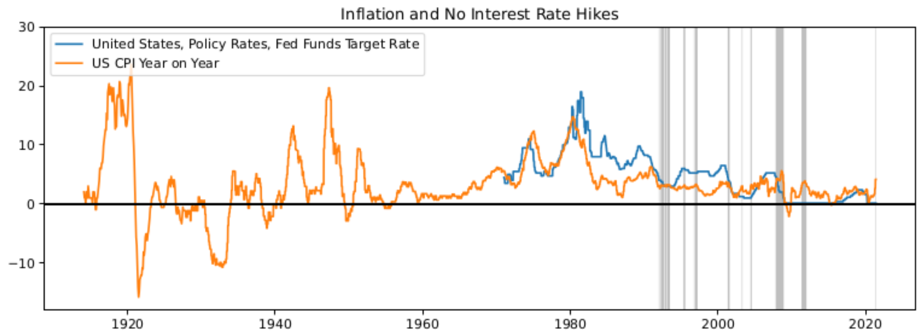


Figure 3.6: High Inflation, No Interest Rate Hikes

Scenario Number	From	To
1	1948-03-31	1948-06-29
2	1957-02-28	1957-03-29
3	1957-06-30	1957-09-27
4	1958-01-31	1958-06-27
5	1967-12-31	1968-03-29
6	1969-09-30	1971-06-29
7	1974-03-31	1976-03-30
8	1979-06-30	1981-09-29
9	1981-12-31	1983-06-29
10	1987-03-31	1987-06-29
11	1989-12-31	1991-10-30
12	1991-12-31	1992-01-30
13	1992-03-31	1992-06-29
14	1995-06-30	1995-07-28
15	2000-12-31	2001-03-30
16	2001-04-30	2001-07-30
17	2003-03-31	2003-04-29
18	2007-10-31	2008-11-28
19	2011-04-30	2011-12-30
20	2021-04-30	2021-05-27

Table 3.3: Individual periods of High Inflation, Low Growth Scenario

term investment strategist, intra-week data often represents noise which is better left out of their analysis.

- **Interpolating Missing Data**. Many financial time series exhibit irregularities in reporting frequency; sometimes data may be reported monthly until a certain data, after which data becomes available daily. Other times, data may simply be missing for certain dates. To combat this problem we use a simple linear interpolation between known values of each time series in order to obtain a weekly time series of values containing only Fridays in every case. This step is carried out before calculating returns. We believe that it is reasonable to assume that on average, the level of a time series moves between two known time points in a linear fashion. Furthermore, we believe it is far more desirable to perform a linear interpolation as opposed to a backward-looking fill of missing values, as this would result in many returns of exact value 0, where there are consecutive Fridays without any data.

- **Calculation of weekly returns:** Returns are calculated as a simple arithmetic return: $r_t = (p_t - p_{t-1})/p_{t-1}$. The use of logarithmic returns was also considered: $\ell_t = \log p_t - \log p_{t-1}$ as these are commonly used, especially in time series modelling due to the fact that log returns are additive. Ultimately we decided it would be more useful to use arithmetic returns as this is a scale which is more familiar to investment teams: logarithmic returns can differ from arithmetic returns by a significant amount when the magnitude of the return is large.

After these preprocessing steps, the return data is filtered so as to include only dates which fall within the scenario of interest. Block bootstrapping is then run on these filtered returns to obtain the results of Section 3.4

3.3.1 Quantities to Estimate by Bootstrapping

We are interested in bootstrapping the mean returns as well as the risk adjusted returns (Sharpe Ratio). These are among the most commonly used indicators of performance in the financial industry and they are among the most important factors looked at by the investment team at Mazars.

Quantity	Formula
Annualized Return	$(1 + \frac{1}{n}\sum_{i=1}^{n} r_i)^{52} - 1$
Annualized Sharpe	$\sqrt{52}(\frac{\overline{r}}{s})$, $\overline{r}$ = sample mean, s = sample standard deviation

Table 3.4: Quantities to Estimate

For both quantities we annualize the returns from weekly to monthly. It is true that the formulas used for annualization in Table 3.4 use an iid assumption in order to make the jump from weekly returns to yearly returns. However, we perform this annualization only to bring the figures to a scale that is more familiar, and hence we believe that such operations are reasonable for this purpose.

3.3.2 Performance Relative To Long Term Average

Aside from knowing how different asset classes performed during scenarios, we would also like to know how these performances compare to the overall performance of these assets overall: both inside and outside the scenario. We aim to obtain an idea of this by bootstrapping from the returns within the scenario, and then the returns overall, and then subtracting the former bootstrapped statistic from the latter. In this way we can create a confidence interval for the performance of an asset class relative to its long term performance.

3.3.3 Indexes

We use a large number of indexes, retrieved from Thomson Reuters Eikon as well as Bloomberg Terminals for our analysis. A full list of indexes used are included in the appendix.

Wherever possible, we aim to use *Total Return Indexes*. While a standard price index like the S&P500 tracks the capital gains of an asset class, a total return index tracks both the capital gains as well as any cashflows such as dividends or interest payments. These features make total return indexes a more accurate representation of the performance of the index for shareholders [15].

We also include a number of *relative* indexes. In the following results these are listed with names similar to "Consumer Staples vs World". The aforementioned index is constructed by dividing the MSCI Consumer Staples Index by the MSCI World Index, and represents the relative performance of consumer staples as compared to the MSCI World. That is, if this index rises, it means that consumer staples are performing particularly well compared to equities overall.

A fundamental challenge for us is finding indexes with enough data to form significant conclusions. In our selection process we chose to use only indexes which have had at least twenty years of data. While having more data is always desirable, we believe that including indexes with less data is still valuable if statistically significant conclusions can be reached. We include the start dates of each index in Appendix A.

3.4 Results

3.4.1 Presentation of Results

Our results consist of bootstrapped confidence intervals for the statistics outlined in Section 3.3.1, first calculated within the relevant scenarios, and then calculated relative to their long term average as in Section 3.3.2.

We use Circular Block Bootstrapping for our analysis, using the optimal block size selection method in [8], for the reasons we outlined in Sections 2.2.6 and 2.3.1. We display results as bar charts with 95% confidence intervals shown in black error bars. All figures displayed are in percent terms.

High Inflation, Low Growth: Mean Return

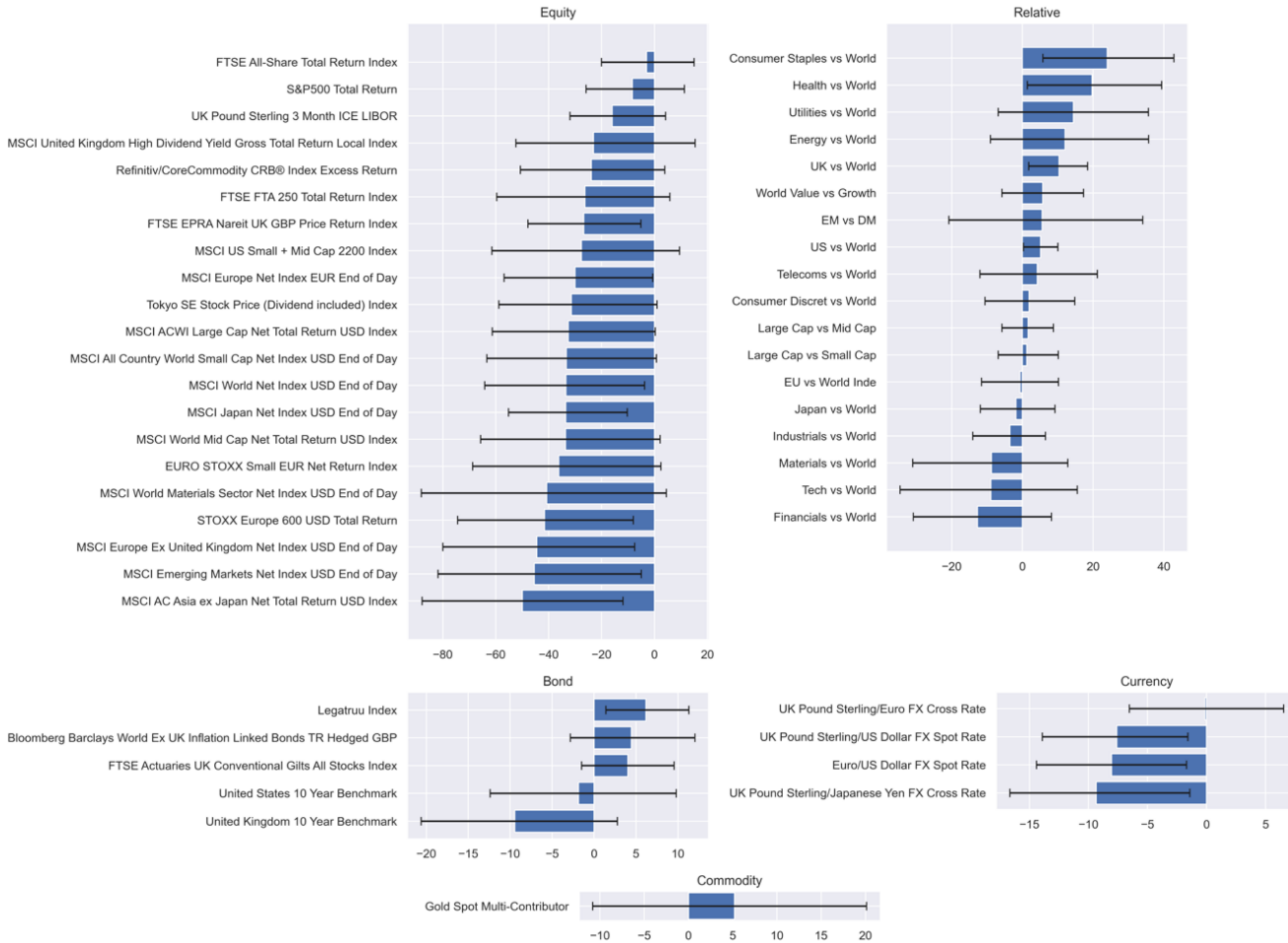


Figure 3.7: Results for High Inflation, Low Growth

High Inflation, Low Growth: Mean return relative to Long Term Average

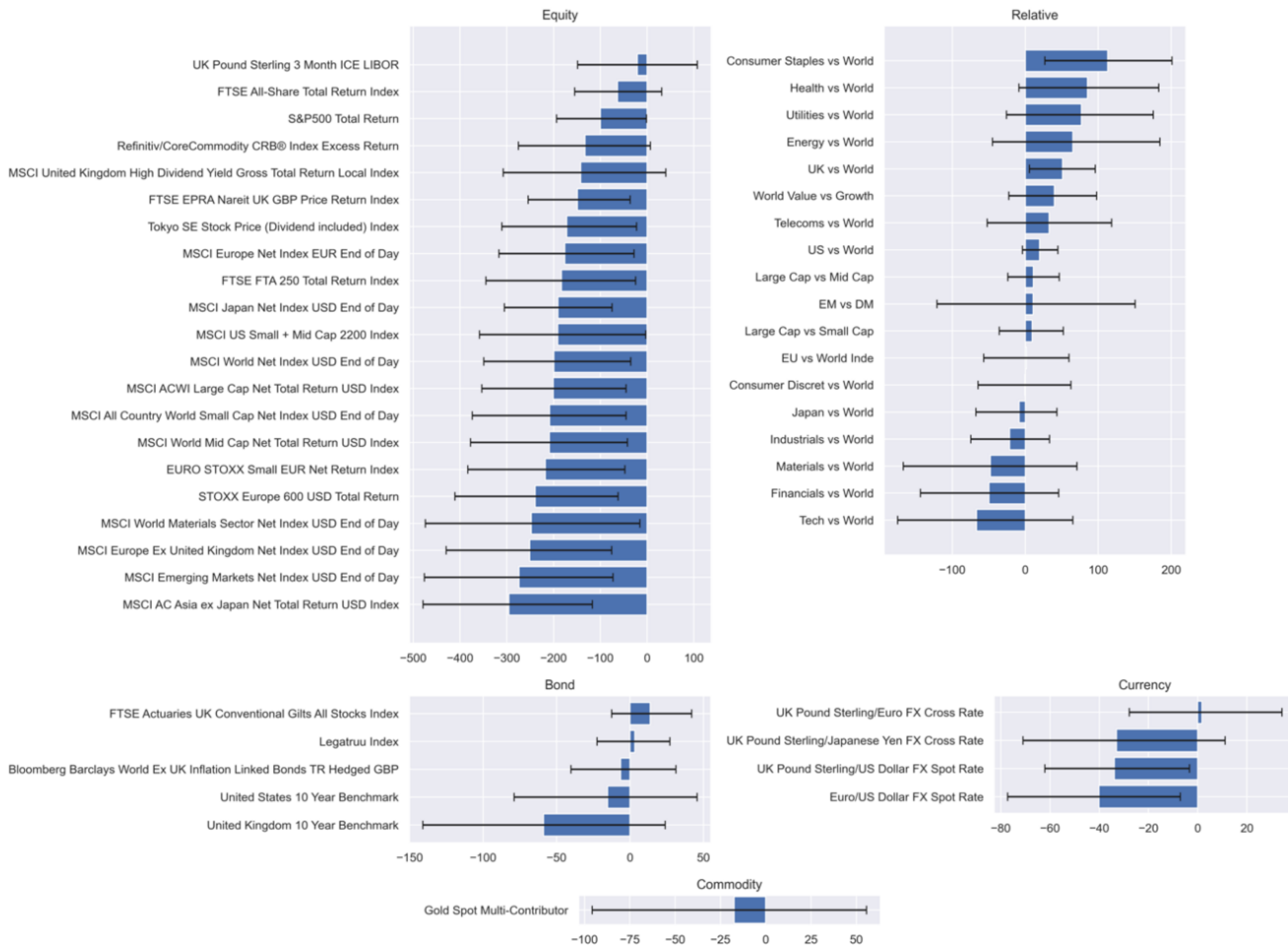


Figure 3.8: Results for High Inflation, Low Growth relative to Long Term Average

High Inflation, High Growth: Mean return

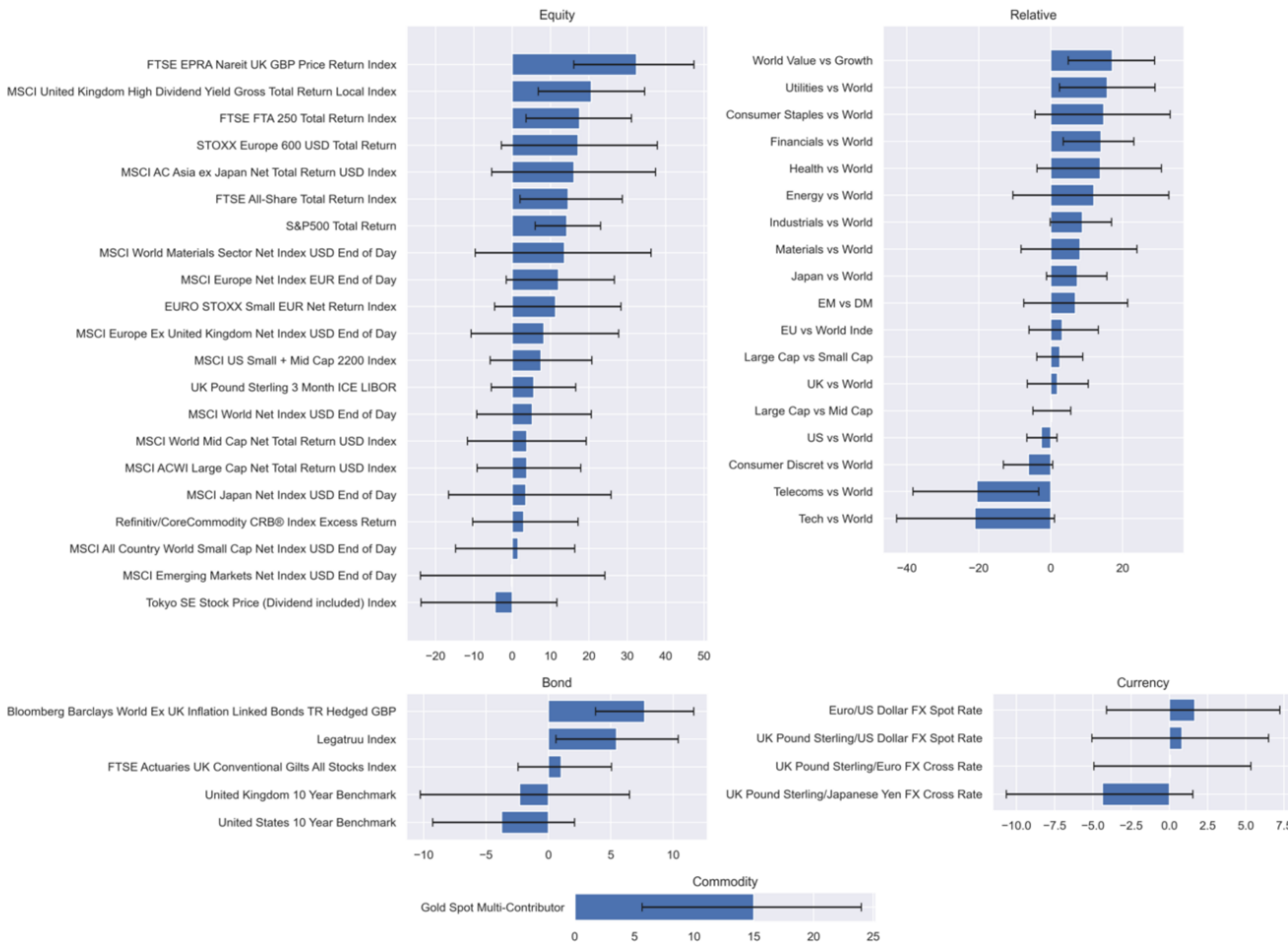


Figure 3.9: Results for High Inflation, High Growth

High Inflation, High Growth: Mean return relative to Long Term Average

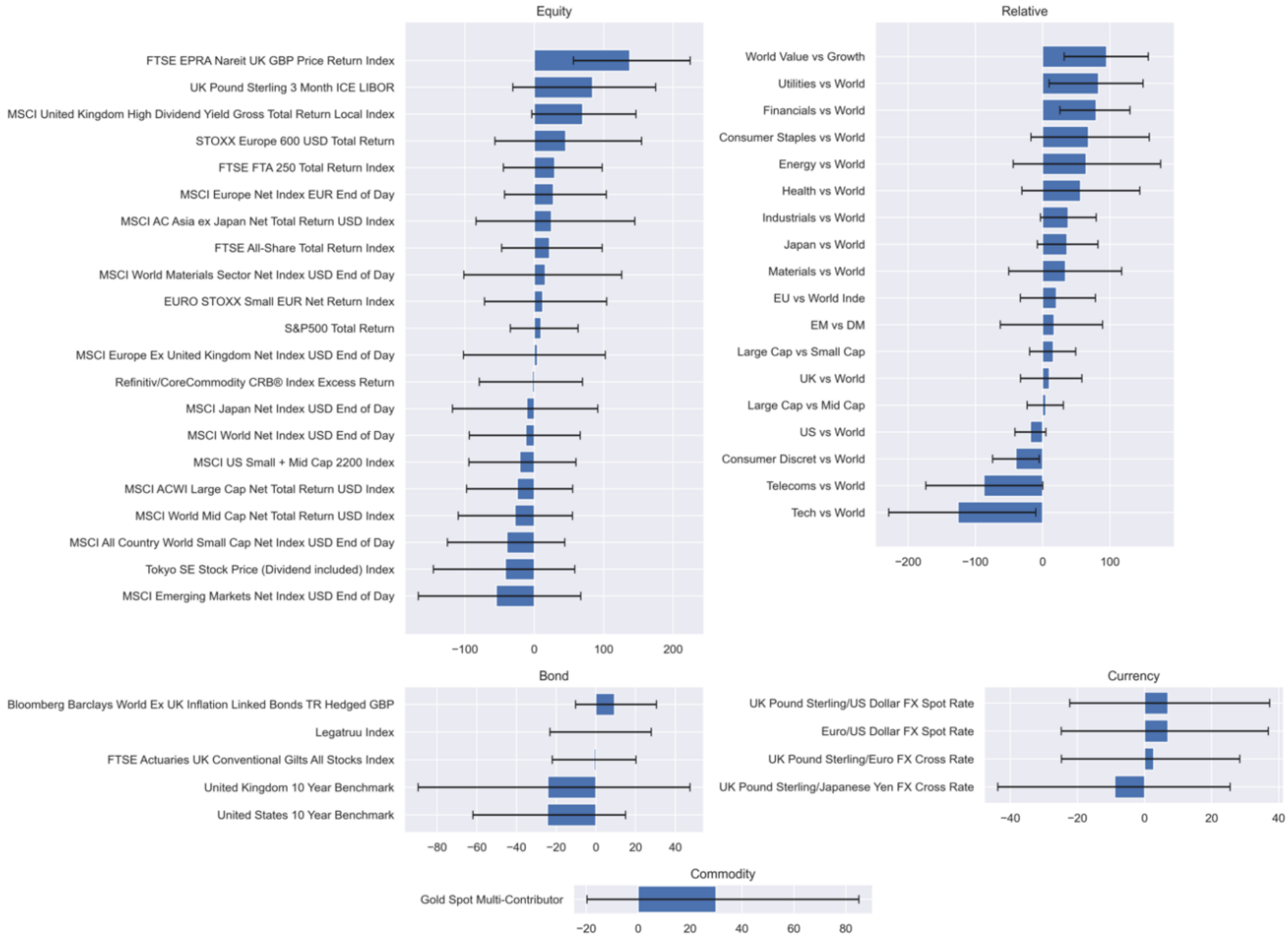


Figure 3.10: Results for High Inflation, High Growth relative to Long Term Average

High Inflation, Interest Rate Hikes: Mean return

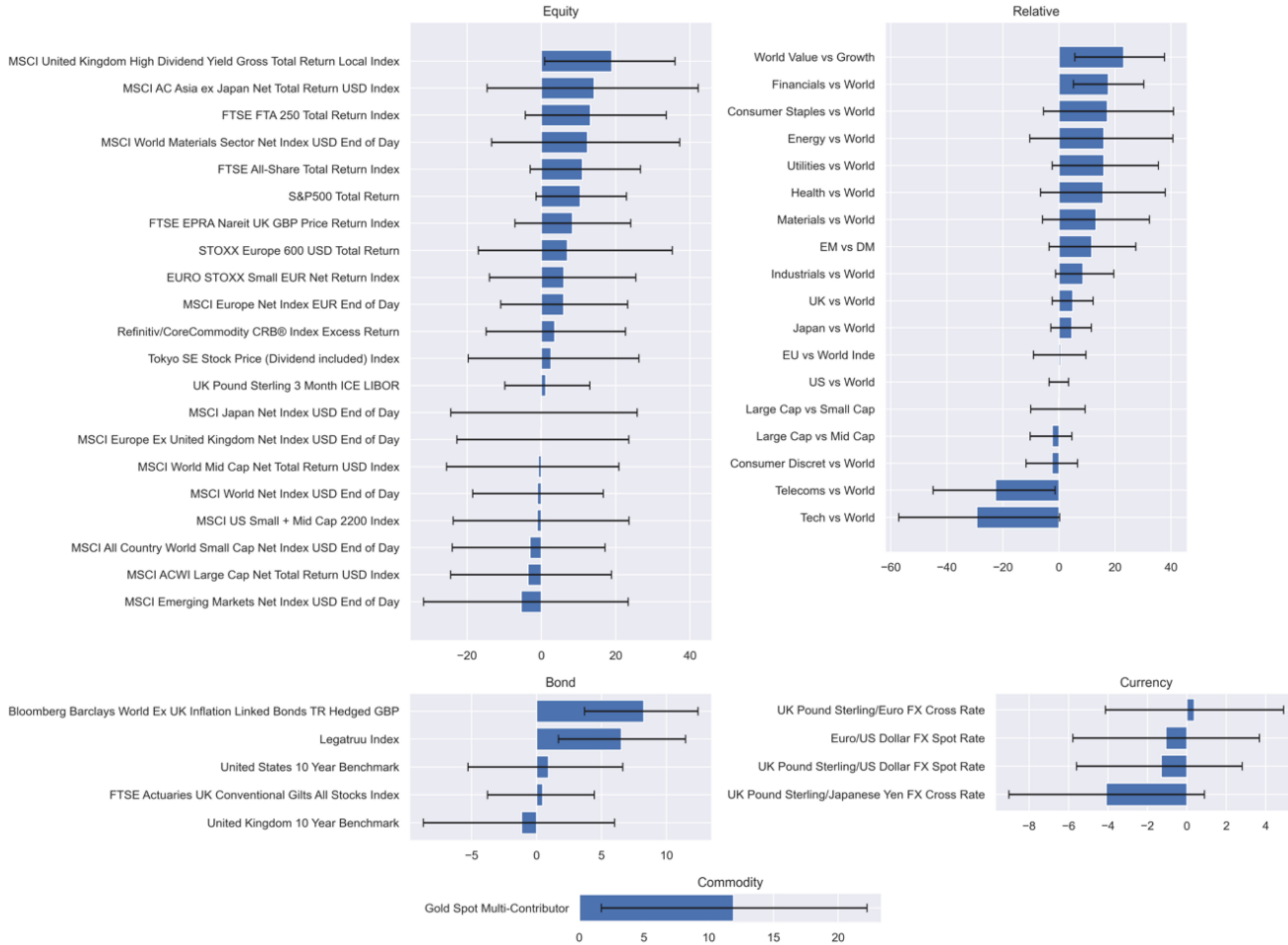


Figure 3.11: Results for High Inflation, Interest Rate Hikes

High Inflation, Interest Rate Hikes: Mean return relative to Long Term Average

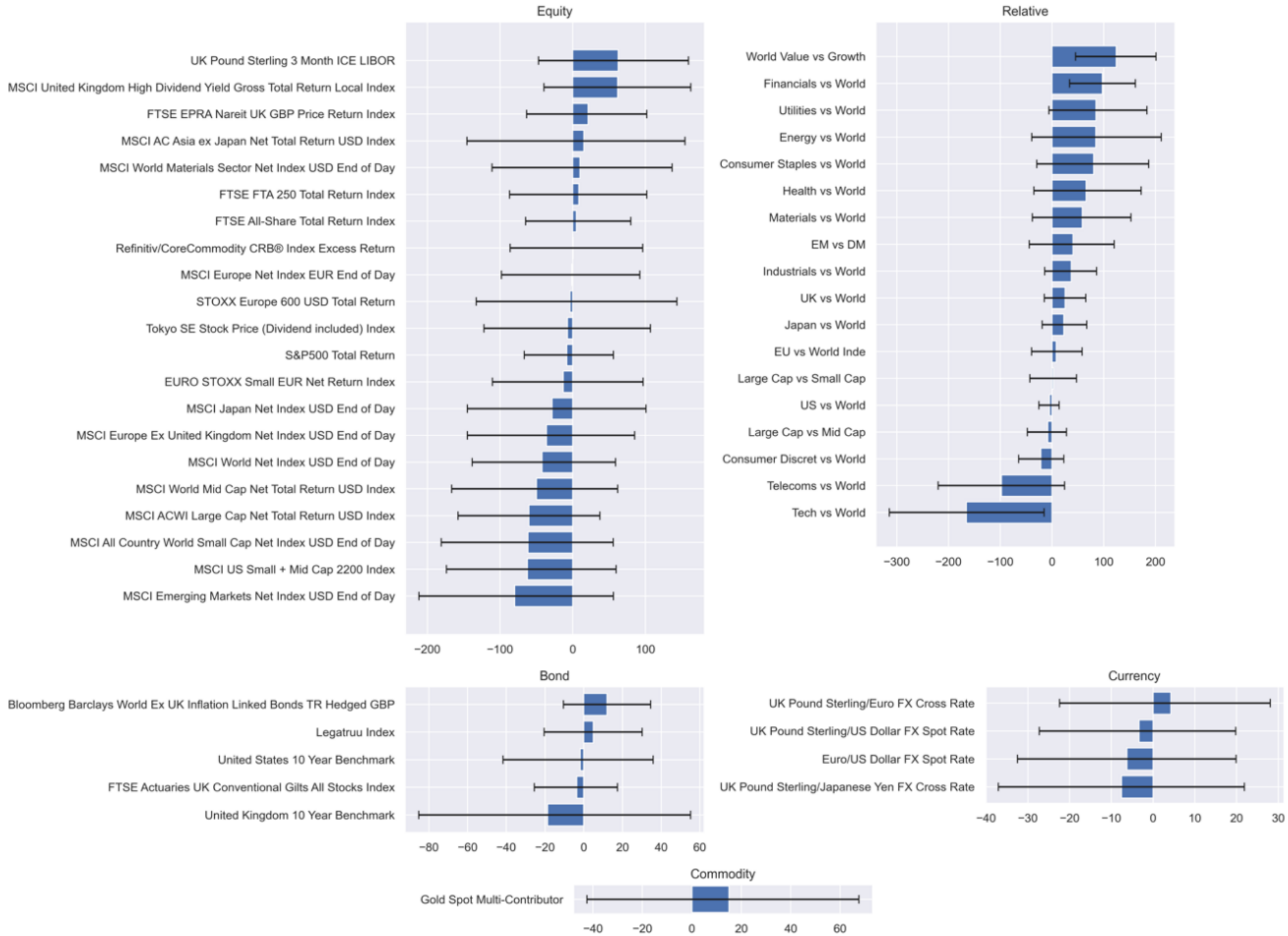


Figure 3.12: Results for High Inflation, Interest Rate Hikes relative to Long Term Average

High Inflation, No Interest Rate Hikes: Mean return

Figure 3.13: Results for High Inflation, No Interest Rate Hikes

High Inflation, No Interest Rate Hikes: Mean return relative to Long Term Average

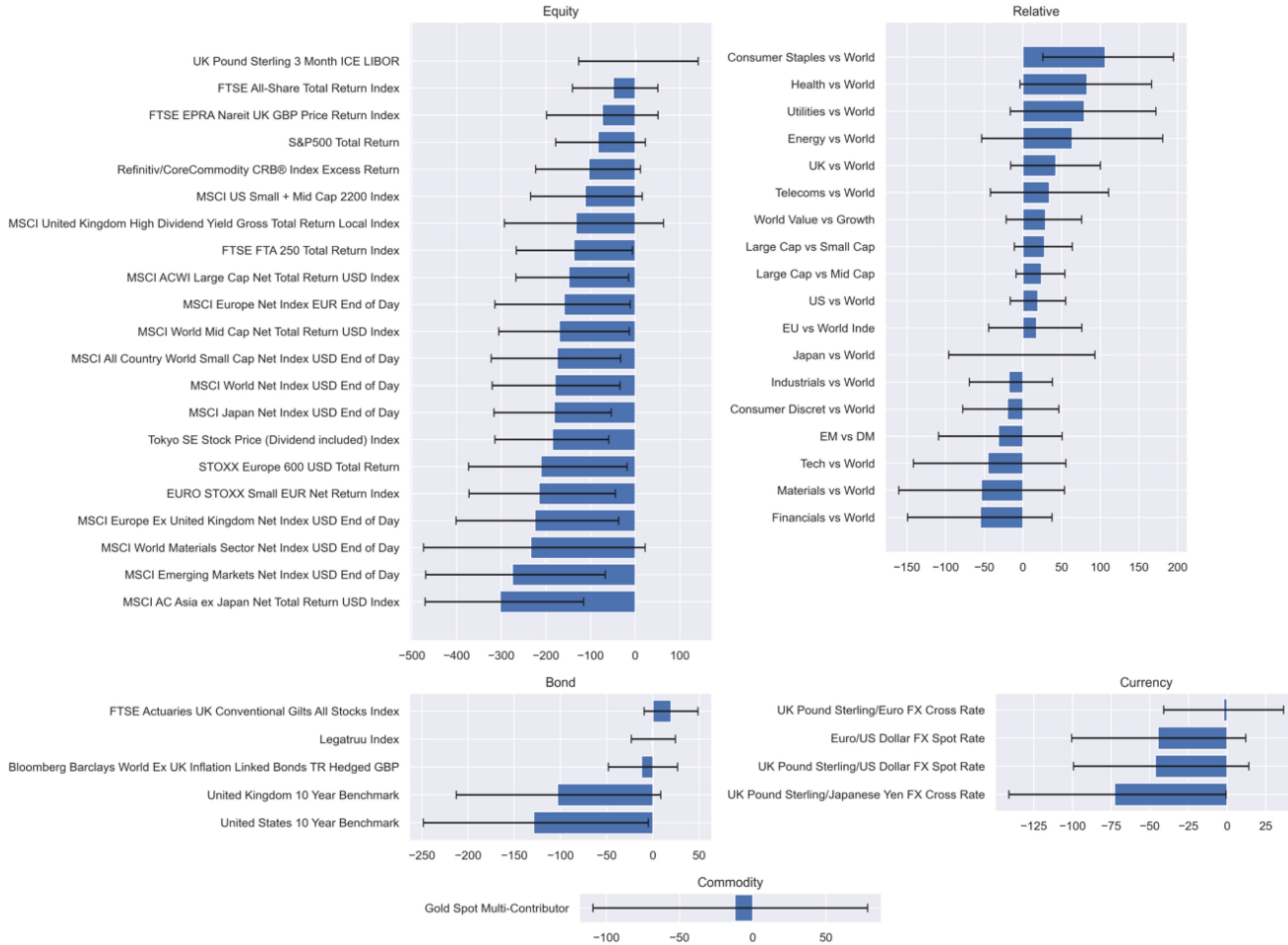


Figure 3.14: Results for High Inflation, No Interest Rate Hikes relative to Long Term Average

High Inflation, Low Growth: Sharpe

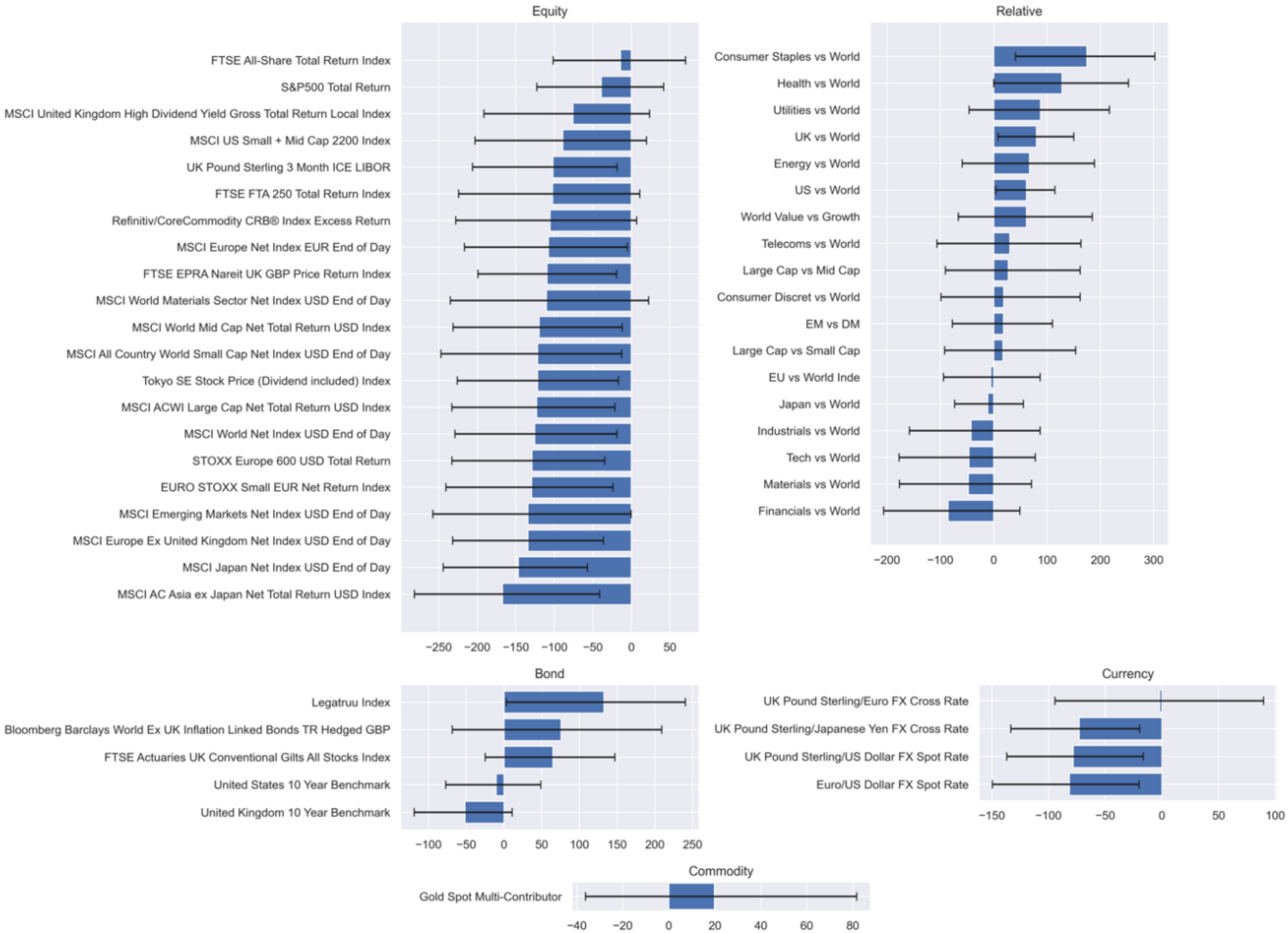


Figure 3.15: Results for High Inflation, Low Growth

High Inflation, Low Growth: Sharpe relative to Long Term Average

Figure 3.16: Results for High Inflation, Low Growth relative to Long Term Average

High Inflation, High Growth: Sharpe

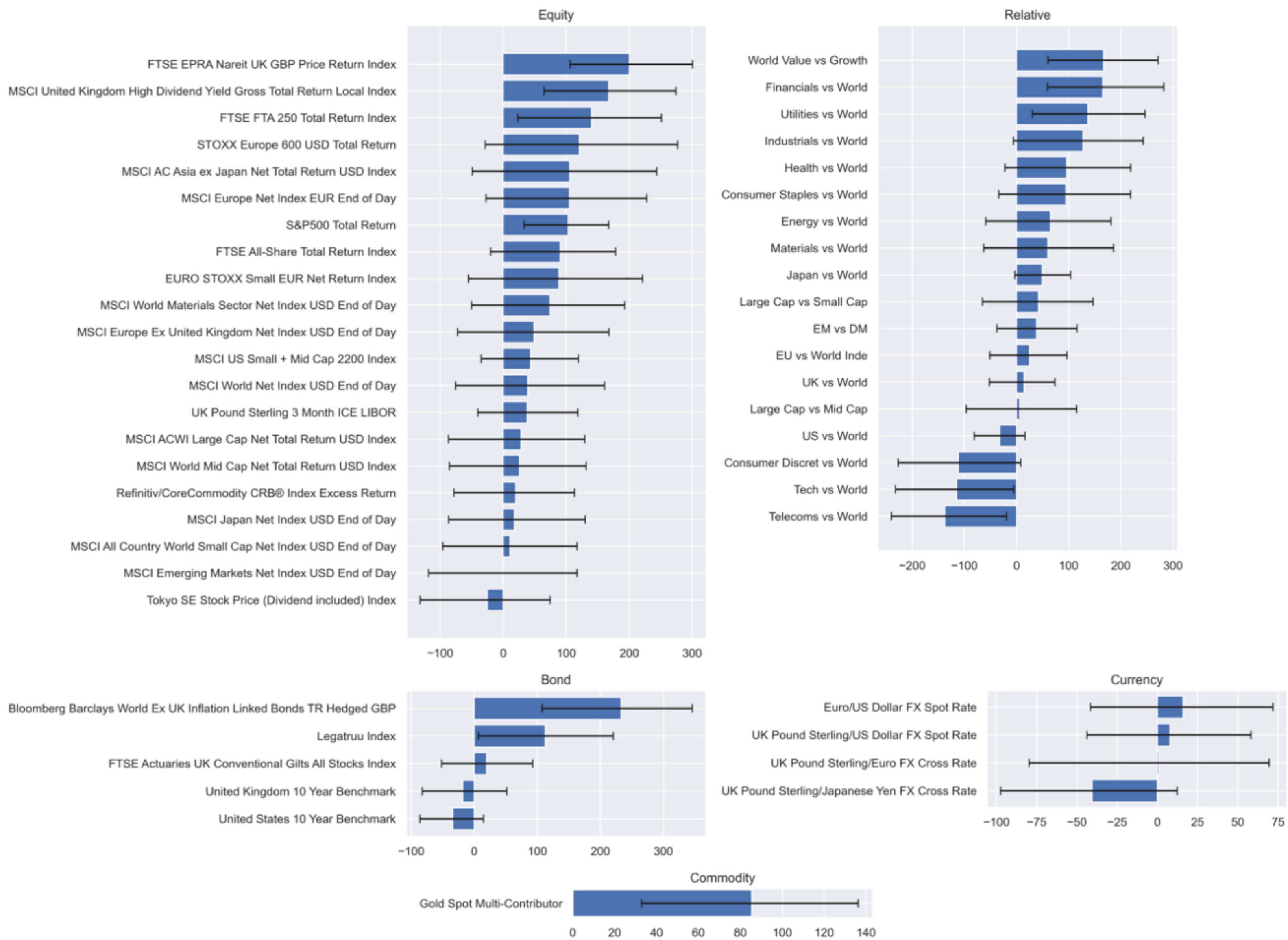


Figure 3.17: Results for High Inflation, High Growth

High Inflation, High Growth: Sharpe relative to Long Term Average

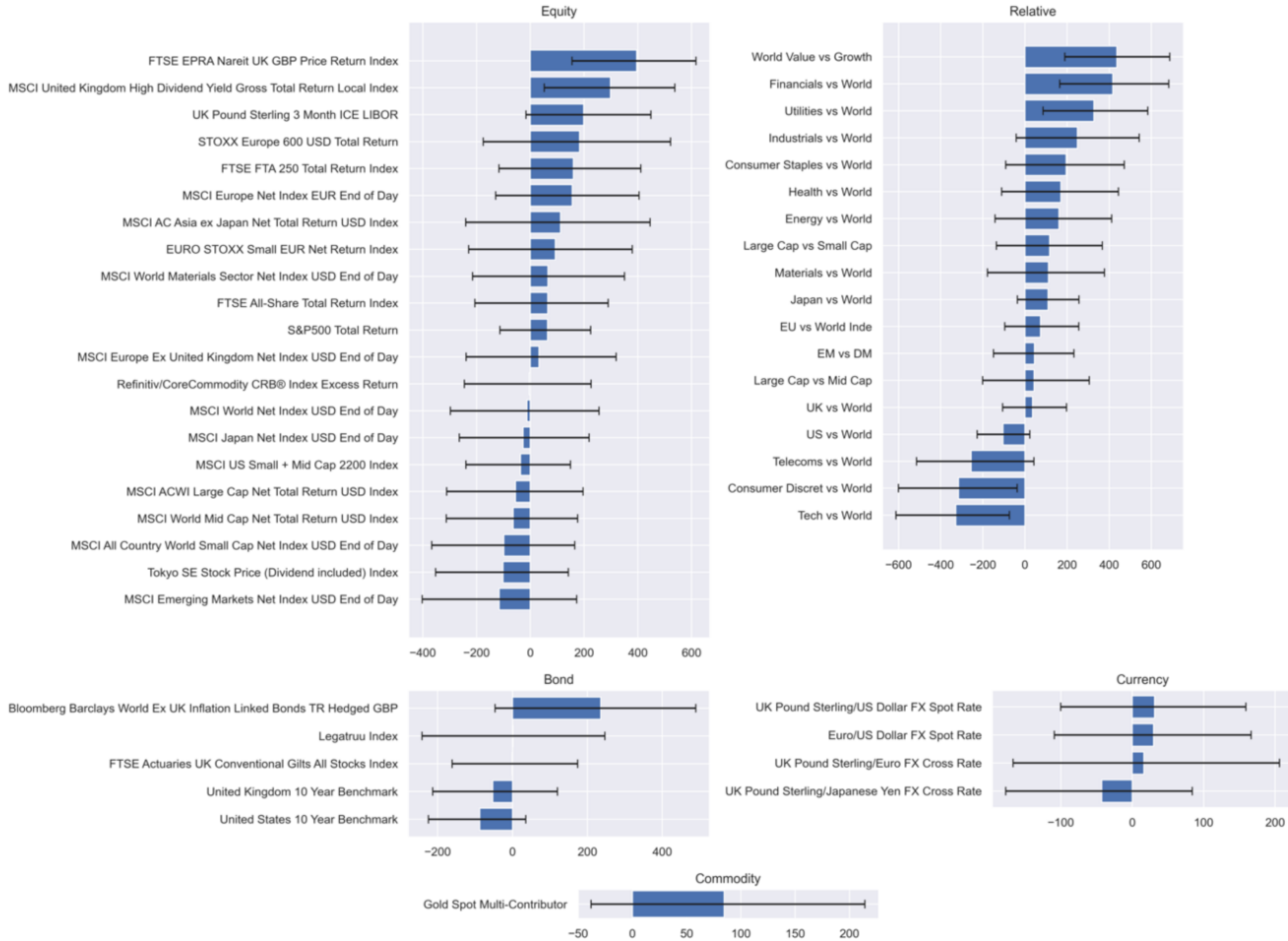


Figure 3.18: Results for High Inflation, High Growth relative to Long Term Average

High Inflation, Interest Rate Hikes: Sharpe

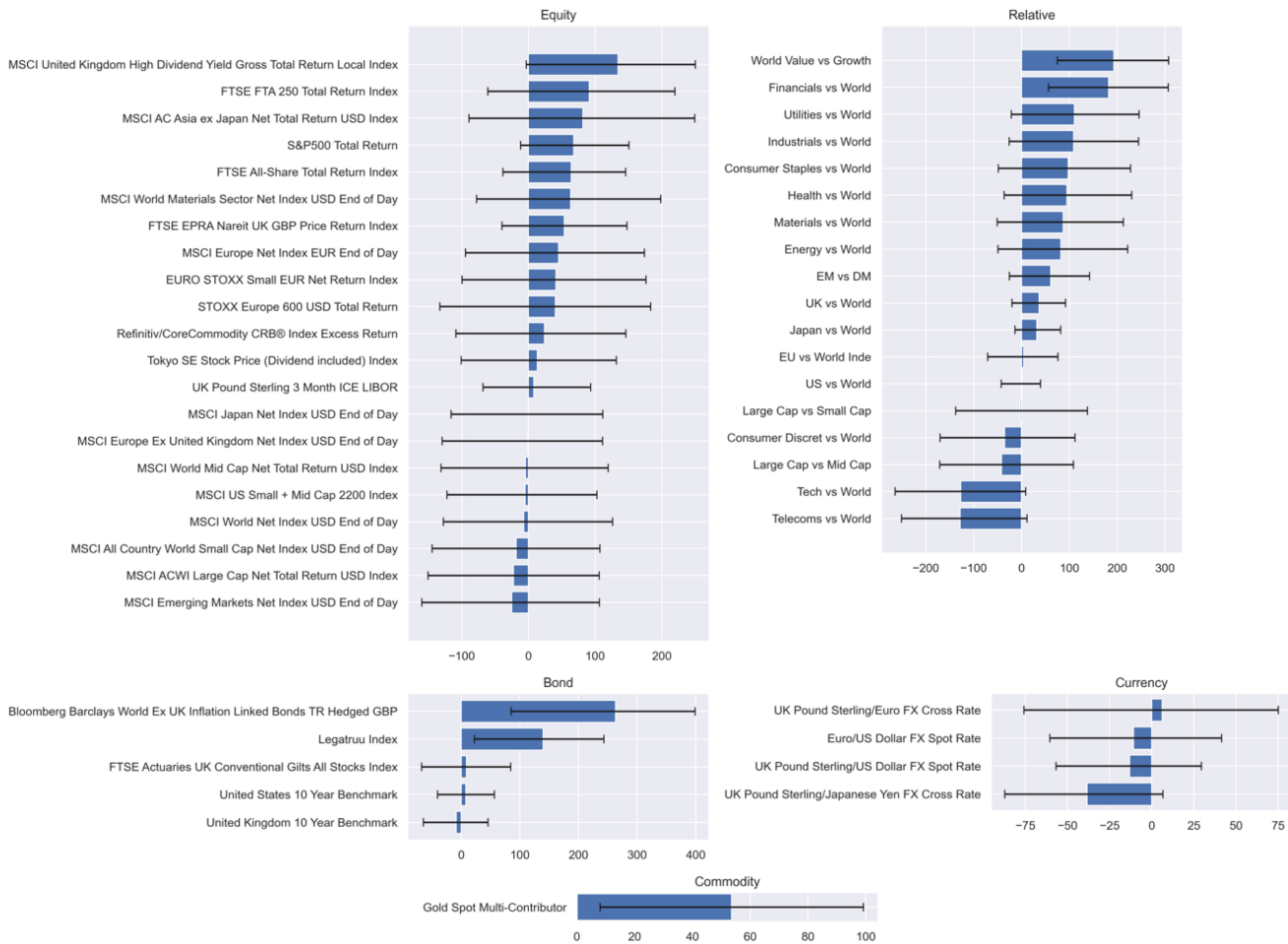


Figure 3.19: Results for High Inflation, Interest Rate Hikes

High Inflation, Interest Rate Hikes: Sharpe relative to Long Term Average

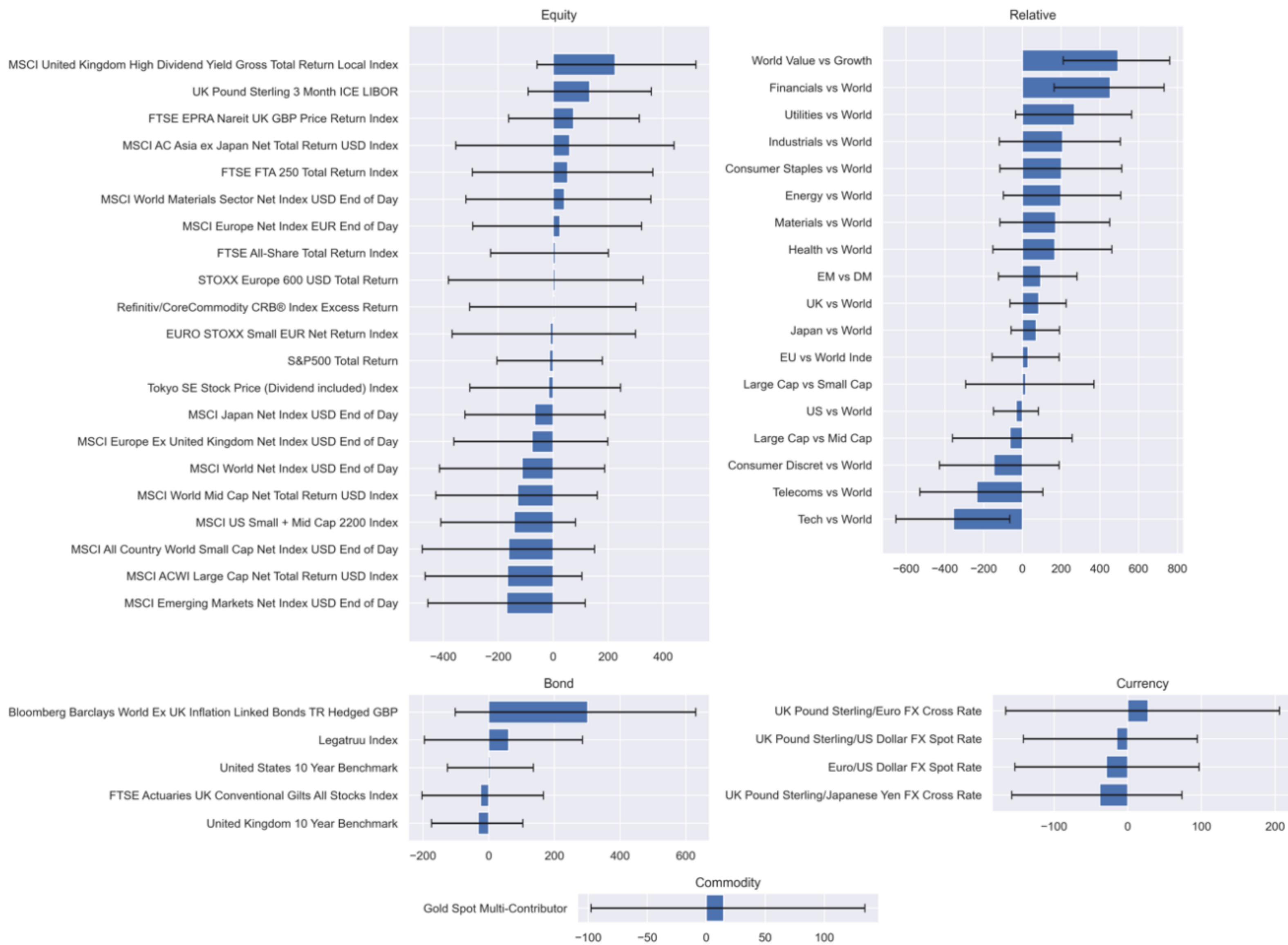


Figure 3.20: Results for High Inflation, Interest Rate Hikes relative to Long Term Average

High Inflation, No Interest Rate Hikes: Sharpe

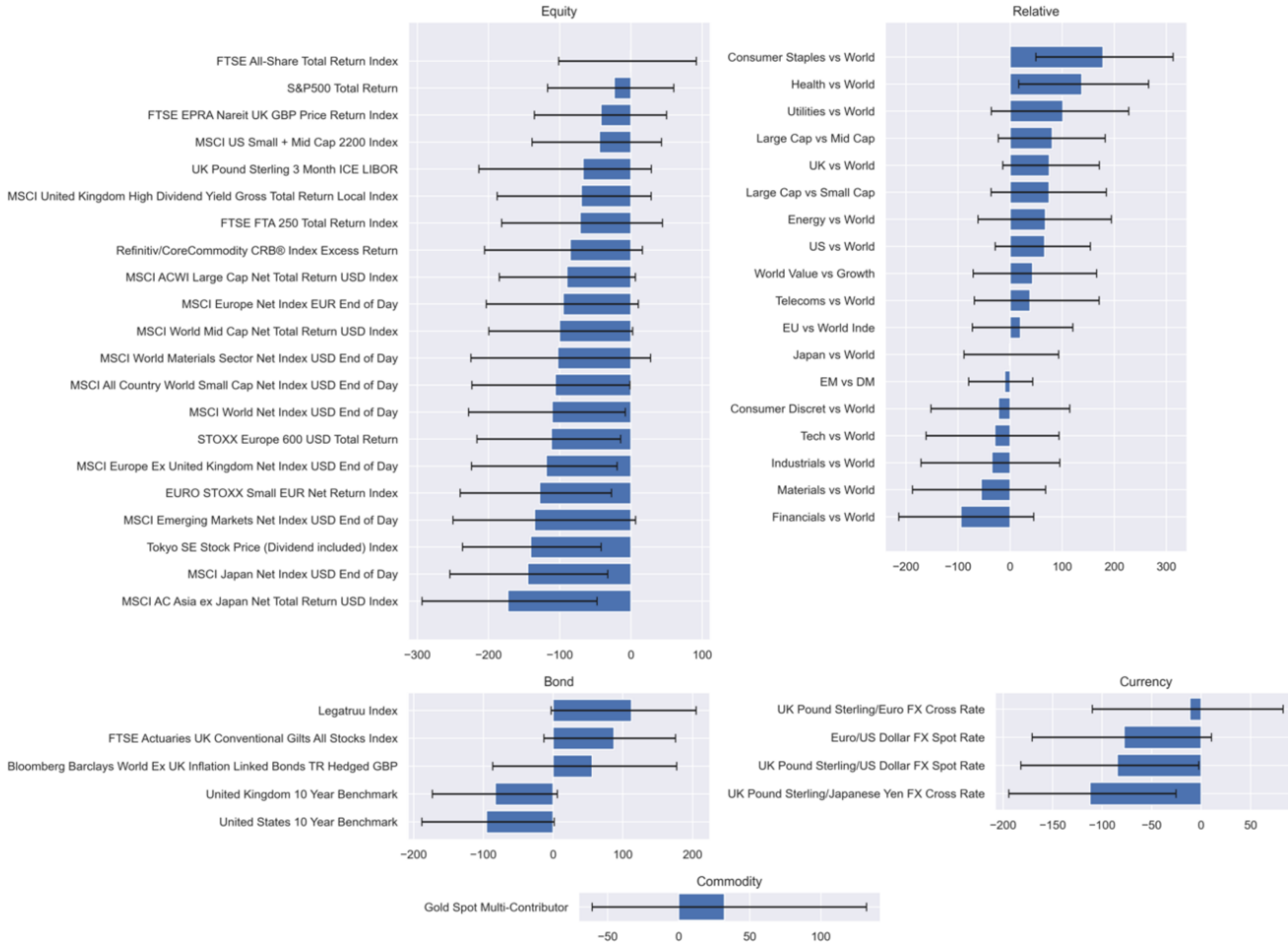


Figure 3.21: Results for High Inflation, No Interest Rate Hikes

High Inflation, No Interest Rate Hikes: Sharpe relative to Long Term Average

Figure 3.22: Results for High Inflation, No Interest Rate Hikes relative to Long Term Average

3.5 Summary of results

In this section we will focus on mean returns as they provide the most clear interpretations. However, we note that looking at historical sharpe ratios does not appear to significantly change our overall conclusions.

3.5.1 High Inflation, Low Growth

In a stagflationary environment, we find that equities perform particularly poorly, both on an absolute basis, and relative to long term average, with statistically significant negative returns in a majority of indexes considered. Within Currencies, we find that the Pound Sterling and Euro underperform relative to the US dollar, and the same for the Pound Sterling relative to the Japanese Yen. Bond indexes are a mixed story, with some indexes showing positive returns and others showing negative or inconclusive results. Within relative indexes, we find that the Consumer Staples vs World, United Kingdom vs World and Health vs World indexes are positive to a statistically significant level.

3.5.2 High Inflation, High Growth

We find that in contrast to a stagflationary environment, many equities classes have positive returns during high inflation and high growth, although not higher relative to their long term average in most cases. Exchange rates do not show positive or negative returns as strongly as the Stagflationary environment, and Gold performs quite well. World Value vs Growth, Utilities vs World and Financials vs World perform well. We believe this is a good demonstration that the specific type of inflation matters when considering past returns, and justifies the use of GDP in the definition of our scenarios.

3.5.3 High Inflation, Interest Rate Hikes

We find it is difficult to make strong conclusions about the positivity or negativity of returns for many of the indexes in this scenario as many confidence intervals are wide, perhaps owning to a lack of data. We find that High Dividend Yield stocks in the UK perform well, along with the World Value vs Growth and Financials vs World indexes. Global Aggregate Bonds (legatruu) and Inflation Linked Bonds also show statistically significant positive returns.

3.5.4 High Inflation, No Interest Rate Hikes

The High Inflation, No Interest Rate Hikes scenario is reminiscent of the stagflationary environment. Equities also perform poorly, Pound Sterling and Euro underperform relative to the US dollar, and the Pound Sterling underperforms relative to the Japanese Yen. Once again, Health Vs World and Consumer Staples vs World show positive returns.

3.6 Utility in Investment Management

We make no assertion that the results that we obtained are likely to hold out of sample. Within Mazars, these results will necessarily be supplemented with further qualitative analysis, based on world news, historical research, analyst opinions, and more. Thus at Mazars, our intention is to use such analysis as the *starting point* for how we will consider questions such as how to invest during inflation. This is because we believe that is is crucial to start from a point of scientific objectivity, giving ourselves the correct prior beliefs, before moving onto the qualitative side.

3.6.1 Individual Periods

When we wish to investigate an individual index further during a specific scenario, we may consider what happened to this index in each separate time period when this scenario held. For example, the High Inflation, Low Growth Scenario, happened 20 separate times in the past; these are outlined in Table 3.3.

If the investment team have determined that a certain index is worth looking into in more detail, we can run circular block bootstrapping on each individual period of a scenario. To make

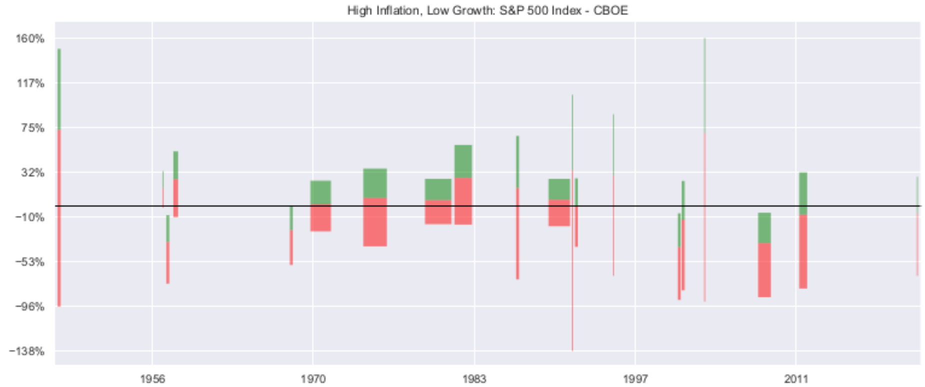


Figure 3.23: Visualization of the S&P 500 during periods of High Inflation and Low Growth

the results of such computations more interpretable we have developed the visualization in 3.23. We place bars at the periods where each individual part of the scenario occurred, each having a width which corresponds to the length of the period. The green part of the bar represents the portion of the confidence interval above the point estimate, and the red part of the bar represents the portion of the confidence interval below the point estimate, with the point estimate being right in the middle. We can also clearly notice from such plots how the width of the confidence interval is affected by the lack of data: scenarios which happened for a shorter period of time show much larger confidence intervals.

We will not include these graphs for every scenario and index in this thesis, but we intend these visualizations to be a invaluable guide when using our analysis for decision making within Mazars.

Chapter 4

Conclusion

During this project, we designed a simple framework for analyzing past returns of asset classes under different economic scenarios. We find that an approach using block bootstrapping is the most suitable for our needs, and implement this approach in order to investigate a number of pressing economic scenarios involving inflation: high economic growth, low economic growth and interest rate hikes. We find that it is possible to make statistically significant conclusions about a large number of indexes, and that the type of inflation is critically important: inflation coupled with low growth has vastly different effects on returns to inflation coupled with high growth.

We argue that such analysis is invaluable to investors faced with unfamiliar scenarios for which they have little experience with, and offers an objective starting point for further analysis. This is a view which is shared by the wealth management team at Mazars, and in part due to the results obtained and the Python tool created (Appendix B), has resulted in the creation of a full time Quantitative Analyst role for the first time within the Wealth Management practice of Mazars UK.

Going forward we believe that there is still scope for improvements and extensions to our analysis. These may include improvements in the definition of existing scenarios, inclusion of additional variables (such as employment), inclusion of indexes with a longer history, as well as possibly more advanced simulation methods. In any case, it is our belief that our framework will be used in Mazars to help guide investment decisions in unfamiliar economic scenarios for years to come.

Appendix A

Index Descriptions

Index	Start Date	Currency
Tokyo SE Stock Price (Dividend included) Index	1993-07-05	JPY
MSCI World Net Index USD End of Day	1998-12-31	USD
FTSE All-Share Total Return Index	1985-12-31	GBP
FTSE FTA 250 Total Return Index	1994-06-24	GBP
FTSE Smallcap Total Return Index GBP End Of Day	1994-06-24	GBP
MSCI Europe Net Index EUR End of Day	1998-12-31	EUR
EURO STOXX Small EUR Net Return Index	1999-01-04	EUR
MSCI Japan Net Index USD End of Day	1998-12-31	USD
MSCI Emerging Markets Net Index USD End of Day	1998-12-31	USD
MSCI World Materials Sector Net Index USD End of Day	1998-12-31	USD
Refinitiv/CoreCommodity CRB® Index Excess Return	1994-01-03	USD
FTSE EPRA Nareit UK GBP Price Return Index	1989-12-29	GBP
UK Pound Sterling 3 Month ICE LIBOR	1986-01-02	
MSCI Europe Ex United Kingdom Net Index USD End of Day	1998-12-31	USD
MSCI ACWI Large Cap Net Total Return USD Index	1995-01-02	NaN
MSCI World Mid Cap Net Total Return USD Index	1994-05-31	NaN
MSCI All Country World Small Cap Net Index USD End of Day	1994-05-31	USD
MSCI AC Asia ex Japan Net Total Return USD Index	2000-12-29	NaN
MSCI United Kingdom High Dividend Yield Gross Total Return Local Index	1998-12-31	NaN
MSCI US Small + Mid Cap 2200 Index	1992-05-29	NaN
STOXX Europe 600 USD Total Return	2001-01-02	NaN
S&P500 Total Return	1988-01-04	USD

Table A.1: Equity Indexes

Index	Start Date	Currency
United Kingdom 10 Year Benchmark	1979-01-02	GBP
United States 10 Year Benchmark	1953-04-15	USD
FTSE Actuaries UK Conventional Gilts All Stocks Index	1985-07-01	GBP
Legatruu Index	1990-01-01	
Bloomberg Barclays World Ex UK Inflation Linked Bonds TR Hedged GBP	1997-01-01	USD

Table A.2: Bond Indexes

Index	Start Date	Currency
UK Pound Sterling/Japanese Yen FX Cross Rate	1975-01-02	JPY
UK Pound Sterling/US Dollar FX Spot Rate	1971-01-04	USD
UK Pound Sterling/Euro FX Cross Rate	1986-05-07	EUR
Euro/US Dollar FX Spot Rate	1975-01-02	USD

Table A.3: Currency Indexes

Index	Start Date	Currency
World Value vs Growth	1998-07-10	USD
Financials vs World	1969-12-31	USD
Industrials vs World	1969-12-31	USD
Tech vs World	1969-12-31	USD
Health vs World	1969-12-31	USD
Utilities vs World	1969-12-31	USD
Consumer Staples vs World	1969-12-31	USD
Consumer Discret vs World	1969-12-31	USD
Telecoms vs World	1969-12-31	USD
Materials vs World	1969-12-31	USD
Energy vs World	1969-12-31	USD
US vs World	1928-01-03	USD
UK vs World	1969-12-31	GBP
EU vs World Inde	1969-12-31	USD
Japan vs World	1969-12-31	USD
EM vs DM	1969-12-31	USD
Large Cap vs Mid Cap	1994-05-31	USD
Large Cap vs Small Cap	1994-05-31	USD

Table A.4: Relative Indexes

Index	Start Date	Currency
Gold Spot Multi-Contributor	1968-03-22	USD

Table A.5: Commodity Indexes

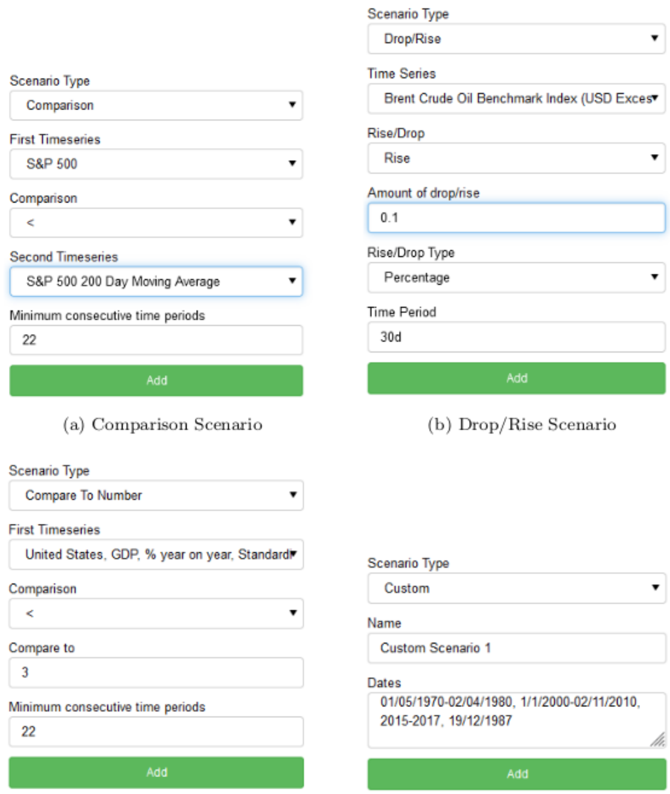
Appendix B

Implementation, user interface and Python Library

It is not our intention that the methods described in this paper should be used for the above scenarios only. Our vision is that these techniques may be used to analyze any economic scenario, as they become relevant.

Therefore at Mazars, we have developed an object-oriented Python library from scratch in which one can define complex economic scenarios and then automatically generate the type of analysis presented in this thesis. Economic scenarios are implemented as *classes* in Python code so that we may make use of inheritance and polymorphism to facilitate efficient code reuse.

As well as a Python interface, a user friendly interface is provided, designed using the `bokeh` library. This interface was designed in order to allow a person without technical knowledge to be able to create their own scenarios, add additional indexes of interest, and produce visualizations similar to those produced above.



(a) Comparison Scenario (b) Drop/Rise Scenario

(c) Compare to a number (d) Custom

Figure B.1: Different Scenarios which can be built out of the box by the scenario builder

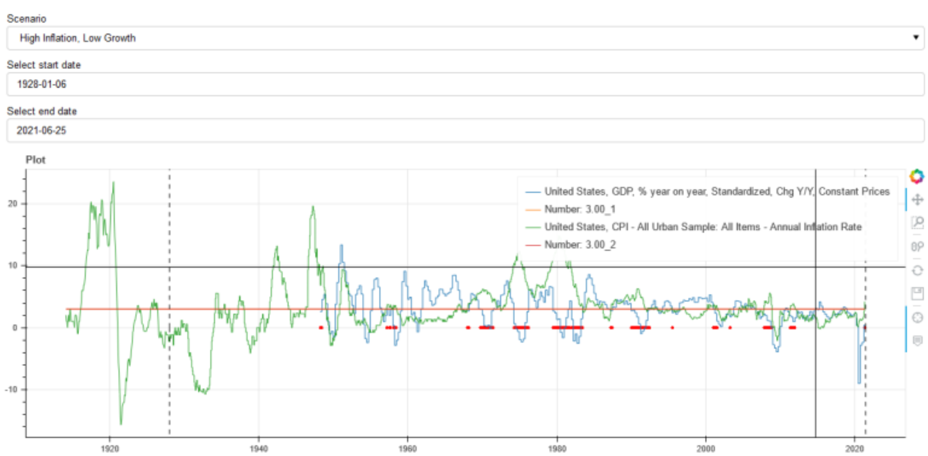


Figure B.2: Interactive plots of the scenario

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