LIBOR DISCONTINUATION

by Ghada Hamieh

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by

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Declaration

The work contained in this thesis is my own work unless otherwise stated.

Signature and date:

Ghada Hamieh

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List of Acronyms

CME	Chicago Mercantile Exchange
ESTER	Euro Short-Term Rate
FCA	Financial Conduct Authority
FRA	Forward Rate Agreement
IBOR	Interbank Offered Rate
ICE	Intercontinental Exchange
ISDA	International Swaps and Derivatives Association
LCH	London Clearing House
LIBOR	London Interbank Offered Rate
LMM	Libor Market Model
OIS	Overnight Indexed Swap
RFR	Risk-Free Rate
SARON	Swiss Average Overnight Rate
SOFR	Secured Overnight Financing Rate
SONIA	Sterling Overnight Index Average
TONA	Tokyo Overnight Average Rate

Contents

1 Introduction

LIBOR, also known as the "world's most important number", has been an active benchmark in the market ever since the 1970s. It was primarily published as BBA LIBOR by the British Bankers' Association on the 1^{st} of January 1986. They wanted to create a standard benchmark that will ensure uniformity for newly traded interest rate instruments back then such as interest rate swaps, FRA and many others. Recently, the role of publishing and administering LIBOR was passed on to the ICE Benchmark Administration. This number now serves as a globally accepted key benchmark for variable interest rates and indicates wholesale unsecured funds borrowing costs between banks. It is widely used in loans, derivatives pricing, economic assessments, and many other things related to interest rates. The methodology to publish LIBOR can be summarized in three stages. First, selected panel banks such as Barclays, JP Morgan Chase, CitiBank Group and few others are asked the following LIBOR submission question: "At what rate could you borrow funds, were you to do so by asking for and then accepting interbank offers in a reasonable market size just prior to 11:00 am?". Then the average of the interest rates reported as an answer is calculated. After that it is published daily by the ICE in five different currencies: GBP, USD, EUR, JPY, CHF and a range of maturities[1].

This benchmark, which was previously seen as immortal, will soon come to an end. Andrew Bailey, CEO of the UK FCA, announced in his speech on "The Future of LIBOR" [1] that it is highly expected that the discontinuation of some IBOR benchmarks will occur in the near future and that they will only support the submission of LIBOR up to 2021. This was the beginning of the journey towards a LIBOR-less world. He then assured this news in another speech "LIBOR: Preparing for an end" [2] stating that LIBOR discontinuation is something that will definitely happen. After that J. Christopher Giancarlo, Chairman of the Commodity Futures Trading Commission also stressed that this discontinuation is inevitable after the end of 2021.

We shall now discuss the main reason that triggered this decision. It was recently agreed that rates must be tied to actual transactions. They must not be based on surveys and judgments of some panel banks to ensure that the rates are genuinely representing the market conditions. This need was mainly because the methodology explained above, that is highly dependent on expert judgments, allowed and even facilitated manipulation of this rate by the banks involved. This was

[1] The active maturities are: ON, 1W, 1M, 2M, 3M, 6M, 12M

evident in the LIBOR scandal in 2012 when some banks were caught reporting false interest rates to benefit their trades and increase their profit. On top of that, some evidence suggested that this was going on undetected for several years.

However, it is so difficult to apply this change in methodology on LIBOR since the underlying market that LIBOR seeks to measure is the market for unsecured short term lending to banks which is no longer sufficiently active or liquid to validate the banks' judgments. Therefore, there is a great need to shift from LIBOR to another system of credible variable interest rate benchmarks that can be calculated based on sufficiently available transactions and liquid money markets. Therefore, the ideal alternative reference rates as explained in [19, page 30] would have to be: strong enough not to be manipulated, reflect accurately on the interest rates market, able to offer as a reference rate in contracts, and benchmark for term lending and funding.

UK, US, Europe, Switzerland and Japan have already selected new overnight RFRs to replace LIBOR as the reference rate in new contracts which are SONIA, SOFR, ESTER, SARON, TONA respectively. All of these satisfy at least two of the following attributes: have short maturity preferably overnight, measure not just wholesale lending costs in inter-bank markets but also incorporate non-bank counter-parties,[2] and are based on secured transactions[3].They are all available now except for ESTER that will be available on October 2019. Moreover, other benefits of using overnight rates are that they free borrowers the exposure to bank credit risk that is embedded in rates with longer tenors and eliminate the cash and derivatives market basis risk. SONIA, which will be our main focus in this paper, is not a new overnight reference rate like the case with SOFR, for example. It has been offered in the market ever since March 1997, and it constitutes a substantial rate in the Sterling financial markets. It is has been administered by the Bank of England since April 2016. As an attempt to strengthen this rate, especially after choosing it to replace the dying LIBOR, they started publishing "reformed SONIA" on the 23^{rd} of April 2018. The main improvement of this benchmark as mentioned in [4] is that now it is based on a significantly more extensive range of overnight unsecured deposits since they included *"bilaterally negotiated transactions alongside brokered transactions"*. Volumes related to the rate now value about £50 billion daily, which makes it indeed way more robust and informative.

[2]Non-bank counter-parties include cash pools, other investment funds, insurance companies ...

[3]Credit transactions in which the lender acquires a security interest in collateral owned by the borrower and is entitled to repossess the collateral in the event of the borrower's default.

The greatest challenge in this pre-discontinuation period is ensuring a smooth transition from LIBOR to the new reference rates in existing contracts that will mature after 2021 in the post-LIBOR era and decreasing the exposure to LIBOR in the upcoming contracts. This would be pretty difficult for most financial institutions since LIBOR is anchored in their pricing infrastructure, hedging and risk management.

Over the past two years, LIBOR transition has been a very hot topic in the financial industry. Many consulting companies and banks published reports explaining the financial and legal threats accompanying LIBOR transition in existing contracts. They also often include news updating on the new reference rates. The FCA members in their turn have been regularly making speeches to update on the LIBOR transition and giving advises ensuring a smooth one. However, very few tackled the impact of this event on quantitative finance which will be at this paper's core. Henrard published a couple of papers ([8],[9], and [10]) highlighting the link between LIBOR transition and quantitative finance. He thoroughly commented on the several choices of the fallback suggested in the ISDA consultation and the undesirable outcomes following these choices, such as value transfer, manipulation, convexity adjustment .... He also demonstrated the change in the pricing scheme due to the expected discontinuation. Mercurio also published a paper [17] highlighting the quantitative side of this issue. He was introducing the instruments [4] used to bootstrap a SOFR curve which is essential now that it will become the new reference rate. He also proposed a multi-curve framework and used it to price these instruments. He then showed how the evaluation of the price of a swap and basis swap would change upon introducing the new fallback in the pricing formulas. Another paper [16] was published by Mercurio and Lyashenko, where they developed an extended LMM to model the dynamics of backward-looking rates which are strong candidates for the choice of the term rate. They then used this model to price some vanilla derivatives referencing RFRs[5].

In this paper, we will first introduce and discuss in details the choices of term rate and spread adjustment suggested in the ISDA consultation. This will help us fully understand the constituents of the LIBOR fallback before working with it. Then we will adopt the multi-curve framework introduced in Mercurio's paper [17]. Using this model, we will derive a closed-form for the forward rate of the LIBOR fallback. In order to build this fallback, we will choose the compounding setting in arrears for the term rate and the historical mean/median approach for the

[4] 1M-Futures, 3M-Futures and SOFR fixed-floating swaps

[5] Futures, Fixed-floating swaps, Cap, Swaption, term-rate basis swap and associated cap

spread adjustment. This will then be used to re-price LIBOR fixed-floating swaps and basis swaps that will mature in the post-LIBOR era. After that, we will analyze the impact of this choice of the fallback on the prices of the swaps and then discuss further the issue of value transfer using historical data. Finally, we will price some RFR linked derivatives that are being traded in large volumes recently as an attempt to accelerate the transition.

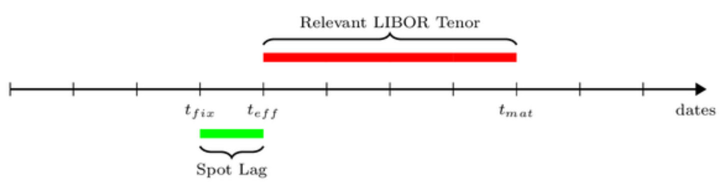
2 Fallback Description

Fallback is a replacement of a discontinued benchmark such as LIBOR by an adjusted RFR and a spread adjustment added on top of it. Financial Stability Board asked ISDA to lead the derivatives industry efforts to develop a fallback rate for LIBOR in existing contracts. Such efforts aim to help market participants understand the fallback preventing a potential market disruption in case of permanent discontinuation of LIBOR.

2.1 LIBOR Dates

In order to discuss the different choices of the fallback , we first need to understand the details of a LIBOR coupon and know the dates associated with the fixing of this LIBOR and the derivatives where it is used.

The dates associated with the fixing are depicted in the diagram below:



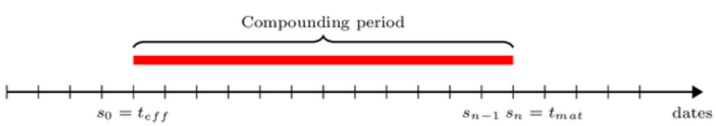
Usually this number is published on the fixing date. However, it is not necessarily effective starting this date. The starting date of the underlying deposit or what is denoted above by effective date is two business days after the fixing date ($t_{eff} = t_{fix} + 2bd$) for the USD LIBOR for example and 0 days for GBP LIBOR which we will be working with throughout this paper. Finally the maturity date depends on the tenor associated with each benchmark. For the LIBOR-linked derivatives' dates, they are usually the same as that of the LIBOR fixing except that the payment date of the former might not be the same as the maturity date of the latter due to non-good business days. This means that the underlying deposit of the LIBOR might be a bit longer than the accrual period of the derivative. An example of such situation is provided by Henrard in [8].

2.2 Choice of term rate

LIBOR is by definition a forward term rate. It represents the interest rate for an upcoming period of time. On the other hand, the suggested replacement were the RFRs which are all so far overnight rates. They represent the cost of lending for one day period or more precisely O/N.

Therefore, in order for these to be used as a replacement for LIBOR in both new and existing contracts, they should be converted into term rates. ISDA and national regulators suggested different methodologies to create this adjusted RFR. The two potential choices that were accepted by almost all parties are :

- **Backward Looking Approach:** This methodology consists of creating a compounded setting-in-arrears rate. In this method the selected RFR is compounded over the same relevant LIBOR tenor period corresponding to the LIBOR it will replace which will ensure that both rates cover the same economic reality. Due to its backward nature it is known at the end of the associated application period. This makes it suitable for derivatives where the payment occurs at the maturity date and not the beginning of the period. The most common example of such derivatives is the fixed-floating swap and basis swap.



The compounding period is divided into daily intervals starting from the effective date of the LIBOR and ending on the maturity date. This rate is calculated using the equation below:

$$(\prod_{i=1}^{n}(1 + \delta_i {I_i}^s) - 1) * \frac{1}{\delta} \tag{2.1}$$

where: δ represents the accrual factor according to the day count convention

$$\delta = \frac{accrual\ period\ in\ days}{365}$$

and ${I_i}^s$ is the SONIA fixing for this day. Applying this method to get the adjusted RFR might be troublesome in some cases due to the dates associated with it. If the payment date of the derivative we are dealing with is before the maturity date of the relevant LIBOR then the underlying rate given by this approach will be unknown at the payment date. Henrard described this problem as "lack of measurability".

- **Forward Looking Approach:** The need for forward looking approach to evaluate the term rate originated mainly from two reasons. First, LIBOR is, as mentioned above, a forward rate, so it is known at the beginning of the underlying deposit period which allows traders to know the payments in advance and ensure cash flow certainty. Therefore, traders would prefer working with an alternative rate that depicts the same nature as LIBOR. Second, in some derivatives, payments are made at the beginning of the period. A typical example of such derivatives is FRA. Payments of about $84 trillion dollars use forward rates and cannot use the above backward rate as a fallback and thus will have to undergo some changes in their term sheets. This choice will also avoid the problem of the dates that we might face using the former approach. How to create a benchmark using this approach is still not decided yet. However, there are now two candidates. They would either create an OIS benchmark or use the forward or spot OIS rates given by:

$$F^{ois}(t, t_{eff}, t_{mat}) = \left(\frac{P^{ois}(t, t_{eff})}{P^{ois}(t, t_{mat})} - 1\right) \frac{1}{(t_{mat} - t_{eff})} \tag{2.2}$$

We will prove in section 3.2.1 that the forward rate of the compounded setting in arrears is actually equal to the forward OIS rate if applied on the same period. Mercurio in [16] described this choice as a market implied prediction of the compounded setting-in-arrears rate. Therefore we can say that from a pricing perspective these two approaches seem to be consistent.

2.3 Choice of spread adjustment

Creating a term rate is not enough to allow RFRs to replace LIBOR. LIBOR is, by nature, a risky rate. This number absorbs a bank's credit risk premium, liquidity and fluctuations in supply and demand in the market. On the other hand, RFRs are overnight rates which implies that they represent the rate of return of investment with almost no financial loss or risk of default given its very short tenor. Therefore, any other rate covering a more extended period must be higher to incorporate the credit risk associated with the given period. ISDA, as a result, suggested adding to this term rate a fixed spread adjustment that would reflect the risky and informative element embedded in LIBOR. The spread adjustment will be calculated one business day before the announcement of the discontinuation and then used once the actual discontinuation takes place. ISDA suggested in its consultation three different choices for this spread :

- **Forward Approach:** In the ISDA consultation document, they proposed that the spread using this approach will be calculated as follows: "*The spread adjustment could be calculated based on observed market prices for the forward spread between the relevant IBOR and the adjusted RFR in the relevant tenor at the time the fallback is triggered.*". Therefore this would require to construct forward spread curves, derived from Forward LIBOR curves and Forward adjusted RFR discount curves, for each relevant tenor that we are interested in finding its fallback. We require them to be up to 30-60 years in order to cover all existing LIBOR-linked contracts, including those with the longest maturities. This spread will thus reflect the market's perception of the future on the day the fallback takes effect and freeze it over the curve from that day onward. It will be present value-neutral on the announcement day. However, it will not be so accurate for further dates since spot rates are volatile and they are unlikely to coincide with forward rates. The formula for this spread at the announcement date t_0 is given by:

$$S(t_0, T) = L(t_0, T, T + lM) - R(t_0, T, T + lM) \tag{2.3}$$

Alternatively, they were suggesting that instead of taking the observation on one day, we can take it on δ days (about 10 to 20 business days) around the announcement date t_0. Therefore this will be the formula:

$$S(t_0, T) = \sum_{s=t_0-\delta}^{t_0} L(s, T, T + lM) - R(s, T, T + lM) \tag{2.4}$$

The disadvantage of such an approach is mainly on the operation side. It involves extensive market data, including very liquid instruments even for high maturities and agreement on the construction methodology such as bootstrapping and interpolation techniques. Due to its nature, it is also susceptible to manipulation, which is the main thing we were trying to eliminate via LIBOR discontinuation.

- **Spot-Spread Approach:** This approach sets a constant spread over the whole curve. This spread would be equal to the spot LIBOR- adjusted RFR spread on the announcement date. The formula here is simple and given by:

$$S(t_0) = L(t_0, t_0 + lM) - R(t_0, t_0 + lM) \tag{2.5}$$

Similar to the approach above they were thinking of doing this calculation over a longer period δ which will change the formula to:

$$S(t_0) = \sum_{s=t_0-\delta}^{t_0} L(s, s + lM) - R(s, s + lM) \tag{2.6}$$

From the implementation side, this method is so simple to grasp and implement. However, it was rejected by most of the respondents to this consultation claiming that it is prone to "*manipulation, extreme conditions and arbitrage*". Moreover, we can consider it as a tiny part of the following approach.

- **Historical mean/median Approach:** The historical mean/median approach used to evaluate this spread is based on taking either the mean or the median of the daily spread between the two term rates in question over a specific look-back period. Denote by the mY: the look-back period and lM: the tenor of the LIBOR used. Therefore, the mean can be evaluated by:

$$S(t_0) = \frac{1}{n}\sum_{i=0}^{n} L(T_i, T_i + lM) - R(T_i, T_i + lM) \tag{2.7}$$

where: T_0=t_0-mY and $T_n = t_0$.

On the other hand, the median is the $\frac{n+1}{2}^{th}$ value in the ordered set A = $\{L(T_i, T_i + lM) - R(T_i, T_i + lM)$; i = 0,. . . ,n }

This methodology for computing the spread is completely based on historical realizations and market conditions before the discontinuation's announcement date and somehow assumes that the past will represent the market's view of the future which is not the case in real life. It also yields value transfer in the market. This issue will be discussed thoroughly in section 4.

There is a big possibility that if the choice of spread adopted is the historical mean/median, then the Spread adjustment wont be equal to the spot spread between the adjusted RFRs and the spot LIBOR on the announcement date. The reason is mainly that the latter is volatile and we cannot guarantee that it will be equal to the long term average we will be using. If we look at graphs 1, 2, and 3 we can see that even after the announcement date of the fallback details the spreads are not converging to a unique average or mean. They are somehow fluctuating around the range of possible spread adjustments but not converging to a unique value. This difference cannot be disregarded.

As an attempt to mitigate an almost inevitable 'Cliff Effect', ISDA suggested having a one-year transitional period $[S_1, S_2]$ that extends from the discontinuation date up to one year on which after that the spread used would be the historical mean/median S(t_0). This method reintroduces the Spot-Spread Approach and combines it with the historical mean/median through a linear interpolation method. In this way, the actual spread will converge gradually to a long term average instead of a sudden change. Therefore the spread in this transitional period will be calculated using the linear interpolation formula suggested by ISDA [12]:

$$S(t,T^*) = \frac{S^2 - t}{S^2 - S^1}(L_f(t_0) - R_f(t_0)) + \frac{t - S^1}{S^2 - S^1}S(t_0) \tag{2.8}$$

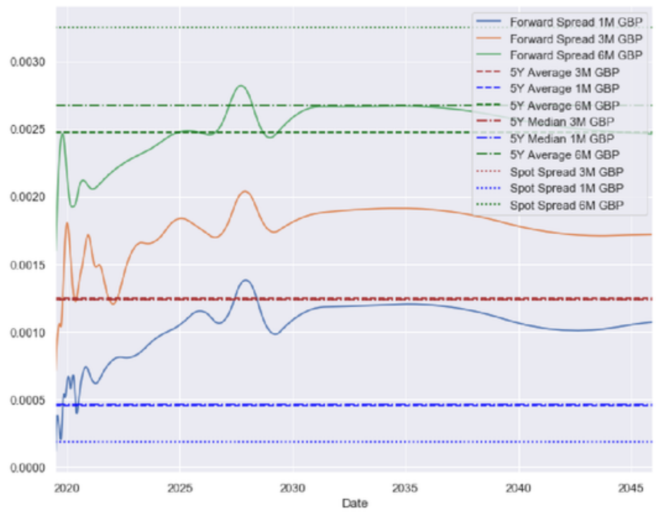
However, many participants were arguing that this method introduces complexity that outweighs its advantages. Therefore it might be omitted. The effect of this transitional period will be discussed later in this paper after implementing it.

One of the main advantages of this approach is that it is easy to implement and almost free of any ambiguity and manipulation.

The choice that was preferred by the vast majority was **Historical mean/ median Approach**. However, this is not yet the final decision, and if it will be, they still didn't decide on the exact look-back period, whether it will be median or mean, and whether they will eliminate the transitional period or not.

2.4 Results and Discussion

In order to have a sense of the difference between each of the above methods, we found the spread using the above methods for 1M GBP LIBOR, 3M GBP LIBOR and 6M GBP LIBOR as of 01/07/2019. The forward approach was made using cubic interpolation method and LogDF as the interpolation variable. The look-back period of the historical mean/median approach is 5 years here. Since we are doing this comparison using the compounding setting in arrears as term rate then we have to compute the historical and spot approach lM before t_0 depending on the LIBOR we are using to ensure that the rates are known. The results are shown in the graph below:



We can see that the spot spread is most likely to be different from the historical average. Moreover, the spread using the forward approach seldom matches with other two approaches, especially in the case of 3M and 1M GBP LIBOR. Each approach reflects on the difference between LIBOR and the RFRs differently.

Since the market preference is leaning more towards the Compounding Setting in Arrears along with the historical mean/median approach as mentioned in the Preliminary Results of ISDA Consultation, these will be our main focus in the paper. In order to study several possible choices for the look-back period and methods of computation, we ran six possible scenarios to visualize the evolution of the possible choices from the start of the look-back period up till 01/07/2019. The six scenarios are: Rolling mean over 10, 7 and 5 years and Rolling median over 10, 7 and 5 years. In all of the above scenarios, we assume that the announcement date is on $t_0 = 01/07/2021$. Therefore the calibration date T^* would be t_0-lM since we are using compounding setting in arrears which are just known at the end of the associated underlying deposit. The daily spread DS(t) is calculated as the difference between the LIBOR fixing published on that day and the corresponding compounded setting-in-arrears rate. The latter is calculated using historical data of the SONIA fixings and equation (2.1). After we get the historical fixings of $L(T_{j-1}, T_j)$ and $R(T_{j-1}, T_j)$ and the daily spread, we calculate the running averages and medians on the three proposed look-back periods.

We can see that the 10Y average is scoring the highest values in the three different cases, and this is due to the period between 2011 and 2013, where there were significantly higher spreads.

Concerning the other look-back periods, there were not any periods of remarkably high spreads in the 7Y period and not in the 5Y period. Therefore, they are closer to each other than to the average calculated over 10 years. If the economic state continues to be more or less stable, then the averages won't differ significantly, and the spread can be anticipated. However, this thing is not guaranteed at all. If any severe economic event occurred soon, a famous example is Brexit, these spreads might diverge. Currently, based on the historical observations, we can say that the 10Y-average incorporates the credit element of LIBOR in its extreme cases. However, the spread adjustment applied to the 5Y and 7Y look-back period represents the moderate risk associated with the LIBOR.

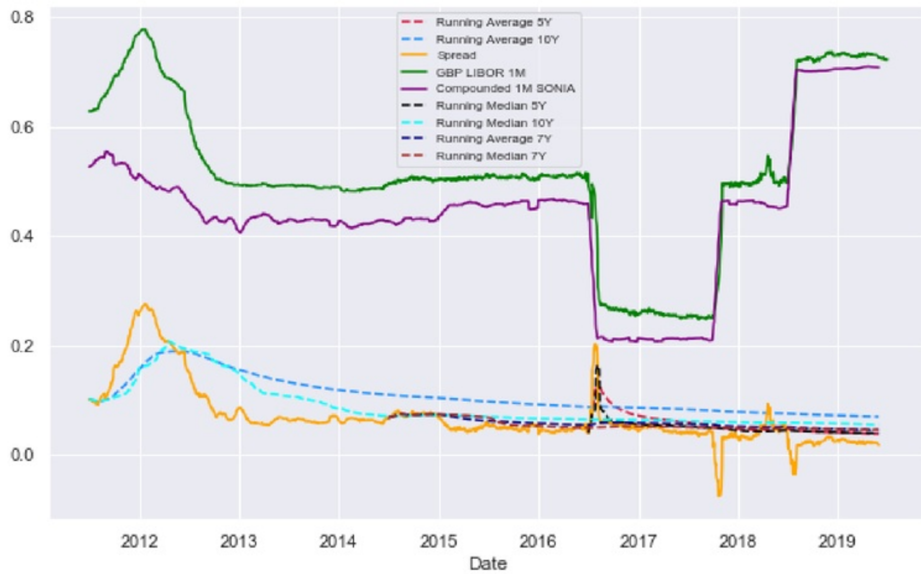


Figure 1: GBP 1M LIBOR

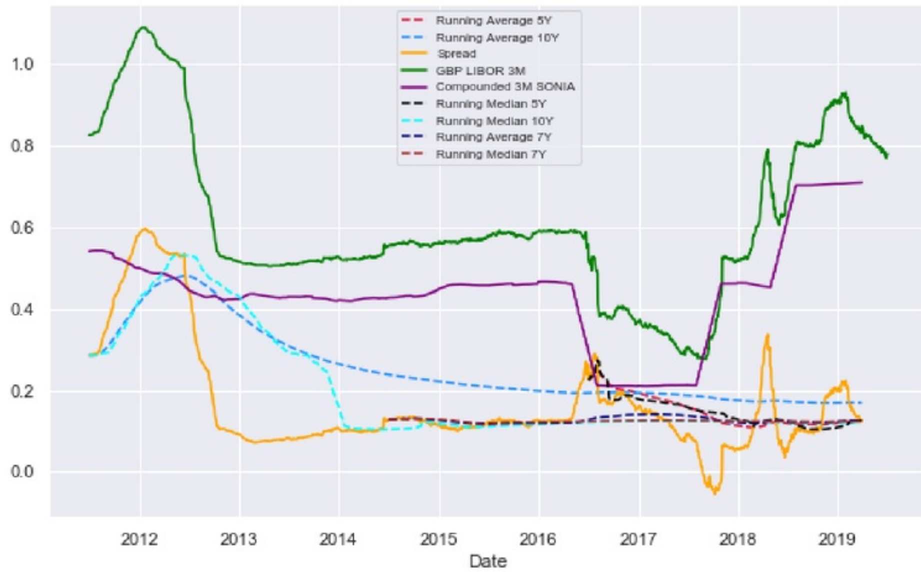


Figure 2: GBP 3M LIBOR

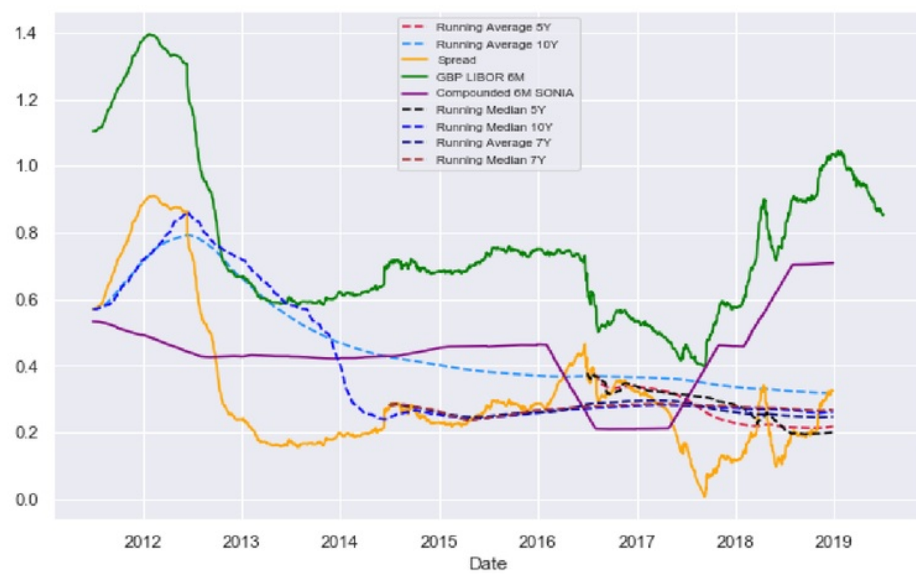


Figure 3: GBP 6M LIBOR

3 Impact on the Valuation Methodology of Derivatives

There is still a huge volume of derivatives referencing LIBOR and maturing beyond 2022 traded in the market especially swaps. The figure below shows the exact amounts of interest rate derivatives referencing LIBOR with different currencies and maturity dates as of the first quarter of 2019.

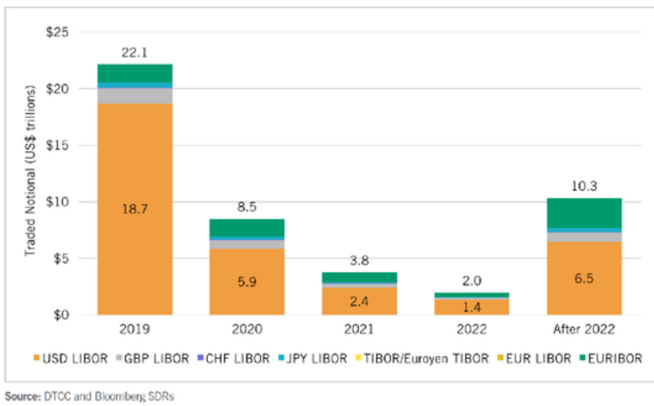


Figure 4: SONIA Futures Data from the ISDA report: Interest Rate Benchmarks Review: First Quarter of 2019, April 2019 [13]

This necessitates changing the valuation methodology of these derivatives to adapt to the information that LIBOR won't exist after the end of 2021 and a new fallback will replace it. This will help us reflect on accurate future cash flows. Some might argue that participants will lessen their exposure to LIBOR-linked products, exit such deals or change the term sheets. Even if that was true, it is still essential to have the real present value of such products even if they will be exited and thus forecast the possible losses or even gains after the disappearance of LIBOR. This will allow us to be prepared for the discontinuation, avoid unpleasant surprises and be well aware of any undesirable consequences and risks. Moreover, this will also be useful for hedging purposes. In this section, we will use a specific model in order to find the closed form of the new valuation methodology of swaps and basis swaps. This will then be used to quantify the impact on the present value. We will use the model proposed in Mercurio's paper [17]. In order to build the LIBOR fallback, we will use the backward looking approach to get the term rate and historical mean approach to get the spread adjustment. This choice is based on the ISDA Consultation results.

3.1 New Payoffs

The change in the valuation scheme mainly originates from a change in the payoff associated with each derivative due to the arrival of vital new information into the market, which is LIBOR discontinuation. We can no longer take the existence of LIBOR for granted as we used to do. The pricing of a LIBOR linked derivative with discounted payoff H as a function of the LIBOR fixings has to change from:

$$V(t) = \mathbb{E}_t[H(\text{LIBOR fixings})] \tag{3.1}$$

to:

$$V(t) = \mathbb{E}_t\left[H(\text{LIBOR fixings})\mathbf{1}_{\{\text{LIBOR exists}\}} + H(\text{LIBOR fallback})\mathbf{1}_{\{\text{LIBOR doesn't exists}\}}\right] \tag{3.2}$$

Similar change is applied on the LIBOR coupon in [8, page 8]. We will now discuss further this change applied on LIBOR Fixed-Floating Swap and LIBOR-SONIA Basis Swap.

3.1.1 LIBOR Fixed-Floating Swap

A standard LIBOR-based swap is a contract that exchanges payments between two differently indexed legs. The floating leg pays at each time T_j, where j = a+1,…,b, the T_{j-1}-spot LIBOR L(T_{j-1}, T_j) multiplied by the accrual factor τ_j. The fixed leg pays the fixed rate K on S_j j = c+1,…,d also multiplied by τ_j. Note that $T_a = S_c$ and $T_b = S_d$.

The discounted payoff of the payer swap is:

$$H = \sum_{j=a+1}^{b} D(t,T_j)\tau_j L(T_{j-1},T_j) - K \sum_{j=c+1}^{d} D(t,S_j)\tau_j \tag{3.3}$$

where D(t,T_j)=$\frac{B(t)}{B(T_j)}$ is the discount factor.

The t-time value of the payer swap where $t < T_{a+1}$is:

$$\begin{aligned}
V(t) &= \mathbb{E}_t[H] \\
&= \sum_{j=a+1}^{b} \mathbb{E}_t[D(t,T_j)L(T_{j-1},T_j)]\tau_j - K \sum_{j=c+1}^{d} \mathbb{E}_t[D(t,S_j)]\tau_j \\
&= \sum_{j=a+1}^{b} P(t,T_j)\, \mathbb{E}_t^{T_j}[L(T_{j-1},T_j)]\tau_j - K \sum_{j=c+1}^{d} P(t,S_j)\tau_j \\
&= \sum_{j=a+1}^{b} P(t,T_j)L_j(t)\tau_j - K \sum_{j=c+1}^{d} P(t,S_j)\tau_j
\end{aligned}$$

This valuation requires $L(T_{j-1}, T_j)$ to be published up to time T_b since $L_j(t)$ by definition is based on these values. However, if the LIBOR discontinuation will take place at a time S^* before T_b then the cash flows will change. It will depend on LIBOR before a certain time T_k included and on the new LIBOR fallback after this time.
The new valuation will be as follows:

$$V(t) = \sum_{j=a+1}^{k} P(t,T_j)L_j(t)\tau_j + \sum_{j=k+1}^{b} P(t,T_j)\hat{L}_j(t)\tau_j - K\sum_{j=c+1}^{d} P(t,S_j)\tau_j \tag{3.4}$$

$\hat{L}_j(t)$ is the forward rate of the LIBOR Fallback.

3.1.2 LIBOR-SONIA Basis Swap

A LIBOR-SONIA basis swap is a floating-floating interest rate swap. One of the legs is the floating leg of a LIBOR fixed-floating swap and the other one is the floating leg of a SONIA referenced swap having same maturity and payment schedule. Consider T_a as the start date and let the payment dates be : $T_{a+1}, \ldots, T_b$. The new payoff will change in a similar way as the one in the above swap. It will become :

$$V(t) = \sum_{j=a+1}^{b} P(t,T_j)R_j(t)\tau_j - \sum_{j=a+1}^{k} P(t,T_j)L_j(t)\tau_j - \sum_{j=k+1}^{b} P(t,T_j)\hat{L}_j(t)\tau_j \tag{3.5}$$

instead of :

$$V(t) = \sum_{j=a+1}^{b} P(t,T_j)R_j(t)\tau_i - \sum_{j=a+1}^{b} P(t,T_j)L_j(t)\tau_j \tag{3.6}$$

3.2 Framework

The model used here is a two-factor multi-curve model where the OIS rate is approximated by the instantaneous rate r(t) and modelled jointly with the forward LIBOR rates.

3.2.1 OIS Rate Dynamics:

In this model r(t) is assumed to follow a one-factor Hull-White model such that :

$$r(t) = x(t) + \alpha(t), \tag{3.7}$$

where, α is a deterministic function, and

$$\mathrm{d}x(t) = -ax(t)\mathrm{d}t + \sigma \mathrm{d}Z(t) \tag{3.8}$$

where a $>$ 0, σ is deterministic, and Z is a standard Brownian motion under the risk neutral measure Q.

The price at time t of the OIS zero-coupon bond with maturity T is given by:

$$P(t,T) = E_t[e^{-\int_t^T r(u)du}] = e^{-\int_t^T \alpha(u)du} A(t,T) e^{-B(t,T)x(t)} \tag{3.9}$$

where,

$$A(t,T) = exp\{\tfrac{1}{2}\int_t^T \sigma^2(u)B^2(u,T)du\}$$
$$B(t,T) = \tfrac{1}{a}[1 - e^{-a(T-t)}]$$
$$e^{-\int_t^T \alpha(u)du} = \tfrac{P(0,T)A(0,t)}{A(0,T)P(0,t)}$$

The calculations done to get the above formula of the zero-coupon bond are provided in the appendix of Mercurio's paper [17]

Using the above computations and assuming daily compounding, we can derive the daily-compounded OIS forward rate for a given interval $[T_{j-1}, T_j]$. This will be the forward rate of the chosen adjusted RFRs in our case.

In the following calculations the interval $[T_{j-1}, T_j]$ is divided into n consecutive days where t_0=T_{j-1} and $t_n = T_j$

The daily compounded OIS rate is:

$$R(T_{j-1}, T_j) = \frac{1}{\tau_j}[\prod_{i=1}^{n}(1 + \tau_i r(t_{i-1})) - 1] \tag{3.10}$$

Then its forward rate is given by:

$$R_j(t) = \frac{1}{\tau_j}\mathbb{E}_t^{T_j}[\prod_{i=1}^{n}(1 + \tau_i r(t_{i-1})) - 1] = \frac{1}{\tau_j P(t,t_n)}\mathbb{E}_t[e^{-\int_t^{t_n} r(t)ds}\prod_{i=1}^{n}(1 + \tau_i r(t_{i-1}))] - \frac{1}{\tau_j}$$

This equality is satisfied by applying the change of measure from the t_n-measure with numeraire P(.,t_n) to the risk neutral measure with numeraire bank account B(.).

Now note that $\mathbb{E}_{t_{i-1}}[e^{-\int_{t_{i-1}}^{t_i} r(s)ds}(1 + \tau_i r(t_{i-1}))] = P(t_{i-1}, t_i)(1 + \tau_i r(t_{i-1})) = 1$

Then using tower property,

$$1 + \tau_j R_j(t) = \frac{1}{P(t,t_n)}\mathbb{E}_t\left[\mathbb{E}_{t_{n-1}}\left[e^{-\int_t^{t_0} r(s)ds}\prod_{i=1}^{n} e^{-\int_{t_{i-1}}^{t_i} r(s)ds}(1 + \tau_i r(t_{i-1}))\right]\right].$$
$$= \frac{1}{P(t,t_n)}\mathbb{E}_t\left[e^{-\int_t^{t_0} r(s)ds}\prod_{i=1}^{n-1} e^{-\int_{t_{i-1}}^{t_i} r(s)ds}(1 + \tau_i r(t_{i-1}) * \mathbf{1}\right].$$

If you repeat this procedure successively over the values of i from n down to 1, we get:

$$1 + \tau_j R_j(t) = \frac{\mathbb{E}_t[e^{-\int_t^{t_0} r(s)ds}]}{P(t,t_n)}$$

Finally, since $t_0 = T_{j-1}$ and $t_n = T_j$ we get the final form of the forward rate as:

$$R_j(t) = \frac{1}{\tau_j}\left\{\frac{P(t,T_{j-1})}{P(t,T_j)} - 1\right\} \tag{3.11}$$

Another approach to reach this result can be found in [7, Theorem 2.4].

We get the same forward rate if we assume continuous compounding instead. The formula for continuous compounding OIS rate is given by:

$$R(T_{j-1},T_j) = \frac{e^{\int_{T_{j-1}}^{T_j} r(u)du} - 1}{\tau_j}$$

Then the forward rate can be calculated by the following steps:

$$\begin{aligned}
1 + \tau_j R_j(t) &= \mathbb{E}_t^{T_j}\left[e^{\int_{T_{j-1}}^{T_j} r(s)ds}\right] \\
&= \frac{1}{P(t,T_j)}\mathbb{E}_t\left[e^{-\int_t^{T_j} r(s)ds} e^{\int_{T_{j-1}}^{T_j} r(s)ds}\right] \\
&= \frac{1}{P(t,T_j)}\mathbb{E}_t\left[e^{-\int_t^{T_{j-1}} r(s)ds}\right] \\
&= \frac{P(t,T_{j-1})}{P(t,T_j)}
\end{aligned}$$

Therefore we reach the conclusion that:

$$R_j^{cont}(t) = R_j^{daily}(t) = \frac{1}{\tau_j}\left[\frac{P(t,T_{j-1})}{P(t,T_j)} - 1\right]$$

3.2.2 Forward LIBOR Dynamics:

In this suggested framework the forward LIBORs will follow, under the associated forward measure Q_j, the shifted Lognormal LMM which is represented as follows:

$$dL_j(t) = \sigma_j(t)[L_j(t) + \alpha_j]dW_j(t) \tag{3.12}$$

σ_j's are considered to be deterministic and α_j's are constant. This model allows non-flat volatility structures while preserving the analytic tractability. We will assume the case of one-factor model i.e. $dW_j(t)dW_i(t) = dt$ for all i, j.

In order to express the forward LIBOR dynamics under the risk neutral measure as well, we will apply the method of change of measure [5]. The drift under the $\mathbb{Q}$- measure is thus given by:

$$\begin{aligned}
\text{Drift}(L_j, Q) &= \frac{d < L_j, \ln\left(\frac{B(.)}{P(.,T_j)}\right) >_t}{dt} \\
&= -\frac{dL_j(t)\ d\ln P(t,T_j)}{dt} \\
&= \rho\sigma_j(t)\sigma(t)B(t,T_j)[L_j(t) + \alpha_j]
\end{aligned}$$

Once we get this drift, we can get the forward rate dynamics under the risk neutral measure by:

$$dL_j(t) = \rho\sigma_j(t)\sigma(t)B(t,T_j)[L_j(t)+\alpha_j]dt + \sigma_j(t)[L_j(t)+\alpha_j]dW(t) \tag{3.13}$$

where $dW(t)dZ(t) = \rho\ dt$

In order to solve for $L_j(t)$ following the above dynamics we will let $\tilde{L}_j(t) = L_j(t) + \alpha_j$. Therefore we get that $\tilde{L}_j(t)$ is a geometric BM and thus :

$$\tilde{L}_j(t) = \tilde{L}_j(0)e^{\int_0^t [\rho\sigma_j B(s,T_j)\sigma(s) - \frac{\sigma^2{}_j(s)}{2}]ds + \int_0^t \sigma_j(s)dW_s} \tag{3.14}$$

The details of the calculation are provided in Appendix A.

3.3 Calibration

We will need to calibrate the model in order to get the necessary volatilities. In what follows we will fix a=0.03, $\rho = 0.5$ and $\alpha_j = \frac{1}{\tau_j}$ as suggested by Mercurio in his paper [18]

3.3.1 Shifted Lognormal LMM Calibration

We will assume throughout our calculations that volatilities are piece-wise constant, time-homogeneous that is:

$$\sigma_j(t) = \sigma_{j-\alpha(t)} \tag{3.15}$$

where $\alpha(t) = \ min\{j : t \leq T_j, j = 0, ..., n\}$, which means that the value of the volatilities depends on the number of reset dates left to maturity.

Then if the average volatility is represented by:

$$\bar{V}_j = \sqrt{\frac{1}{T_{j-1}}\int_0^{T_{j-1}} \sigma_j{}^2(t)dt} \tag{3.16}$$

$$\hat{\sigma_{j-1}}^2 = [T_{j-1}\bar{V_j}^2 - \sum_{i=2}^{j-1} \hat{\sigma_{j-i}}^2(T_i - T_{i-1})] * \frac{1}{T_1} \tag{3.17}$$

with an initial condition that: $\hat{\sigma_1} = \bar{V}_2$

In order to derive the value of $\bar{V}_j$ from market data, Mercurio in [18, page 18 Appendix B] suggested to use Black's formula for caplets in order to calibrate. If we assume the case of ATM caplet volatilities we can derive from black's formula the formula for calibration.

We start with :

$$(L_j(0)+\alpha_j)\left[2\Phi\left(\frac{\sqrt{T_{j-1}}\bar{V}_j}{2}\right)-1\right] = L_j(0)\left[2\Phi\left(\frac{\sqrt{T_{j-1}}\sigma_j^{ATM}}{2}\right)-1\right]$$

and thus we get:

$$\bar{V_j} = \frac{2}{\sqrt{T_{j-1}}}\Phi^{-1}\left(\frac{L_j(0)}{2(L_j(0)+\alpha_j)}[2\Phi(\frac{{\sigma_j}^{ATM}\sqrt{T_{j-1}}}{2}) - 1] + \frac{1}{2}\right) \tag{3.18}$$

${\sigma_j}^{ATM}$ used here is the ATM black caplet volatilities. In order to use the same formula but using ATM normal caplet volatilities we can transform these normal vols into black ones using the following equivalence formula:

$$BL(L, K^{ATM}, V) = L\Phi\left(\frac{\log(\frac{L}{K^{ATM}}) + \frac{v^2}{2}}{v}\right) - K^{ATM}\Phi\left(\frac{\log(\frac{L}{K^{ATM}}) - \frac{v^2}{2}}{v}\right)$$

$$= L(2\Phi(\frac{v}{2} - 1)$$

$$NB(L, K^{ATM}, V) = (L - K^{ATM})\Phi(d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-\frac{{d_1}^2}{2}}$$

$$= \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}$$

However, we know that $BL(L, K^{ATM}, V) \approx NB(L, K^{ATM}, V)$
Thus, ${\sigma_{BL}}^{ATM} \approx 2\Phi^{-1}\left[\frac{{\sigma_{NB}}^{ATM}\sqrt{T-t}}{\sqrt{2\pi}2L} + \frac{1}{2}\right]$
Finally, we plug in the above equation for calibration.

3.3.2 Minimal Basis Volatility

The second parameter that should be calibrated is the volatility in the OIS model. The OIS model will be calibrated jointly with the LMM based on the minimal basis volatility model.
Almost all banks assume that the LIBOR-OIS basis is deterministic and thus these two have perfect positive correlation (ρ=1). However, in his paper [18], Mercurio argues that this assumption is not realistic and that this basis must be treated as stochastic. In order to visualize this thing, first we will take the case of multiplicative basis B_j:

$$[1 + \tau_j F_j(t)][1 + \tau_j B_j(t)] = 1 + \tau_j L_j(t) \tag{3.19}$$

Thus the basis is given by the following equation:

$$B_j(t) := (\frac{1 + \tau_j L_j(t)}{1 + \tau_j F_j(t)} - 1)\frac{1}{\tau_j} \tag{3.20}$$

where j:=1,...,n

In order to study the distribution of the historical data of this basis spread, we will take the historical daily fixings of the LIBOR and historical 3M OIS-forward rates. After plotting its KDE we can see that there is no reason to assume a deterministic basis.

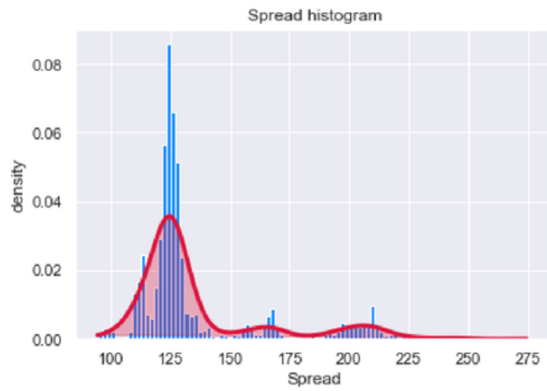


Figure 5: Study of the basis distribution

In the minimal basis volatility model suggested by Mercurio, the correlation will be set to a value less than 1. Since the basis now is assumed stochastic then this might lead to large and unrealistic scenarios for this basis. In order to prevent this, we will choose $\sigma(t)$ that minimizes the variance of the basis.

To derive this $\sigma(t)$, we will start first by deriving the dynamics of the basis $B_j(t)$ using ito's formula and the dynamics of the associated $L_j(t)$ and $F_j(t)$.

$$dL_j(t) = \sigma_j(t)[L_j(t) + \alpha_j]dW_j(t) \tag{3.21}$$

$$dF_j(t) = (F_j(t) + \frac{1}{\tau_j})(B(t,T_j) - B(t,T_{j-1}))\sigma(t)dZ_j(t) \tag{3.22}$$

Therefore we get that the dynamics of the basis is as follows:

$$dB_j(t) = \ldots dt + [B_j(t) + \frac{1}{\tau_j}][\sigma_j(t)\frac{L_j(t) + \alpha_j}{L_j(t) + \frac{1}{\tau_j}}dW_j(t) - (B(t,T_j) - B(t,T_{j-1})\sigma(t)dZ_j(t)] \tag{3.23}$$

$$\mathbb{V}ar(dB_j(t)) = \{\sigma_j(t)^2 + (B(t,T_j) - B(t,T_{j-1})^2\sigma(t)^2 - 2\rho(B(t,T_j) - B(t,T_{j-1})\sigma_j(t)\sigma(t)\}dt$$

Minimizing this volatility with respect to $\sigma(t)$ and assuming that $\sigma(t)$ is time-dependent and not constant we get that:

$$\sigma(t) = \frac{\rho\sigma_n(t)}{B(t,T_n) - B(t,T_{n-1})} \tag{3.24}$$

This will ensure the minimization of the B_n volatility at any time t.

3.4 Closed Form

The time t forward of the LIBOR fallback is given by:

$$\hat{L}_j(t) = \mathbb{E}_t^{T_j}[\hat{L}(T_{j-1}, T_j)]$$

The methodology used to compute $\hat{L}(T_{j-1}, T_j)$ is still not agreed on, so for now we cant have a unique formula for the fallback. In this paper we will choose the term rate to be the compounded setting in arrears driven by the backward looking approach and the historical mean with three different look-back periods for the credit spread adjustment $S(T^*)$

$$\hat{L}(T_{j-1}, T_j) = R(T_{j-1}, T_j) + S(T^*) \tag{3.25}$$

and thus,

$$\hat{L}_j(t) = R_j(t) + \mathbb{E}_t^{T_j}[S(T^*)] \tag{3.26}$$

where $R_j(t) = \mathbb{E}_t^{T_j}[R(T_{j-1}, T_j)]$ given by (3.11)

Therefore our new valuation now is based on our choice of the spread adjustment. We can also notice that the swaps changed from being independent of the curve dynamics to model dependent derivatives due to the introduction of the spread.

Based on our choice of spread adjustment we get that:

$$S(T^*) = \frac{1}{n+1} \sum_{i=1}^{n+1} L(T_i, T_i + lM) - R(T_i, T_i + lM) \tag{3.27}$$

where $T_0 = T^* - mY$, m = 5,7, or 10, $T_n = T^* = t_0 - lM$, and l = 3 or 6 months which are the two LIBOR tenors we will be working with.

Therefore,

$$\begin{aligned}
(n+1)\,\mathbb{E}_t^{T_j}[S(T^*)] &= \mathbb{E}_t^{T_j}\left[\sum_{i=0}^{n} L(T_i, T_i + lM) - R(T_i, T_i + lM)\right] \\
&= \sum_{i=0}^{i_{t-lM}} L(T_i, T_i + lM) - R(T_i, T_i + lM) \\
&+ \sum_{i=i_{t-lM}+1}^{i_t - 1} L(T_i, T_i + lM) - \mathbb{E}_t^{T_i}[R(T_i, T_i + lM)] \\
&+ \sum_{i=i_t}^{n} \mathbb{E}_t^{T_i}[L(T_i, T_i + lM)] - \mathbb{E}_t^{T_i}[R(T_i, T_i + lM)]
\end{aligned}$$

The reason we divided the summation that way is that after today t = T_{i_t} and up to the calibration day T^*, the fixing of the LIBORs and the lM compounded SONIA are not known, so we will use their expected values knowing the information up to today under the T_j forward measure. Moreover, the value of the lM compounded SONIA can only be known one day before the payment date. Therefore these values are only known up to T_{t-lM} included since any rate after that has the payment after i_t and thus not known just using the information up to i_t .

In order to get the first part of the spread we take the historical data of the lM LIBOR and SONIA fixings. The lM LIBOR fixings are used as they are and we get the lM compounded SONIA using (2.1) Then we evaluate the spread between these two quantities. These results are already presented in figures 2 and 3.

Now if we want to compute $\mathbb{E}_t^{T_j}[R(T_i, T_i + lM)]$ when $t \in (T_i, T_i + lM]$ the the computations will be as follows :

$$\mathbb{E}_t^{T_j}[R(T_i, T_i + lM)] = \left[\prod_{k=i-1}^{i_t} (1 + \delta_k I_k{}^s)\, \mathbb{E}_t^{T_j}[e^{\int_t^{T_i + lM} r(s)ds}] - 1 \right] * \frac{1}{\delta} \tag{3.28}$$

where I_k^s is the daily fixing of SONIA.
The calculations needed to evaluated $\mathbb{E}_t^{T_j}[e^{\int_t^{T_i+lM} r(s)ds}]$ will be shown below.

Finally, in order to get the closed form of the last part of the spread, we will derive the closed form of : $\mathbb{E}_t{}^{T_j}(L(T_i, T_i + lM))$ and $\mathbb{E}_t{}^{T_j}(R(T_i, T_i + lM))$

$$\begin{aligned}
\mathbb{E}_t{}^{T_j}(L(T_i, T_i + lM)) &= \mathbb{E}_t{}^{T_j}\left(\frac{P(t,T_j)}{P(T_i,T_j)}\frac{P(T_i,T_j)}{P(t,T_j)}L(T_i, T_i + lM)\right) \\
&= \mathbb{E}_t\left(\frac{B(t)}{B(T_i)}\frac{P(T_i,T_j)}{P(t,T_j)}L(T_i, T_i + lM)\right) \\
&= \mathbb{E}_t\left(e^{-\int_t^{T_i} r(s)ds}\frac{P(T_i,T_j)}{P(t,T_j)}L(T_i, T_i + lM)\right) \\
&= \frac{1}{P(t,T_j)}\,\mathbb{E}_t\left(e^{-\int_t^{T_i} r(s)ds}\,\mathbb{E}_{T_i}(e^{-\int_{T_i}^{T_j} r(s)ds})L(T_i, T_i + lM)\right) \\
&= \frac{1}{P(t,T_j)}\,\mathbb{E}_t\left(\mathbb{E}_{T_i}(e^{-\int_t^{T_i} r(s)ds}e^{-\int_{T_i}^{T_j} r(s)ds}L(T_i, T_i + lM)\right) \\
&= \frac{1}{P(t,T_j)}\,\mathbb{E}_t\left(e^{-\int_t^{T_j} r(s)ds}L(T_i, T_i + lM)\right) \\
&= \frac{1}{P(t,T_j)}\,\mathbb{E}_t\left(e^{-\int_t^{T_j} r(s)ds}(\tilde{L}(T_i, T_i + lM) - \alpha_i)\right) \\
&= \frac{1}{P(t,T_j)}\,\mathbb{E}_t\left(e^{-\int_t^{T_j} r(s)ds}(\tilde{L}(T_i, T_i + lM))\right) - \frac{\alpha_i}{P(t,T_j)}\,\mathbb{E}_t\left(e^{-\int_t^{T_j} r(s)ds}\right) \\
&= A - \alpha_i
\end{aligned}$$

The second equality holds due to a change of measure from the T_j forward measure to the risk neutral measure. Moreover, tower property was used to reach the 6^{th} equality since we have that $t < T_{i-1}$.

So,

$$\mathbb{E}_t^{T_j}(L_i(T_i)) = (L_i(t) + \alpha_i)e^{\int_t^{T_i}(\rho\sigma_i(s)\sigma(s)(B(s,T_i+lM)-B(s,T_j))ds} - \alpha_i \tag{3.29}$$

where $L_i(T_i) = L(T_i, T_i + lM)$ The detailed computations are represented in Appendix B.

Now in order to evaluate $\mathbb{E}_t^{T_j}(R(T_i, T_i + lM))$, we will approximate the actual daily compounded setting in arrears OIS rate by the continuous-time one which is given by:

$$R(T_i, T_i + lM) = \frac{e^{\int_{T_i}^{T_i+lM} r(u)du} - 1}{\tau_i} \tag{3.30}$$

This approximation will yield simpler and more compact expressions.
Therefore, $\mathbb{E}_t^{T_j}(R(T_i, T_i + lM)) = \mathbb{E}_t^{T_j}(\frac{e^{\int_{T_i}^{T_i+lM} r(s)ds} - 1}{\tau_i})$

$$\begin{aligned}
\mathbb{E}_t^{T_j}(e^{\int_{T_i}^{T_i+lM} r(s)ds}) &= \frac{1}{P(t,T_j)}\mathbb{E}_t(e^{-\int_t^{T_j} r(s)ds}P(T_i + lM, T_j)e^{\int_{T_i}^{T_i+lM} r(s)ds}) \\
&= \frac{1}{P(t,T_j)}\mathbb{E}_t(e^{-\int_t^{T_j} r(s)ds + \int_{T_i}^{T_i+lM} r(s)ds}) \\
&= \frac{1}{P(t,T_j)}e^{\mu_1 + \frac{\sigma_1^2}{2}}
\end{aligned}$$

Since the exponent is normally distributed so the expectation can be treated as the moment generating function of X$\sim \mathcal{N}(\mu_1, \sigma_1^2)$

$$\mathbb{E}_t^{T_j}(e^{\int_{T_i}^{T_i+lM} r(s)ds}) = e^{\int_{T_i}^{T_i+lM} \alpha(s)ds \, + \, (B(t,T_i+lM)-B(t,T_j))x(t) + \int_{T_i}^{T_i+lM} \sigma^2(s)B^2(s,T_i+lM)(\frac{1}{2}-B(s,T_j))ds}$$
$$e^{-B(T_i,T_i+lM)\int_t^{T_i} \sigma^2(s)e^{-a(T_i-s)}(B(s,T_j)+\frac{1}{2}B(T_i,T_i+lM)e^{-a(T_i-s)})ds}$$

The full derivation steps of will also be provided in Appendix B.

If we want to use the historical mean with the transitional period then the forward of the LIBOR fallback using this spread is:

$$\begin{aligned}
\hat{L}_j(t) &= R_j(t) + \mathbb{E}_t^{T_j}(S(t,t_0)) \\
&= R_j(t) + \frac{S^2 - t}{S^2 - S^1}\left(\mathbb{E}_t^{T_j}(L_{lM}(t_0)) - \mathbb{E}_t^{T_j}(R_{lM}(t_0))\right) + \frac{t - S^1}{S^2 - S^1}\mathbb{E}_t^{T_j}(S(t_0))
\end{aligned}$$

where t_0 is the announcement date

3.5 From caps to Asian Options

As clear from our calculations and change of the pricing scheme shown above, LIBOR derivatives have become more like path-dependent derivatives. This path-dependence is easier to deal with in the case of linear payoffs such as swaps than in non-linear payoffs such as caps. We will show below how will the pricing formula change for caps when we introduce the LIBOR fallback and the complications it thus yields. The discounted payoff of a cap can be expressed as the sum of that of the caplets that forms it. Therefore it is sufficient to study the change in the price of a caplet in an interval $[T_{j-1}, T_j]$. We start by a brief definition of a caplet. A caplet can be considered as a call option on an interest rate benchmark. The discounted payoff of a caplet is usually:

$$D(t, T_j)\tau_j \left(L(T_{j-1}, T_j) - K\right)^+ \tag{3.31}$$

Now, if $T_{i-1} \geq S^*$ then the payoff will be:

$$D(t, T_j)\tau_j \left(R(T_{j-1}, T_j) + S(T^*) - K\right)^+ \tag{3.32}$$

Therefore the new price is :

$$V(t) = \mathbb{E}_t \left(e^{\int_t^{T_j} r(s)ds} \tau_j \left(R(T_{j-1}, T_j) + S(T^*) - K\right)^+ \right)$$

but,

$$\begin{aligned} S(T^*) = &\sum_{i=0}^{i_{t-lM}} \left(L(T_i, T_i + lM) - R(T_i, T_i + lM)\right) + \sum_{i=i_{t-lM}+1}^{i_t - 1} \left(L(T_i, T_i + lM) - R(T_i, T_i + lM)\right) \\ &+ \sum_{i=i_t}^{n} \left(L(T_i, T_i + lM) - R(T_i, T_i + lM)\right) \\ &= K_1 - \sum_{i=i_{t-lM}+1}^{i_t - 1} R(T_i, T_i + lM) + \sum_{i=i_t}^{n} \left(L(T_i, T_i + lM) - R(T_i, T_i + lM)\right) \end{aligned}$$

where K_1 is the known part of the spread from historical data and the other part is still unknown and thus considered as stochastic. We don't apply change of measure in this case in order to have a uniform measure (risk neutral measure) for all these stochastic figures. If we let X = K - K_1, then the time-t price of the caplet is:

$$\begin{aligned} &V(t) = \\ &\mathbb{E}_t \left(e^{\int_t^{T_j} r(s)ds} \tau_j \left(R(T_{j-1}, T_j) - \sum_{i=i_{t-lM}+1}^{i_t - 1} R(T_i, T_i + lM) + \sum_{i=i_t}^{n} \left(L(T_i, T_i + lM) - R(T_i, T_i + lM)\right) - X \right)^+ \right) \\ &= \mathbb{E}_t(e^{\int_t^{T_j} r(s)ds} \tau_j (A - X)^+) \end{aligned}$$

However, in this case A is no longer lognormal as it was in the old usual pricing formula which makes it harder to compute this price and somehow impossible to get a closed-form pricing formula. In other words we cant proceed as before in order to reach the Black's formula. It is the same case as in the Asian options, thus we can use the same tools we use to compute the price of an Asian option. One famous tool is Monte Carlo simulation which is generally used to price path-dependent options. This method will allow us to generate a large number of random possible paths for the prices incorporated in the discounted payoff. Then by Central Limit Theorem the average of the simulated paths will converge to the actual price. Brief illustration of how to use this method here is provided in Appendix C.

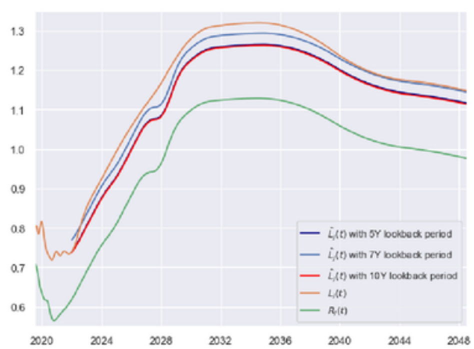
4 Value Transfer

One of the main issues that ISDA highlighted in the consultation and tried to prevent while choosing a suitable spread adjustment is value transfer once the fallback is applied. The forward approach, as mentioned in Section 2.3, will almost avoid this value transfer around the announcement date since it will be present value-neutral. However, the historical mean/median approach, which will most likely be the one used, will induce some value transfer even with the transitional period. Value transfer will be associated with sudden changes in the prices of contracts in case the market forward spread doesn't coincide with the historical spread on the announcement day.

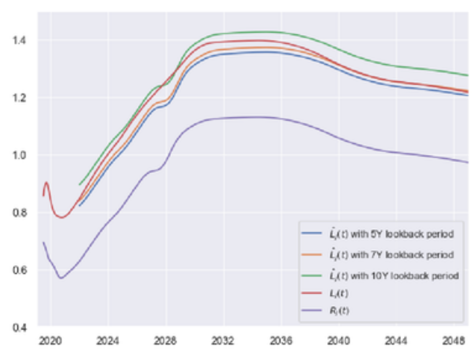
4.1 Impact on the Present Value

Once market participants knew that the future of LIBOR is no longer guaranteed, they discovered that there valuation methodology of LIBOR-linked products should change to incorporate this uncertainty in the existence of this figure. This led to the formula in (3.2). However, at that time, there was not any information about what will replace LIBOR in this equation. Therefore, this change was just qualitative, and no actual money or value transfer could be involved. On the 27^{th} of November, when the results of the consultation were announced, the fallback framework was more transparent, and the range of choices and unknown elements in this framework became narrower. Therefore, one can now assume a set of scenarios for the fallback and change the payoff accordingly; this will impact the present value and allow us to have an idea of how much the value transfer will be. In order to quantify the amount of this value transfer, we will price the swap spreads using the closed form we derived above and compare it with the old valuation assuming no LIBOR discontinuation. We will consider three scenarios. In the three of them, the term rate is the compounded setting in arrears, the spread adjustment is calculated using the historical mean approach, and the announcement date is on the 1^{st} of July 2021. The only difference is that the look-back period differs. We will consider 5,7 and 10 years look-back periods. We think that the choice of the look-back period will have the major effect on the prices. This will be applied on 3M GBP LIBOR and 6M GBP LIBOR.

As a first step, we will evaluate the forward curve of the LIBOR Fallback $\hat{L}_j(t)$ according to equation (3.26). As we can notice from the equation, the shift from $R_j(t)$ to $\hat{L}_j(t)$ is not parallel, and it is dependent on the payment date T_j. This is not so clear in the graphs below since the dependence on T_j is minor relatively. This shift will be parallel after the discontinuation and will be equal to the constant spread from the historical approach. We can also notice that the forward curves of the fallback using 5 years and 7 years are closer to each other than to the one using 10 year look-back period. This is logical since if we refer to figures 2 and 3, we can notice that in the period between the beginning of the 7 year look-back period and that of 5-year look-back period the historical spreads were somehow stable and no significant fluctuations were recorded especially for the 3M GBP LIBOR. Therefore, their corresponding averages will not differ much.



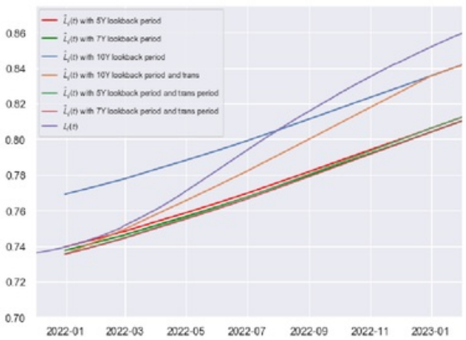
(a) GBP 3M LIBOR.



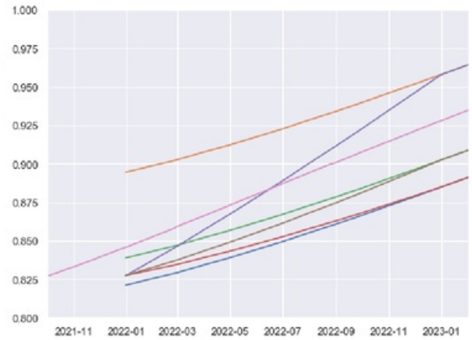
(b) GBP 6M LIBOR.

Figure 6: Forward Curves

If we want to apply the one year transitional period, the change in the forward curve will just be up to one year from the discontinuation date. No matter what our choice of the look-back is, $\hat{L}_j(t)$ will start from the same point since the spread in all cases will be equal to the spot spread.

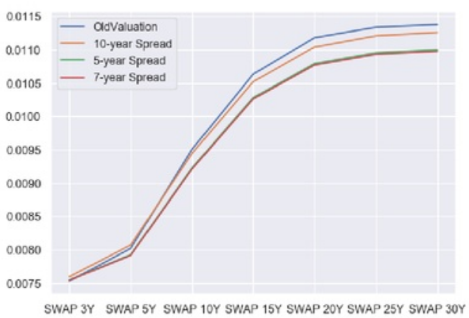


(a) GBP 3M LIBOR.

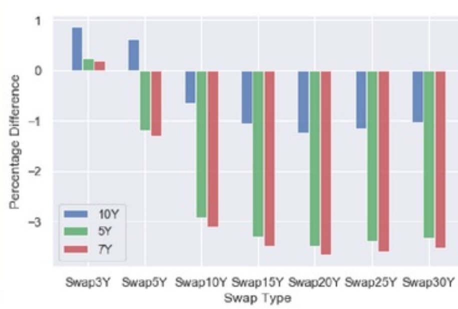


(b) GBP 6M LIBOR.

Now we will use these results to price the LIBOR fixed-floating swaps. We will find the fixed rate of each swap which is in linear association with the price of the swap. We get the following results:

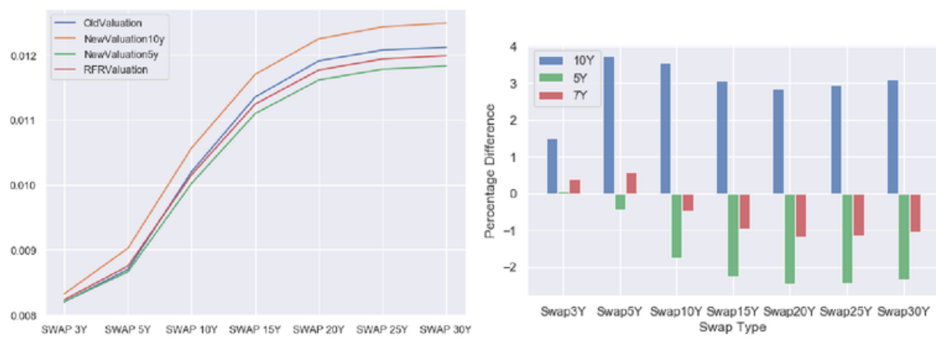


(c) Swap Prices



(d) Percentage Difference

Figure 7: 3M GBP LIBOR

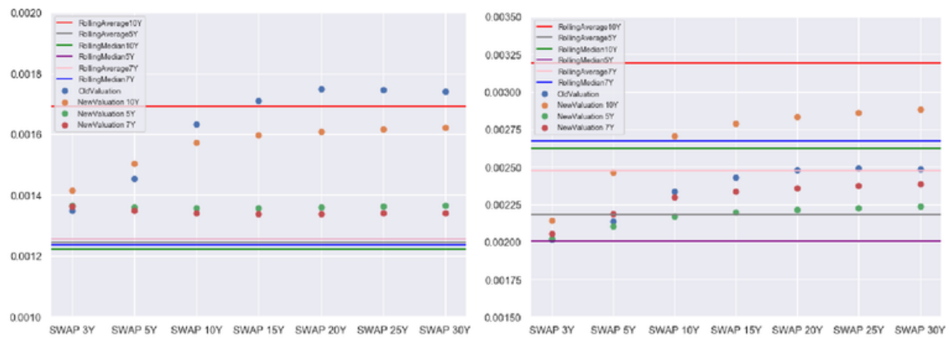


(a) Swap Prices (b) Percentage Difference

Figure 8: 6M GBP LIBOR

We can see that value transfer will occur in all cases but with different percentages. For 3M GBP LIBOR, the swap rate calculated using the fallback is, in most cases, less than the swap rate calculated using the old valuation. This will result in a potential loss for the fixed-rate receiver and gain in the case of floating rate receiver. However, it is not the same situation for 6M GBP LIBOR. If we get the spread using 10 years look-back period, the swap rate is always higher than that using the old valuation. For the other look-back periods it is almost always less but with fewer percentages compared to the 3M GBP LIBOR case. Therefore, here the loss is always for the floating rate receiver in case they decide on using 10Y look-back period and almost always a loss for the fixed rate receiver in the other two cases. If we refer to graph 3, we can see that the rolling average over the 10 year look-back period is in most cases higher than the daily spread and thus is expected to overestimate the LIBOR/adjusted RFR spread. This is evident also in those results and the results of the basis spread below. Analyzing these results yields the conclusion that in order to minimize the value transfer, 10 years look-back period is a better choice for the 3M GBP LIBOR and 7Y look-back period for the 6M GBP LIBOR. However, in order to ensure consistency, it was agreed that the approach would be unified for all LIBORs. Therefore, we expect that the value transfer will be more intense in some LIBOR rates than others. This value transfer might not seem so significant if we look on the change of fixed rates of each swap alone; however, from a large portfolio perspective, the impact would be notable.

We also repriced the basis spread and compared them to the running averages and medians at that day:



(a) 3M GBP LIBOR (b) 6M GBP LIBOR

Figure 9: Basis Swap

A similar analysis can be applied to the basis spread. We would just like to add that the spread curve with respect to maturities is flattening gradually and not flat from the beginning, as expected if we are assuming that we will have a constant spread in the future, since part of the spread is still dependent on the actual forward spread between LIBOR and SONIA until discontinuation date.

Another way to evaluate the value transfer by studying the impact on the present value was demonstrated in Henrard's blog [11]. He created a "LIBOR Fallback Transformer". This transformer will take the portfolio of legacy swaps and then change its present value according to the chosen adjusted RFR and spread. In order to get a reasonable range for the possible spreads, he calculated the running averages and medians using several choices for the look-back period and announcement date and chose the range accordingly. He then used this range along with another range for the spot spread (needed for the one-year transitional period) to evaluate the present value of a portfolio via this Fallback transformer. He was thus able to compute the P&L for each couple spread adjustment - spot spread in these two ranges. The amount of value transfer he detected was noteworthy.

4.2 Discussion on Historical Data

The mere fact that we were able to change the valuation methodology to each of (3.4) and (3.5) means that value transfer has already the tendency to start. Another catalyst for the value transfer is the fact that the preferred choice for the spread adjustment is the historical one. The majority of the historical data that will be used to compute the latter is already available in the market especially if the look-back period will be 10 years, in which case about 80% of the data is already known. Even though the exact details of the fallback are not known yet, which leaves a margin of uncertainty, but the market now has enough information to create a credible range of the potential spreads and use it to impact the prices. All of this will lead us to the hypothesis that value transfer has already taken place and will continue to occur whenever new information concerning the fallback is revealed.

We so far represented a theoretical reason that motivates this hypothesis. In order to visualize this impact we will study the historical basis spreads of the 3M GBP LIBOR-SONIA basis swaps since they are directly related to the spread between the LIBOR fixings and adjusted RFRs. We take 30Y, 10Y and 5Y basis swap. The primary information about the spread adjustment were announced by ISDA on 27^{th} of November 2018.

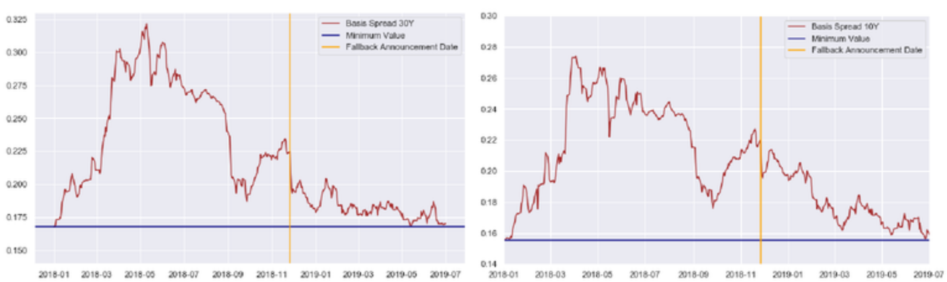
It is clear in the graph above that all the spreads in the period just before this date were higher than the range of rolling averages and medians. However, on the day the consultation results were announced, there was a sudden and significant drop in the spreads where they reached a level closer to the range of rolling averages and medians. After this drop, the spreads kept on the same trend and are gradually converging to the range of historical spreads. This drop was most significant in the 30Y basis swap since it is mostly dependent on the spread. Only 2 years will be the actual LIBOR/SONIA spread and the other 28 years will be the fixed, most probably historical, spread calculated on the announcement day.

Although this observation is somehow intuitive due to the nature of this derivative, but it is interesting to see it translated in a formula. In order to see this, we will write down the basis spread (K) formula when taking into consideration the LIBOR fallback. To get this formula we will solve for the spread that sets the price of a basis swap expressed in (3.5) to zero.

$$
\begin{aligned}
K &= \frac{\sum_{j=a+1}^{k} P(t,T_j)L_j(t)\tau_j + \sum_{j=k+1}^{b} P(t,T_j)\hat{L}_j(t)\tau_j - \sum_{j=a+1}^{b} P(t,T_j)R_j(t)\tau_j}{\sum_{j=a+1}^{b} P(t,T_j)\tau_j} \\
&= \frac{\sum_{j=a+1}^{k} P(t,T_j)\left(L_j(t) - R_j(t)\right)\tau_j + \sum_{j=k+1}^{b} P(t,T_j)\left(\hat{L}_j(t) - R_j(t)\right)\tau_j}{\sum_{j=a+1}^{b} P(t,T_j)\tau_j} \\
&= \frac{\sum_{j=a+1}^{k} P(t,T_j)\left(L_j(t) - R_j(t)\right)\tau_j + \sum_{j=k+1}^{b} P(t,T_j)\left(\mathbb{E}_t^{T_j}[S(T^*)]\right)\tau_j}{\sum_{j=a+1}^{b} P(t,T_j)\tau_j}
\end{aligned}
$$

The last step is achieved by using (3.26)

Therefore, the evolution of the prices for high maturities such as 30Y and 10Y are the ones that shows more clearly the impact of the announcement of the primary details of the fallback on market prices.



(a) 30 Y maturity

(b) 10Y maturity

Figure 10: Basis Spread

From the above figures, we can see that not just the sudden drop that implies value transfer, but also the fact that the spread has reached, after this announcement, its minimum value from 2018 up to now. This was the case for both maturities, although it is more evident in the case of 30Y. Now that we can already sense the value transfer in the market prices, we will go back to our study of the present values to see if this is apparent there too. The difference between the prices calculated via market prices using the old valuation and that using the new valuation adapted to each look-back period is not as high as expected especially compared to the intensity of value transfer depicted above. Moreover, the present value of the basis spread for all the maturities are either within or very close to the range of the historical spreads at the same day represented in the graphs which match our analysis of historical observations. This is logical since if we refer to 2 and

3, we can see that after the end of November 2018, the actual daily spread is fluctuating within the range of running historical averages and medians. We cannot detect a unique convergence towards a specific value since the exact details of the fallback are still unknown, but the fact that they are already within the range makes us think that the market is already acting upon the suggested fallback.

Hence, it seems that the value transfer that was expected to occur once the fallback is triggered is already occurring gradually. Finally note that the precise volume of the value transfer cannot be computed since the whole details of the fallback are still not revealed.

5 Valuation of SONIA Vanilla Derivatives

As a result of the LIBOR discontinuation market participants are motivated to lessen their exposure to LIBOR. That is why we expect to sense now a movement towards trading RFR linked derivatives. In the Financial Stability Report of Bank of England [3], they highlighted the risks of the continuing reliance on LIBOR, and there was a clear emphasis on the importance of reducing exposure to LIBOR after noticing that many new contracts are still referencing LIBOR. This reliance on LIBOR will lead to conduct, systemic and many other types of risk. They insisted that the transition must be accelerated and that the FPC will monitor the progress closely.

5.1 SONIA Futures

The interest in SONIA futures grew significantly lately. They are now offered by CME, ICE and CurveGlobal. Andrew Bailey in his speech [2] also pointed out the significant expansion in the volumes of the RFRs futures. Interest in SONIA in specific increased from nearly zero in mid 2018 up to approximately 129 billion by the end of June this year. The graph provided below reveals this expansion in the last couple of months:

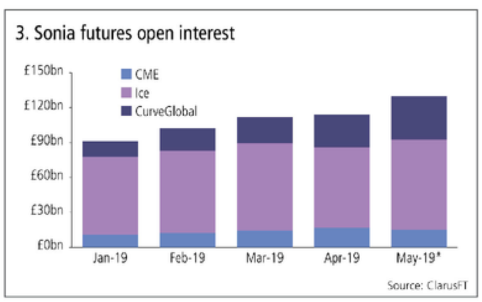


Figure 11: SONIA Futures Data from Amir Khwaja ,Swaps data: a new era of competition in interest rate futures , Risk.net, June 2019 [15]

5.1.1 3M SONIA Futures

The underlying rates are daily compounded SONIA interest rate during the associated reference quarter denoted as $[T_{j-1}, T_j)$ and they are approximated by the following formula:

$$R(T_{j-1}, T_j) = \frac{1}{\tau_j}\{e^{\int_{T_{j-1}}^{T_j} r(u)du} - 1\} \tag{5.1}$$

Futures convexity adjustment is calculated to move from the forward rate to the futures rate which is the expected value under the risk neutral measure.

Therefore in this case:

$$C_j(0) = \mathbb{E}(R(T_{j-1}, T_j)) - \mathbb{E}^{T_j}(R(T_{j-1}, T_j)) = {f_j}^{3M}(0) - R_j(0) \tag{5.2}$$

Using the same model we represented in our work above for the OIS rate we can, as suggested by Mercurio in [17, page 9], get the closed form of the futures rate and thus that of the convexity adjustment.

$$\begin{aligned}
1 + \tau_j {f_j}^{3M}(0) &= \mathbb{E}(e^{\int_{T_{j-1}}^{T_j} r(u)du}) \\
&= \mathbb{E}(e^{\int_{T_{j-1}}^{T_j} x(u)du}) e^{\int_{T_{j-1}}^{T_j} \alpha(u)du} \\
&= \frac{P(0, T_{j-1}) A(0, T_j)}{A(0, T_{j-1}) P(0, T_j)} \mathbb{E}(e^{\int_{T_{j-1}}^{T_j} x(u)du}) \\
&= \frac{P(0, T_{j-1}) A(0, T_j)}{A(0, T_{j-1}) P(0, T_j)} \mathbb{E}(\mathbb{E}_{T_{j-1}}(e^{\int_{T_{j-1}}^{T_j} x(u)du})) \\
&= \frac{P(0, T_{j-1}) A(0, T_j) A(T_{j-1}, T_j)}{A(0, T_{j-1}) P(0, T_j)} \mathbb{E}(e^{B(T_{j-1}, T_j) x(T_{j-1})}) \\
&= \frac{P(0, T_{j-1}) A(0, T_j) A(T_{j-1}, T_j)}{A(0, T_{j-1}) P(0, T_j)} e^{\frac{1}{2} B^2(T_{j-1}, T_j) \int_0^{T_{j-1}} \sigma^2(u) e^{-2a(T_{j-1}-u)} du}
\end{aligned}$$

Therefore, we get that

$$\begin{aligned}
C_j(0) &= {f_j}^{3M}(0) - (\frac{P(0, T_{j-1})}{P(0, T_j)} - 1)\frac{1}{\tau_j} \\
&= \frac{P(0, T_{j-1})}{\tau_j \; P(0, T_j)} (\frac{A(0, T_j) A(T_{j-1}, T_j)}{A(0, T_{j-1})} e^{\frac{1}{2} B^2(T_{j-1}, T_j) \int_0^{T_{j-1}} \sigma^2(u) e^{-2a(T_{j-1}-u)} du} - 1)
\end{aligned}$$

5.1.2 1M SONIA Futures

In the case of 1M SONIA Futures, they are traded with an underlying rate different than that in the 3M future case. The underlying rate here is the approximation of the arithmetic average of daily SONIA during the delivery month. If we suppose that the delivery month is represented by [T-δ,T] where δ is approximately the year fraction of one month, then the rate is:

$$\frac{1}{\delta} \int_{T-\delta}^{T} r(u)du \tag{5.3}$$

The future rate of the above underlying rate is its expectation under the risk neutral measure

$\mathbb{Q}$. Therefore it is computed as suggested in [17, page 8]in this case as follows:

$$
\begin{aligned}
{f_s}^{1m} &= \frac{1}{\delta}\mathbb{E}\left\{\int_{T-\delta}^{T} r(u)du\right\} \\
&= \frac{1}{\delta}\int_{T-\delta}^{T} \alpha(u)du \\
&= \frac{1}{\delta}\log\frac{P(0,T-\delta)}{P(0,T)} + \frac{1}{\delta}\log\frac{A(0,T)}{A(0,T-\delta)}
\end{aligned}
$$

The convexity adjustment here is thus: $\frac{1}{\delta}\log\frac{A(0,T)}{A(0,T-\delta)}$

5.1.3 Pricing Results

In the calculations of the price of the futures above we assumed constant volatility. We will choose the constant volatility proposed in the paper [17] which is :

$$\sigma = \frac{\rho\sigma_n(0)}{B(0,T_n) - B(0,T_{n-1})}$$

We evaluate the convexity adjustment of SONIA futures contracts with different delivery intervals:

Table 1: Convexity Adjustments for 3M SONIA as of 01/07/2019

Delivery Interval	$[T_1, T_2)$	$[T_3, T_4)$	$[T_5, T_6)$	$[T_7, T_8)$	$[T_9, T_{10})$
Convexity Adjustment	6.48575e-07	3.98734e-06	6.9035e-06	2.07189e-05	3.39751e-05

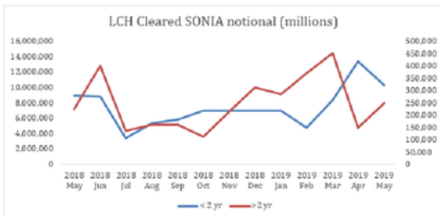
Table 2: Convexity Adjustments for 1M SONIA as of 01/07/2019

Delivery Interval	$[S_1, S_2)$	$[S_3, S_4)$	$[S_5, S_6)$	$[S_7, S_8)$	$[S_9, S_{10})$
Convexity Adjustment	6.34271e-07	2.84634e-06	1.5071e-05	2.14139e-05	3.2457e-05

5.2 SONIA Fixed-Floating Swap

In his speech, Andrew Bailey highlighted the importance of SONIA indexed swaps in the UK. The notional of outstanding cleared SONIA swaps now exceed ten trillion pounds. This rapid growth in the SONIA indexed swaps market is promising and a sign that we have increasing liquidity. This market is now of great importance, mainly because it will most probably be used to create a forward SONIA term rate.

In order to see the evolution of this market, we will provide the reader with the volumes of cleared SONIA indexed swaps:



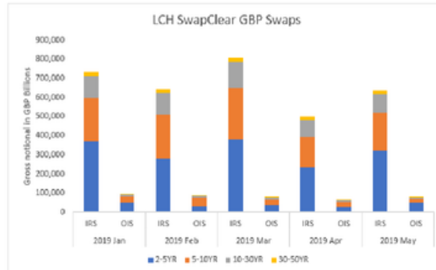


Figure 12: Swaps Data from John Feeney, SONIA and SOFR trading and Term Risk Free Rates, Clarus Financial Technology, June 2019

It is clear from the graphs provided above that trades referencing SONIA with maturities less than 2 years significantly outweigh the trades with maturities greater than 2 years. Also, if we compare the latter to the volume of high maturity LIBOR IRS traded, we can see that they still need much time to catch up. Therefore the liquidity in SONIA trades we were talking about is not sufficient as it is pooling exclusively in short maturities and is not even along the curve. This indicates that a robust term market in RFRs needed to create the term rate is still in its primary stages due to this lack of liquidity in the longer-dated derivatives market. On the other hand, traders will not be motivated to transition to this market unless they have a term rate to transition to. Hence, this sounds more like a circularity problem.

Due to its growing importance in the derivatives market, we will explain the valuation methodology of these swaps and show some swap rates for swaps of different maturities and schedules.

In this swap a floating leg pays the daily compounding of SONIA rates which is now considered as the adjusted RFR replacing LIBOR at each time T_i where i = a+1,...,b. This rate will be approximated by the continuous compounding of SONIA which is as follows:

$$R(T_{j-1}, T_j) = \frac{e^{\int_{T_{j-1}}^{T_j} r(u)du} - 1}{\tau_j} \tag{5.4}$$

The forward rate of the underlying floating rate is given by equation (3.11). Even if the daily compounded rate of SONIA was used instead of this approximation, the value of the swap wont change since we proved in Section 3.2.1 that they have the same forward rate. The fixed leg pays a fixed rate K on the dates S_j j = c+1,...,d also multiplied by τ_j.

Thus the swap value at time t is as follows:

$$V(t) = \sum_{i=a+1}^{b} P(t, T_i)R_i(t)\tau_i - K \sum_{j=c+1}^{d} P(t, T_j)\tau_j \tag{5.5}$$

Table 3: SONIA Swap Rates as of 01/07/2019

Swap Tenor	3Y	5Y	10Y	15Y	20Y	25Y	30Y
Swap Rate in %	0.61777	0.65611	0.78664	0.89215	0.94287	0.95877	0.96323

6 Conclusion

In this paper, we worked with a GBP LIBOR fallback consisting of compounded setting in arrears SONIA rate along with a spread adjustment using historical mean with different look-back periods. Then a two-factor multi-curve model was used to reprice some swaps upon the change of their payoffs. This allowed us to quantify the value transfer that will occur due to LIBOR discontinuation by studying the impact on the present value.

"The wise driver steers a course to avoid a crash rather than relying on a seat-belt.", said Andrew Bailey in his speech. This was a clear invitation for all market participants to shift from LIBOR to the alternative reference rates as soon as possible. LIBOR discontinuation will be a severe crash in the financial industry that a fallback, although will mitigate the possible contract frustration, we think will not be enough to save the market participants from all the aftermath. The fallback until now suffers from lack of clarity and some trades referencing LIBOR still do not have a clear definition, especially exotic ones. Therefore, firms must start to cover and address all the risks of exposure to LIBOR-linked instruments maturing after discontinuation and try to decrease their reliance on them.

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Appendix A

Due to the shifted LMM dynamics of forward LIBOR under the forward measure we get the following dynamics for the forward LIBOR under the risk neutral measure:

$$dL_j(t) = \rho\sigma_i(t)\sigma B(t,T_j)[L_j(t)+\alpha_j]dt + \sigma_j(t)[L_j(t)+\alpha_j]dW_t$$

Therefore if we let $\tilde{L}_j(t) = L_j(t)+\alpha_j$, we get that $\tilde{L}_j(t)$ is a geometric Brownian motion.

$$d\tilde{L}_j(t) = \tilde{L}_j(t)\left(\rho\sigma_i(t)\sigma(t)B(t,T_j)dt + \sigma_j(t)dW_t\right)$$

$$\frac{d\tilde{L}_j(t)}{\tilde{L}_j(t)} = \rho\sigma_i(t)\sigma(t)B(t,T_j)dt + \sigma_j(t)dW_t$$

$$\begin{aligned} d\ ln(\tilde{L}_j(t)) &= \frac{d\tilde{L}_j(t)}{\tilde{L}_j(t)} - \frac{1}{2}\frac{1}{\tilde{L}_j^2(t)}(d\tilde{L}_j(t))^2 \\ &= \left(\rho\sigma_i(t)\sigma(t)B(t,T_j) - \frac{\sigma_j^2(t)}{2}\right)dt + \sigma_j(t)dW_t \end{aligned}$$

Integrate both sides:

$$ln(\tilde{L}_j(t)) = ln(\tilde{L}_j(0)) + \int_0^t \left(\rho\sigma_i(t)\sigma(t)B(t,T_j) - \frac{\sigma_j^2(t)}{2}\right)dt + \int_0^t \sigma_j(t)dW_t$$

Therefore,

$$\tilde{L}_j(t) = \tilde{L}_j(0)e^{\int_0^t\left(\rho\sigma_i(t)\sigma(t)B(t,T_j)-\frac{\sigma_j^2(t)}{2}\right)dt+\int_0^t\sigma_j(t)dW_t}$$

Appendix B

In order to derive the closed form of the expectation of LIBOR under the forward measure we will use the following results :

1) r(t) = x(t) + α(t)

2) $\int_t^T x(s)ds$ is Gaussian $\sim N$ (B(t,T)x(t),$\int_t^T \sigma^2(s)B^2(s,T)ds)$

3) $\tilde{L}_j(t) = \tilde{L}_j(0)e^{\int_0^t(\rho\sigma_i(s)\sigma(s)B(s,T_i)-\frac{\sigma_i(s)^2}{2})ds+\int_0^t\sigma_i(s)dW(s)}$, and $\tilde{L}_j(t) = L_j(t)+\alpha_j$

4) $\alpha(t)$ is deterministic

5) Tower property

6) Sum of two guassian is again guassian

7) Independence of BM increments

8) x(T) = x(t)$e^{-a(T-t)} + \int_t^T \sigma(u)e^{-a(T-u)}dZ(u)$

In what follows we will assume that $T_i - T_{i-1} = 1M$

$\mathbb{E}_t{}^{T_j}(L_i(T_{i-1})) = A - \alpha_j$
$A = \frac{\tilde{L_i}(t)e^{\int_t^{T_{i-1}}(\rho\sigma_i(s)\sigma(s)B(s,T_i)-\frac{\sigma_i(s)^2}{2})ds}}{P(t,T_j)}\mathbb{E}_t\left(e^{-\int_t^{T_j}r(s)ds}e^{\int_t^{T_{i-1}}\sigma_i(s)dW(s)}\right)$
But, $-\int_t^{T_j}r(s)ds + \int_t^{T_{i-1}}\sigma_i(s)dW(s)$ is normally distributed so the expectation above can be considered as the MGF of $N(\mu,\sigma)$

$$\mu = \mathbb{E}_t\left(-\int_t^{T_j}r(s)ds + \int_t^{T_{i-1}}\sigma_i(s)dW(s)\right) = \mathbb{E}_t\left(-\int_t^{T_j}x(s)+\alpha(s)ds\right) = -\int_t^{T_j}\alpha(s)ds - B(t,T_j)x(t)$$

$$\begin{aligned}\sigma^2 &= V\left(-\int_t^{T_j}r(s)ds + \int_t^{T_{i-1}}\sigma_i(s)dW(s)\right)\\ &= V\left(-\int_t^{T_j}r(s)ds\right) + V\left(\int_t^{T_{i-1}}\sigma_i(s)dW(s)\right) - 2Cov\left(\int_t^{T_j}r(s)ds, \int_t^{T_{i-1}}\sigma_i(s)dW(s)\right)\\ &= \int_t^{T_j}\sigma^2(s)B^2(s,T_j)ds + \int_t^{T_{i-1}}\sigma_i{}^2(s)ds - 2Cov\left(\int_t^{T_j}\sigma(s)B(s,T_j)dZ(s), \int_t^{T_{i-1}}\sigma_i(s)dW(s)\right)\\ &= \int_t^{T_j}\sigma^2(s)B^2(s,T_j)ds + \int_t^{T_{i-1}}\sigma_i{}^2(s)ds - 2\rho\int_t^{T_{i-1}}\sigma_i(s)\sigma(s)B(s,T_j)ds\end{aligned}$$

So, $A = \frac{\tilde{L_i}(t)e^{\int_t^{T_{i-1}}(\rho\sigma_i(s)\sigma(s)B(s,T_i)-\frac{\sigma_i{}^2}{2})ds}}{P(t,T_j)}e^{\mu+\frac{\sigma^2}{2}}$

So,

$\mathbb{E}_t{}^{T_j}(L_i(T_{i-1})) = \frac{\tilde{L_i}(t)e^{\int_t^{T_{i-1}}(\rho\sigma_i(s)\sigma(s)(B(s,T_i)-B(s,T_j))ds}}{P(t,T_j)}e^{-\int_t^{T_j}\alpha(s)ds-B(t,T_j)x(t)+\frac{1}{2}\int_t^{T_j}\sigma^2(s)B^2(s,T_j)ds} - \alpha_i$
but, $e^{-\int_t^{T_j}\alpha(s)ds-B(t,T_j)x(t)+\frac{1}{2}\int_t^{T_j}\sigma^2(s)B^2(s,T_j)ds} = P(t,T_j)$ Therefore they cancel in the equation.

We will use similar tools to find the expectation of the forward of the term rate $R_i(t)$
We will need to compute: $\mathbb{E}_t{}^{T_j}\left(\frac{e^{\int_{T_{i-1}}^{T_i}r(s)ds}-1}{\tau}\right)$

$$\mathbb{E}_t{}^{T_j}\left(e^{\int_{T_{i-1}}^{T_i}r(s)ds}\right) = \frac{1}{P(t,T_j)}\mathbb{E}_t\left(e^{-\int_t^{T_j}r(s)ds+\int_{T_{i-1}}^{T_i}r(s)ds}\right)$$

but the exponent is normally distributed so again we have the MGF of N(μ_1,σ_1)

$$\begin{aligned}\mu_1 &= \mathbb{E}_t\left(-\int_t^{T_j}r(s)ds \; + \; \int_{T_{i-1}}^{T_i}r(s)ds\right)\\ &= -\mathbb{E}_t\left(\int_t^{T_j}x(s)ds\right) - \mathbb{E}_t\left(\int_t^{T_j}\alpha(s)ds\right) + \mathbb{E}_t\left(\int_{T_{i-1}}^{T_i}x(s)ds\right) + \mathbb{E}_t\left(\int_{T_{i-1}}^{T_i}\alpha(s)ds\right)\\ &= -B(t,T_j)x(t) - \int_t^{T_j}\alpha(s)ds + \int_{T_{i-1}}^{T_i}\alpha(s)ds + (B(t,T_i)-B(t,T_{i-1}))x(t)\end{aligned}$$

$\sigma_1{}^2 = V(\int_{T_{i-1}}^{T_i} r(s)ds) \ + \ V(\int_t^{T_j} r(s)ds) \ - \ 2Cov(\int_{T_{i-1}}^{T_i} r(s)ds, \int_t^{T_j} r(s)ds)$

but, $V(\int_{T_{i-1}}^{T_i} r(s)ds) = B^2(T_{i-1}, T_i) \int_t^{T_{i-1}} \sigma^2(s) e^{-2a(T_{i-1}-s)} ds + \int_{T_{i-1}}^{T_i} \sigma^2(s) B^2(s, T_i) ds$

So,

$$\begin{aligned}
\sigma_1{}^2 &= B^2(T_{i-1}, T_i) \int_t^{T_{i-1}} \sigma^2(s) e^{-2a(T_{i-1}-s)} ds \ + \ \int_{T_{i-1}}^{T_i} \sigma^2(s) B^2(s, T_i) ds \ + \ \int_t^{T_j} \sigma^2(s) B^2(s, T_j) ds \\
&- 2Cov\left(\int_t^{T_j} \sigma(s) B(s, T_i) dZ \ + \ B(t, T_j) x(t), \int_{T_{i-1}}^{T_i} \sigma(s) B(s, T_j) dZ \ + \ B(T_{i-1}, T_i) x(T_{i-1})\right) \\
&= B^2(T_{i-1}, T_i) \int_t^{T_{i-1}} \sigma^2(s) e^{-2a(T_{i-1}-s)} ds \ + \ \int_{T_{i-1}}^{T_i} \sigma^2(s) B^2(s, T_i) ds \\
&+ \ \int_t^{T_j} \sigma^2(s) B^2(s, T_j) ds \ - \ 2 \int_{T_{i-1}}^{T_i} \sigma^2(s) B(s, T_i) B(s, T_j) ds \\
&- \ 2Cov\left(\int_t^{T_j} \sigma(s) B(s, T_i) dZ, B(T_{i-1}, T_i) \int_t^{T_{i-1}} \sigma(s) e^{-a(T_{i-1}-s)} dZ(s)\right) \\
&= B^2(T_{i-1}, T_i) \int_t^{T_{i-1}} \sigma^2(s) e^{-2a(T_{i-1}-s)} ds \ + \ \int_{T_{i-1}}^{T_i} \sigma^2(s) B^2(s, T_i) ds \\
&+ \ \int_t^{T_j} \sigma^2(s) B^2(s, T_j) ds \ - \ 2 \int_{T_{i-1}}^{T_i} \sigma^2(s) B(s, T_i) B(s, T_j) ds \\
&- \ 2B(T_{i-1}, T_i) \int_t^{T_{i-1}} \sigma^2(s) B(s, T_j) e^{-a(T_{i-1}-s)} dZ(s)
\end{aligned}$$

Finally, $\mathbb{E}_t{}^{T_j}\left(e^{\int_{T_{i-1}}^{T_i} r(s)ds}\right) = \frac{1}{P(t,T_j)} e^{\mu_1 + \frac{\sigma_1^2}{2}}$

Appendix C

We will use the following results from the Hull-White model used above:

$$\int_t^T x(u)du = B(t,T)x(t) + \int_t^T \sigma(u) B(u,T) dZ_u$$

$$\int_T^S x(u)du = B(T,S)x(T) + \int_T^S \sigma(u) B(u,S) dZ_u$$

$$x(T) = e^{a(T-t)} x(t) + \int_t^T \sigma(u) e^{a(T-u)} dZ_u$$

In general we have:

$$\begin{aligned}
R(T,S) &= \frac{e^{\int_T^S r(u)du} - 1}{\tau_{T,S}} \\
&= \frac{e^{\int_T^S x(u)du} e^{\int_T^S \alpha(u)du} - 1}{\tau_{T,S}}
\end{aligned}$$

Therefore if we use the above equations along with ito's isometry we get that:

$$\begin{aligned}1+\tau_{T,S}R(T,S) &== e^{\int_T^S x(u)du}e^{\int_T^S \alpha(u)du}\\ &= e^{B(T,S)x(T)+\int_T^S \sigma(u)B(u,S)dZ_u+\int_T^S \alpha(u)du}\\ &= e^{B(T,S)e^{a(T-t)}x(t)+B(T,S)\mathcal{N}(0,\int_t^T \sigma^2(u)e^{2a(T-u)}du)+\mathcal{N}(0,\int_T^S \sigma^2(u)B^2(u,S)du)+\int_T^S \alpha(u)du}\end{aligned}$$

In order to simulate the discount factor we will use:

$$e^{\int_t^{T_j} r(u)du} = e^{B(t,T_j)x(t)+\mathcal{N}(0,\int_t^{T_j} \sigma^2(u)B^2(u,T_j)du)\int_t^{T_j} \alpha(u)du}$$

Moreover, keep in mind that between i $= i_{t-lM}+1$ and i $= i_t - 1$, the compounded rate is still not known and will be estimated by:

$$\left[\prod_{k=i-1}^{t}(1+\delta_k I_k{}^s)[e^{\int_t^{T_i+lM} r(s)ds}]-1\right]*\frac{1}{\delta}$$

Note that we are also required to simulate $L(T_i, T_i + lM)$. To do so we will take the form of the forward LIBOR derived from the shifted Lognormal model given by $\tilde{L}_j(t)$ in (3.14). Then,

$$L(T_i, T_i+lM) = \tilde{L}_i(T_i) - \alpha_i = \tilde{L}_i(0)e^{\int_0^{T_i}[\rho\sigma_i B(s,T_i+lM)\sigma(s)-\frac{\sigma^2{}_i(s)}{2}]ds+\mathcal{N}(0,\int_0^{T_i}\sigma_i^2(u)du)} - \alpha_i \quad (6.1)$$

During the simulation we will use: $\mathcal{N}(\mu, \sigma^2) = \mu + \sigma\mathcal{N}(0,1)$ and dW_tdZ_t=ρdt which can be achieved using Choleski's decomposition.

Using these tools we will generate different paths for the payoff and then take their average.

LIBOR DISCONTINUATION

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