IMPERIAL

DEPARTMENT OF MATHEMATICS

GUIDE TO OPTIONAL MODULES

These notes should be read in conjunction with your student handbook. Some of the information may be subject to alteration.

Updated information will be posted on the Maths Central Blackboard site and on your course blackboard page.

MSc Applied Mathematics OVERVIEW

MSc students may take modules of two different types:

1. Those listed below, which are often also year-3 undergraduate modules or year-4 (MSci) modules. You always need to follow the 4-year version (starts with MATH7XXX).
2. Other (up to two) modules. These are:
   a. Level 7 Modules from the MSc in Pure Mathematics.
   b. The undergraduate module “MATH70049 Introduction to Statistical Learning”;
   c. Under exceptional circumstances, up to 2 modules may be taken from other Departments (if, for instance, such a module is directly relevant for the MSc project). When choosing out of department modules, make sure you have enough ECTS to fulfill the taught element of the degree.
      Permission will only be granted after internal discussion during the first week, subject to MSc Programme Director approval and with permission from the other Department.

Students will not be allowed to take modules from MSc Statistics, MSc Mathematics and Finance or MSc Machine Learning and Data Science.

Advice on the choice of options

Students are advised to read these notes carefully and to discuss their option selections with their Personal Tutor.

It is anticipated that lecturers will give advice on suitable books at the start of each module. Students should contact the proposed lecturers if they desire any further details about module content in order to make their choice of course options.

You will not be committed to your choice of most optional modules until the completion of your module selection in late October, although the expectation is that any substantial modification of your initial choices should be requested within the first three weeks of term. The exception to this is that students do become committed to the completion of certain modules that have more than 10% CW these are examined only by project at some stage during the module, as will be made clear by the lecturer at the start of the module.

Module assessment and examinations

Most MATH7 modules are examined by one written examination of 2.5 hours in length.

Some of the modules may have an assessed coursework/progress test element, limited in most cases to 10% of overall module assessment. Some modules have a more substantial coursework component (for example, 25 percent) and others are assessed entirely by coursework. Details can be found in the tables below. Precise details of the number and nature of coursework assignments will be provided at the start of each module.

Students should bear in mind that single-term modules assessed by projects usually require extra time-commitment during that term. Students should note that, in principle, 1 ECTS represents around 25 hours of effort, and so a single 7.5 ECTS module represents 187.5 hours of effort: completing this in a single term is a substantial task. Thus, the Department strongly advises students to take **at most** one such module in a term and not allowed to do more than 2 of these modules per term.

Note: Students who take modules which are wholly assessed by project will be deemed to be officially registered on the module through the submission of a specified number of pieces of assessed work for that module. Thus, once a 15% is reached in these modules, a student will be committed to completing it. In contrast, students only become committed to modules with summer examinations when they choose the modules for examinations in late October and February.

MSc Applied Mathematics Module List

Note that not all of the individual modules listed below are offered every session and the Department reserves the right to cancel a particular module if, for example, the number of students attending that module does not make it viable. Similarly, some modules are occasionally run as 'Reading/Seminar Courses'.

All MATH7 modules are equally weighted and are worth 7.5 ECTS unless otherwise specified.

In the tables below, the % Exam indicates the weighting attached to the final written exam in Summer (May/June), unless otherwise indicated. The % CW indicates the weighting attached to any assessed work that is completed during the module. This may include in-class tests, projects, or problem sets to be turned in.

The groupings of modules below have been organised to indicate some natural affinities and connections. We recall that a minimum of 4 modules from the A Group are required for students enrolled in the Scientific Computing & Machine Learning stream.

FLUIDS

Module Code	Module Titles	Group	Term	Lecturer	% exam	% CW
MATH70001	Fluid Dynamics 1		1	Professor X. Wu	90	10
MATH70002	Fluid Dynamics 2		2	Professor J. Mestel	90	10
MATH70051	Vortex Dynamics		2	Professor D. Crowdy	90	10
MATH70052	Hydrodynamic Stability		2	Professor X. Wu	90	10

MATHEMATICAL METHODS

Module Code	Module Titles	Group	Term	Lecturer	% exam	% CW
MATH70004	Asymptotic Methods		1	Dr O. Schnitzer	90	10
MATH70005	Optimisation	A	1	Dr D. Kalise	90	10
MATH70006	Applied Complex Analysis		2	Dr. X. Guan	90	10
MATH70141	Introduction to Game Theory		1	Dr S. Brzezicki	90	10

DYNAMICS

Module Code	Module Titles	Group	Term	Lecturer	% exam	% CW
MATH70007	Dynamics of Learning and Iterated Games		1	Professor S. van Strien	40 (Oral)	60
MATH70008	Dynamical Systems		1	Prof J. Lamb	90	10
MATH70009	Bifurcation Theory		2	Prof D. Turaev	90	10
MATH70053	Random Dynamical Systems and Ergodic Theory		2	Prof J. Lamb	40 (Oral)	60
MATH70023	Computational Dynamical Systems	A	1	Dr E. Keaveny	50	50

MATH70146	Advanced Topics in Dynamical Systems		2	Professor S. van Strien	40 (Oral)	60
MATH70003	Introduction to Geophysical Fluid Dynamics		2	Prof P. Berloff	90	10
MATH70143	Dynamics, Symmetry, and Integrability		2	Prof D. Holm	90	10

BIOLOGY

Module Code	Module Titles	Group	Term	Lecturer	% exam	% CW
MATH70014	Mathematical Biology		1	B. Bassols Cornudella	90	10
MATH70137	Mathematical Biology 2: Systems Biology		2	Dr O. Karin	90	10

MATHEMATICAL PHYSICS

Module Code	Module Titles	Group	Term	Lecturer	% exam	% CW
MATH70015	Quantum Mechanics 1		1	Dr E-M Graefe	90	10
MATH70016	Special Relativity and Electromagnetism		1	Dr G. Pruessner	90	10
MATH70017	Tensor Calculus and General Relativity		2	Dr C. Ford	90	10
MATH70018	Quantum Mechanics 2		2	Dr R. Barnett	90	10
MATH70147	Statistical Mechanics		1	Dr T. Bertrand	50	50

APPLIED PDEs

Module Code	Module Titles	Group	Term	Lecturer	% exam	% CW
MATH70054	Introduction to Stochastic Differential Equations and Diffusion Processes	A	1	Prof P. Bressloff	90	10
MATH70019	Theory of Partial Differential Equations		1	Dr M. Sorella	90	10
MATH70135	Advanced Partial Differential Equations 1		1	Prof G. Pavliotis	90	10
MATH70021	Advanced Partial Differential Equations 2		2	Prof M. Coti Zelati	90	10

NUMERICAL ANALYSIS and COMPUTATION

Module Code	Module Titles	Group	Term	Lecturer	% exam	% CW
MATH70022	Finite Elements: Numerical Analysis and Implementation	A	2	Prof C. Cotter & Prof D. Ham	50	50
MATH70024	Computational Linear Algebra	A	1	Dr A. Broms	50	50
MATH70025	Computational Partial Differential Equations	A	2	Dr S. Mughal	0	100
MATH70026	Methods for Data Science	A	2	Dr B. Bravi	0	100
MATH70134	Mathematical Foundations of Machine Learning	A	1	Dr N. Boulle	0	100

MATH70031	Probability Theory 2		2	Dr B. Dagallier	90	10
MATH70148	Probabilistic Generative Models	A	2	Dr F. Tobar	60	40

FINANCE

Module Code	Module Titles	Group	Term	Lecturer	% exam	% CW
MATH70012	Mathematical Finance: An Introduction to Option Pricing		1	Dr P. Siorpaes	90	10
MATH70130	Stochastic Differential Equations in Financial Modelling		1	Prof D. Brigo	90	10

STATISTICS

Module Code	Module Titles	Group	Term	Lecturer	% exam	% CW
MATH70049	Introduction to Statistical Learning		2	Professor G. Nason	100	
MATH70139	Spatial Statistics		2	Dr A Sykulski	90	10

GEOMETRY

Module Code	Module Titles	Group	Term	Lecturer	% exam	% CW
MATH70140	Geometric Complex Analysis		2	Dr D. Cheraghi	90	10

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Module descriptions

MATH70001: Fluid Dynamics 1

Brief Description

Fluid dynamics investigates motions of both liquids and gases. Being a major branch of continuum mechanics, it does not deal with individual molecules, but with an 'averaged' motion of the medium (i.e. collections of molecules). The aim is to predict the velocity, pressure and temperature fields in flows arising in nature and engineering applications. In this module, the equations governing fluid flows are derived by applying fundamental physical laws to the continuum. This is followed by descriptions of various techniques to simplify and solve the equations with the purpose of describing the motion of fluids under different conditions.

Learning Outcomes

On successful completion of this module, you will be able to

- state the underlying assumptions of the continuum hypothesis;
- compare and contrast the different frameworks that can be used to describe fluid motion and to identify the connections between them;
- derive exact solutions of the Navier-Stokes equations and justify the physical and made in obtaining them;
- perform simplifications arising under the assumption of inviscid flow which permit the integration of the Euler equations, leading to results such as Bernoulli's equation and Kelvin's circulation theorem;
- demonstrate a sound understanding of the method of conformal mappings and be able to use this method to analyse various two-dimensional inviscid flows;
- choose the appropriate conformal mapping to solve inviscid flow problems in complicated geometries;
- predict the shape of the flow streamlines for such problems.

Module Content

The module is composed of the following sections:

1. Introduction: The continuum hypothesis. Knudsen number. The notion of fluid particle. Kinematics of the flow field. Lagrangian and Eulerian frameworks. Streamlines and pathlines. Strain rate tensor. Vorticity and circulation. Helmholtz's first theorem. Streamfunction.
2. Governing Equations: Continuity equation. Stress tensor and symmetry, Constitutive relation. The Navier- Stokes equations.
3. Exact Solutions of the Navier-Stokes Equations: Couette and Poiseuille flows. The flow between two coaxial cylinders. The flow over an impulsively started plate. Diffusion of a potential vortex.
4. Inviscid Flow Theory: Integrals of motion. Kelvin's circulation theorem. Potential flows. Bernoulli's equation. Cauchy-Bernoulli integral for unsteady flows. Two-dimensional flows. Complex potential. Vortex, source, dipole and the flow past a circular cylinder. Adjoint mass. Conformal mapping. Joukovskii transformation. Flows past aerofoils. Lift force. The theory of separated flows. Kirchhoff and Chaplygin models.

MATH70002: Fluid Dynamics 2

Brief Description

In this module, we deal with a wide class of realistic problems by seeking asymptotic solutions of the governing Navier-Stokes equations in various limits. We shall start with the "slow, small or sticky" case, when the Reynolds number is low and we obtain the linear Stokes equations. Then we consider the lubrication limit, and show how a thin layer of fluid is able to exert enormous pressures and prevent moving solid bodies from touching. Next we shall consider the "fast and vast" limit of high Reynolds number, which is characteristic of most flows we encounter in everyday life. In the final part of the module we consider a mixture of advanced topics, including flight, bio-fluid-dynamics and an introduction to flow stability.

Learning Outcomes

On successful completion of this module you will be able to:

- simplify and solve the governing Navier-Stokes equations in situations where there is a short lengthscale in one of the coordinate directions;
- apply the general properties of low Reynolds number flows to predict the drag on slow-moving bodies, like a solid sphere or spherical bubble, and appreciate the causes of the 'Stokes paradox';
- analyse lubrication-like flows in thin layers;
- derive the boundary-layer equations and identify self-similar solutions for flows at large Reynolds number;
- determine stability criteria for various fundamental flows;
- model animal locomotion at low and high Reynolds numbers;
- interpret results from advanced fluid mechanics textbooks and research papers;
- independently appraise and evaluate some advanced topics in viscous fluid mechanics.

Module Content

The module is composed of the following sections:

1. Low-Reynolds-number flows: Dynamic Similarity. Properties of the Stokes equations. Uniqueness and minimal dissipation theorems. The analysis of the flow past a solid sphere and spherical bubble. Stokes paradox.
2. Lubrication Theory: Derivation of Reynolds' lubrication equation and examples. Hele-Shaw and thin film flows.
3. High-Reynolds-number flows; Boundary-layer theory: The notion of singular perturbations. Derivation of boundary-layer equations. Blasius flow, Falkner-Skan solutions and applications. Von Mises variables and their application to periodic boundary layers. Prandtl-Batchelor Theorem for flows with closed streamlines.
4. Introduction to hydrodynamic stability: Importance of stability. Rayleigh-Taylor and Kelvin-Helmholtz instabilities. Circular flow stability criterion.
5. Swimming and Flight; Animal locomotion: Scallop theorem. Resistive Force Theory. Introduction to 3D-aerofoil theory. Flight strategies.
6. Advanced Topics: Current research in areas such as convection and magnetohydrodynamics.

MATH70003: Introduction to Geophysical Fluid Dynamics

Brief Description

This is an advanced-level fluid-dynamics course with geophysical flavours. The lectures target upper-level undergraduate and graduate students interested in the mathematics of planet Earth, and in the variety of motions and phenomena occurring in planetary atmospheres and oceans. The lectures provide a mix of theory and applications.

Learning Outcomes

On successful completion of this module you will be able to:

- demonstrate a deep understanding of the foundations of geophysical fluid dynamics;
- model a broad range of natural phenomena associated with the atmosphere and ocean;
- appreciate the main concepts and terminology used in the field;
- derive the boundary layer equations for flow in a rotating frame and justify the relative importance of various terms in the equations of motion;
- describe, select appropriately and apply a range of methods and techniques for solving practical problems;
- independently appraise an advanced topic in geophysical fluid dynamics;
- evaluate results from research papers in the field of geophysical fluid dynamics.

Module Content

The module is composed of the following sections:

1. Introduction and basics;
2. Governing equations (continuity of mass, material tracer, momentum equations, equation of state, thermodynamic equation, spherical coordinates, basic approximations);
3. Geostrophic dynamics (shallow-water model, potential vorticity conservation law, Rossby number expansion, geostrophic and hydrostatic balances, ageostrophic continuity, vorticity equation);
4. Quasigeostrophic theory (two-layer model, potential vorticity conservation, continuous stratification, planetary geostrophy);
5. Ekman layers (boundary-layer analysis, Ekman pumping);
6. Rossby waves (general properties of waves, physical mechanism, energetics, reflections, mean-flow effect, two-layer and continuously stratified models);
7. Hydrodynamic instabilities (barotropic and baroclinic instabilities, necessary conditions, physical mechanisms, energy conversions, Eady and Phillips models);
8. Ageostrophic motions (linearized shallow-water model, Poincare and Kelvin waves, equatorial waves, ENSO “delayed oscillator”, geostrophic adjustment, deep-water and stratified gravity waves);
9. Transport phenomena (Stokes drift, turbulent diffusion);
10. Nonlinear dynamics and wave-mean flow interactions (closure problem and eddy parameterization, triad interactions, Reynolds decomposition, integrals of motion, enstrophy equations, classical 3D turbulence, 2D turbulence, transformed Eulerian mean, Eliassen-Palm flux).

MATH70004: Asymptotic Methods

Brief Description

This advanced course presents a systematical introduction to asymptotic methods, which form one of the cornerstones of modern applied mathematics. The foundation of asymptotic approximations is laid down first. The key ideas and techniques for deriving asymptotic representations of integrals, and for constructing appropriate solutions to differential equations will be explained. The techniques introduced find wide applications in engineering and natural sciences.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the foundation upon which asymptotic approximations are based;
- describe a variety of asymptotic methods and for each method acquire a thorough understanding of the key ideas involved and their mathematical nature;
- demonstrate basic skills in applying each of these methods to solve classical problems;
- combine, modify and extend methods to unfamiliar problems, such as those that emerge from research topics or practical applications;
- outline how asymptotic methods can in principle be applied to a wide variety of problems;
- interpret results from advanced textbooks and research papers on asymptotic methods;
- construct advanced solution techniques by selecting an appropriate combination of different asymptotic methods to solve higher-dimensional problems.

Module Content

1. Asymptotic approximations (fundamentals)
   Order notation. Diverging series, asymptotic expansions. Parameter expansions, overlap regions, distinguished limits and uniform approximations.Stokes phenomenon.
2. Introduction to perturbation methods
   Asymptotic solution of algebraic equations with a small parameter. Regular vs. singular perturbations. Method of dominant balance. Local analysis of ordinary differential equations.
3. Asymptotic analysis of integrals
   Method of integration by parts. Integrals of Laplace type: Laplace's method, Watson's Lemma. Integrals of Fourier type: method of stationary phase. Integral in the complex plane: method of steepest descent. Method of splitting the range of integration.
4. Matched asymptotic expansion
   Inner and outer expansions, matching principles, notions of 'boundary layer' and interior layer. Composite approximation. Application to relaxation oscillations.
5. Methods of multiple scales
   WKB approximations including turning-point problems and eigenvalue quantisation. Secular terms and solvability conditions. Poincare-Lindstedt method for periodic solutions. Multiscale method for quasi-periodic solutions. Application to weakly perturbed oscillators, nonlinear resonance, parametric resonance.
6. A selection of topics from the following: Stokes phenomenon, hyperasymptotics, expansions involving logarithmic terms, homogenisation.

MATH70005: Optimisation (A)

Brief Description

This module is an introduction to the theory and practice of mathematical optimization and its many applications in mathematics, data science, and engineering. The module aims at endowing students with the necessary mathematical background and a thorough methodological toolbox to formulate optimization problems and developing an algorithmic approach to its solution. The module is structured into five parts: (i) formulation and classification of problems; (ii) unconstrained optimization; (iii) stochastic and nature-inspired optimization; (iv) convex optimization; (v) introduction to optimal control and dynamic optimization. The assessed coursework for this module involves a series of computational tasks.

Learning Outcomes

On successful completion of this module you will be able to

- formulate a mathematical optimization problem by identifying a suitable objective and constraints;
- identify the mathematical structure of an optimization problem and, based on this classification, choose an appropriate methodological approach;
- develop a mathematical and computational appreciation of convexity as a fundamental feature in optimization;
- implement different computational optimization algorithms such as gradient descent and related variants;
- analyse the results of a computational optimization method in terms of optimality guarantees, sensitivities, and performance.
- interpret the role played by optimization in its application to computational data science;
- design optimal control approaches relevant to tackling large-scale nonlinear problems.

Module Content

1. Mathematical preliminaries
2. Unconstrained optimization
3. Gradient descent methods
4. Linear and non-linear least squares problems
5. Stochastic gradient descent
6. Nature-inspired optimization
7. Convex sets and functions
8. Convex optimization problems and stationarity
9. KKT conditions
10. Duality
11. Introduction to dynamic optimization and optimal control.

This final topic is linked to the Mastery Material for MSci students which will involve the study of some of the following solution techniques:

- shooting and multiple shooting methods;
- the reduced gradient approach;
- two-point boundary value solvers for optimal control;
- dynamic Programming and the Hamilton-Jacobi PDE;
- the linear-quadratic regulator and the Riccati equation.

This will be examined by way of an extra question on the May examination paper.

MATH70006: Applied Complex Analysis

Brief Description

The aim of this module is to learn tools and techniques from complex analysis and the theory of orthogonal polynomials that can be used in mathematical physics. The course will focus on mathematical techniques, though will also discuss relevant physical applications, such as electrostatic potential theory. The course incorporates computational techniques in the lectures.

Learning Outcomes

On successful completion of this module, you will be able to:

- apply the technique of contour deformation for calculating integrals;
- appreciate the connection that exists between computational tools such as quadrature and orthogonal polynomials and complex analysis;
- evaluate singular integral equations with Cauchy and logarithmic kernels;
- use the Wiener-Hopf method to solve a class of integral equations;
- compute matrix functions using contour inegration;
- interpret results from advanced textbooks and research papers;
- independently appraise and evaluate an advanced topic in complex analysis.

Module Content

This module covers the following topics:

- Revision of complex analysis: complex integration, Cauchy's theorem and residue calculus;
- Singular integrals: Cauchy, Hilbert, and log kernel transforms;
- Potential theory: Laplace's equation, electrostatic potentials, distribution of charges in a well;
- Riemann–Hilbert problems: Plemelj formulae, additive and multiplicative Riemann–Hilbert problems;
- Orthogonal polynomials: recurrence relationships, solving differential equations, calculating singular integrals;
- Integral equations: integral equations on the whole and half line, Fourier transforms, Laplace transforms;
- Wiener–Hopf method: direct solution, solution via Riemann–Hilbert methods;
- Singularities of differential equations: analyticity of solutions, regular singular points, hypergeometric functions.

MATH70007: Dynamics of Learning and Iterated Games

Brief Description

Recently there has been considerable interest in modelling learning. The settings to which these models are applied is wide-ranging. Examples include optimization of strategies of populations in ecology and biology, iterated strategies of people in a competitive environment and learning models used by technology companies such as Google.

This module is aimed at discussing a number of such models in which learning evolves over time and which have a game theoretic background. The module will use tools from the theory of dynamical systems and will aim to be rigorous. Topics will include replicator systems, best response dynamics and fictitious games, reinforcement learning and no-regret learning.

Learning Outcomes

On successful completion of this module, you will be able to:

- analyse 2D replicators systems for one and two player games;
- work comfortably with the notions of Nash, Correlated Equilibrium, Course Correlated Equilibrium and Evolutionary Stable Strategies;
- explain the notion of reciprocity in relation to Iterated Prisoner Dilemma games;
- appreciate the connection between Reinforcement Learning and replicator systems;
- outline the idea behind no regret learning models and the Blackwell approachability theorem;

- derive the proofs behind the methods that are used in the final project;
- appraise and interpret results from advanced textbooks and research papers.

Module Content

The module will cover the following topics:

- Replicator systems;
- Rock-paper-scissor games;
- Iterated prisoner dilemma games;
- Best response dynamics;
- Two player games;
- Fictitious games as a learning model;
- Reinforcement learning;
- No regret learning

MATH70008: Dynamical Systems

Brief Description

The theory of Dynamical Systems is an important area of mathematics which aims at describing objects whose state changes over time. For instance, the solar system comprising the sun and all planets is a dynamical system, and dynamical systems can be found in many other areas such as finance, physics, biology and social sciences. This course provides a rigorous treatment of the foundations of discrete-time dynamical systems.

Learning Outcomes

On successful completion of this module, you will be able to:

- demonstrate a familiarity with the basic concepts of topological dynamics;
- provide an outline of the ergodic theory of dynamical systems;
- appreciate the concept of symbolic dynamics through which topological and probabilistic dynamical properties can be understood;
- demonstrate an understanding of precise mathematical characterisations of chaotic dynamics;
- apply the above context in a number of one-dimensional settings, in particular in the context of piecewise affine expanding maps;
- independently appraise and evaluate advanced topological and probabilistic dynamical properties, beyond the foundations;
- independently develop and interpret examples in two and higher dimensions.

Module Content

The module covers the following topics:

- Introduction: orbits, periodic orbits and their local stability;
- Topological dynamics: invariant sets and limit sets, coding and sequence spaces, topological conjugacy, transitivity and mixing;
- Chaotic dynamics: sensitive dependence, topological entropy, topological Markov chains;
- Ergodic theory: sigma-algebras and measures, invariant measures, Poincaré recurrence, ergodicity and Birkhoff's Ergodic Theorem, Markov measures and metric entropy;
- Additional reading material in line with M4 objectives.

MATH70009 Bifurcation Theory

Brief Description

This module serves as an introduction to bifurcation theory, concerning the study of how the behaviour of dynamical systems such as ODEs and maps changes when parameters are varied.The goal is to acquaint the students with the foundations of the theory, its main discoveries and the universal methods behind this theory that extend beyond its remit.

Learning Outcomes

On successful completion of this module, you will be able to:

- exploit basic dimension reduction methods (invariant manifold and invariant foliations);
- apply the method of normal forms;
- demonstrate a sound knowledge of the basics of stability theory;
- appreciate the role of control parameters and to construct bifurcation diagrams;
- describe the mathematical framework associated with classical local and global bifurcations;
- interpret results from advanced textbooks and research papers on bifurcation theory;
- independently appraise and evaluate the transition from periodic to quasiperiodic regimes and to chaos via destruction of quasiperiodicity.

Module Content

The following topics will be covered:

1. Bifurcations on a line and on a plane;
2. Centre manifold theorem; local bifurcations of equilibrium states;
3. Local bifurcations of periodic orbits – folds and cusps;
4. Homoclinic loops: cases with simple dynamics, Shilnikov chaos, Lorenz attractor;
5. Saddle-node bifurcations: destruction of a torus, intermittency, blue-sky catastrophe;
6. Routes to chaos and homoclinic tangency.

MATH70012: Mathematical Finance: An Introduction to Option Pricing

Brief Description

The mathematical modelling of derivatives securities, initiated by Bachelier in 1900 and developed by Black, Scholes and Merton in the 1970s, focuses on the pricing and hedging of options, futures and other derivatives, using a probabilistic representation of market uncertainty. This module is a mathematical introduction to this theory, in a discrete-time setting. We will mostly focus on the no-arbitrage theory in market models described by trees; eventually we will take the continuous-time limit of a binomial tree to obtain the celebrated Black- Scholes model and pricing formula.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the fundamental principles involved in pricing derivatives;
- describe and critically analyse simple market models and explore their qualitative properties;
- confidently perform calculations involving pricing and hedging in discrete market models;
- demonstrate a familiarity with some key concepts in modern probability theory and apply them to perform computations;

- outline a mathematical formulation describing the behaviour of a number of financial derivatives;
- construct dynamic programming techniques to solve problems where inter-temporal relations are important;
- appraise and critically evaluate one or more of the advanced topics listed below.

Module Content

The module will cover the following topics:

financial derivatives, arbitrage, no-arbitrage pricing, self-financing portfolios, non-anticipative trading strategies, hedging of derivatives, domination property, complete markets, 'risk-neutral' probabilities, the fundamental theorems of asset pricing, conditional probability and expectation, filtrations, Markov processes, martingales, change of measure.

Extra mastery component will include the following advanced topics: utility, optimal investment.

MATH70014: Mathematical Biology

Brief Description

Mathematical biology entails the use of mathematics to model biological phenomena in order to understand these systems, as well as predict their behaviour. It is an incredibly diverse field utilising the complete mathematical toolbox to ascertain insight into many areas of biology and medicine including population dynamics, physiology, epidemiology, cell biology, biochemical reactions, and neurology. This module aims to provide a foundational course in the subject area relying primarily on tools from applied dynamical systems, applied PDEs, asymptotic analysis and stochastic processes.

Learning Outcomes

On successful completion of this module you will be able to:

- translate biological phenomena into the language of mathematics;
- appreciate canonical problems in epidemiology, ecology, biochemistry and physiology;
- analyse sets of ordinary differential equations especially in the non-linear setting;
- critically analyse sets of partial differential equations especially when either travelling-wave solutions or pattern forming phenomena might emerge;
- utilise the concept of stochastic population processes for exact and approximate solutions;
- use the techniques of order-of-magnitude reasoning and dimensional analysis;
- interpret results from the research literature on Mathematical Biology and analyze how the syllabus content relates to this wider body of work;
- appraise and evaluate an advanced topic in Mathematical Biology from a selection of case studies.

Module Content

Examples and topics include:

1. One-dimensional systems: existence and uniqueness; fixed points and their stability; bifurcations; logistic growth; SIS epidemic model; spruce budworm model; law of mass action; Michaelis-Menten enzyme dynamics.
2. Multidimensional systems: existence, uniqueness, fixed point stability; two-dimensional systems; SIS model for two populations; genetic control systems; population competition models; predator-prey dynamics and the Lotka-Volterra model.

3. Oscillations and bifurcations: Poincaré-Bendixson Theorem; oscillations in predator-prey models; relaxation oscillators; Fitzhugh-Nagumo model; fixed point bifurcations; Hopf bifurcations and limit cycles.
4. Spatial dynamics: reaction-diffusion equations; Fisher-Kolmogorov equation; travelling waves in predator-prey systems; spatial SIS model; spread of rabies in a fox population; Turing instabilities; pattern formation in one and two dimensions.
5. Stochastic processes: continuous-time Markov chains; simple birth and death processes; stationary probability distributions; logistic growth process; branching processes and drug resistance; multivariate processes; stochastic enzyme dynamics; stochastic predator-prey dynamics.

MATH70015: Quantum Mechanics I

Brief Description

Quantum mechanics is one of the most successful theories in modern physics and has an exceptionally beautiful underlying mathematical structure. It provides the basis for many areas of contemporary physics, including atomic and molecular, condensed matter, high-energy particle physics, quantum information theory, and quantum cosmology, and has led to countless technological applications. This module aims to provide an introduction to quantum phenomena and their mathematical description. We will use tools and concepts from various areas of mathematics and physics, such as classical mechanics, linear algebra, probability theory, numerical methods, analysis and geometry.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate Schrödinger's formulation of quantum mechanics, wave functions and wave equations;
- construct the mathematical framework of quantum mechanics, including the 4 postulates of quantum mechanics and the Dirac notation;
- solve the eigenvalue problem for basic one-dimensional quantum systems;
- exploit the method of stationary states to deduce the time-evolved quantum state from the initial state of a system;
- communicate fluently using the Dirac notation;
- interpret results from advanced quantum mechanics textbooks and research papers;
- independently appraise and evaluate an advanced (more contemporary) topic in quantum mechanics from those listed in the syllabus below.

Module Content

The module will cover the following topics:

1. Hamiltonian dynamics;
2. Schrödinger equation and wave functions;
3. stationary states of one-dimensional systems;
4. mathematical foundations of quantum mechanics;
5. quantum dynamics;
6. angular momentum.

A selection of topics among the following additional optional topics will be covered depending on students interests:

1. approximation techniques;

2. explicitly time-dependent systems;
3. geometric phases;
4. numerical techniques;
5. many-particle systems;
6. cold atoms;
7. entanglement and quantum information.

MATH70016: Special Relativity and Electromagnetism

Brief Description

This module presents a beautiful mathematical description of a physical theory of great historical, theoretical and technological importance. It demonstrates how advances in modern theoretical physics are being made and gives a glimpse of how other theories (say quantum chromodynamics) proceed. This module does not follow the classical presentation of special relativity by following its historical development, but takes the field theoretic route of postulating an action and determining the consequences. The lectures follow closely the famous textbook on the classical theory of fields by Landau and Lifshitz.

Learning Outcomes

On successful completion of this module you will be able to

- demonstrate an understanding of the relation between space and time and apply Lorentz transforms;
- appreciate the structure of special relativity as derived from the principle of least action;
- determine relativistic particle trajectories;
- derive Maxwell's equations from first principles and apply them to variety of interactions of charges and fields;
- critically analyse various solutions of the electromagnetic wave equations;
- describe electrostatic interactions and motion using Coulomb's law;
- construct an expansion of electrostatic interactions in terms of multipoles.

Module Content

This course follows closely the following book: L.D. Landau and E.M. Lifschitz, Course on Theoretical Physics Volume 2: Classical Theory of Fields.

1. Special relativity: Einstein's postulates, Lorentz transformation and its consequences, four vectors, dynamics of a particle, mass-energy equivalence, collisions, conserved quantities.
2. Electromagnetism: Magnetic and electric fields, their transformations and invariants, Maxwell's equations, conserved quantities, wave equation.

MATH70017: Tensor Calculus and General Relativity

Brief Description

This module provides an introduction to General Relativity. Starting with the rather simple Mathematics of Special Relativity the goal is to provide you with the mathematical tools to formulate General Relativity. Some examples, including the Schwarzschild space-time are considered in detail.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the application of tensors in special relativity;
- demonstrate a working knowledge of tensor calculus;
- explain the concepts of parallel transport and curvature;
- formulate and solve the geodesic equation for a given space-time metric;
- derive Einstein's field equations and analyse Schwarzschild's solution;
- interpret results from advanced general relativity textbooks and research papers;
- appraise and critically evaluate two of the extensions and applications listed below.

Module Content

This module will cover the following topics:

1. Special Relativity
2. Tensors in Special Relativity
3. Tensors in General Coordinates Systems
4. Parallel Transport and Curvature
5. General Relativity
6. The Schwarzschild Spacetime
7. Variational Methods
8. Extensions and Applications (selected from gravitational waves, Einstein-Hilbert action, cosmology, Einstein- Cartan theory, differential geometry)

MATH70018: Quantum Mechanics 2

Brief Description

Quantum mechanics (QM) is one of the most successful theories in modern physics and has an exceptionally beautiful underlying mathematical structure. Assuming some prior exposure to the subject (such as Quantum Mechanics I), this module aims to provide an intermediate/advanced treatment of quantum phenomena and their mathematical description. Quantum theory combines tools and concepts from various areas of mathematics and physics, such as classical mechanics, linear algebra, probability theory, numerical methods, analysis and geometry.

Learning Outcomes

On successful completion of this module, you will be able to:

- outline key aspects of quantum mechanics at the intermediate/advanced level;
- harness the power of symmetry in understanding quantum mechanics;
- describe many-particle quantum mechanical systems, and demonstrate familiarity with the formalism of second quantisation;
- solve complex quantum mechanical problems using the machinery introduced in this module;
- use the knowledge gained here as a solid foundation for a research project in quantum mechanics;
- interpret results from advanced quantum mechanics textbooks and research papers;
- appraise and evaluate a topic in quantum mechanics from the syllabus at an advanced level.

Module Content

This module will cover the following core topics:

1. quantum mechanics in the momentum basis;
2. the Heisenberg picture;

3. the use of symmetry and general transformations in quantum mechanics;
4. Elements of Quantum Computation;
5. perturbation theory;
6. adiabatic processes;
7. second quantisation;
8. introduction to many-particle systems;
9. Fermi and Bose statistics.

Additional topics include: WKB theory, the Feynman path integral, quantum magnetism.

MATH70019: Theory of Partial Differential Equations

Brief Description

In this module, students are exposed to different phenomena which are modelled by partial differential equations. The course emphasizes the mathematical analysis of these models and briefly introduces some numerical methods.

Learning Outcomes

On successful completion of this module you will be able to:

- appreciate how to formally differentiate complicated finite dimensional functionals and simple infinite dimensional functionals;
- describe, select and use a variety of methods for solving partial differential equations;
- outline how various partial differential equations respect conservation laws;
- utilize energy methods to critically analyse the stability of solutions to PDEs;
- develop the general method of characteristics and derive the eikonal equation;
- justify the proper use of the calculus of variations in classical settings.

Module Content

The module is composed of the following sections:

1. Introduction to PDEs
   1.1. Basic Concepts
   1.2. Gauss Theorem
2. Method of Characteristics
   2.1. Linear and Quasilinear first order PDEs in two independent variables.
   2.2. Scalar Conservation Laws
   2.3. Hamilton-Jacobi Equations. General Method of Characteristics.
3. Diffusion
   3.1. Heat equation. Maximum principle
   3.2. Separation of variables. Fourier Series.
4. Waves
   4.1. The 1D wave equation
   4.2. 2D and 3D waves.
5. Laplace-Poisson equation
   5.1. Dirichlet and Neumann problems.
   5.2. Introduction to calculus of variations. The Dirichlet principle.
   5.3. Finite Element Method.
   5.4. Lagrangians and the minimum action principle.

MATH70021: Advanced Partial Differential equations 2

Brief Description

The focus of the course is the theory of nonlinear partial differential equations (PDEs) and their modern treatment through analytical techniques. The emphasis is on methods (such as fixed point theorems, Fourier analysis) and how they apply to classical problems involving fluid mechanics and wave propagation.

Learning Outcome

On successful completion of this module you will be able to:

- appreciate the concepts of distribution (differentiation, convergence);
- manipulate the main properties of the Sobolev space H^m for integer m (inbeddings and compactness theorems, Poincaré inequality);
- derive the variational formulation of a specific elliptic boundary value problem and to provide the reasoning leading to the proof of the existence and uniqueness of the solution;
- develop the spectral theory of an elliptic boundary value problem;
- solve a parabolic boundary value problem using the spectral theory of the associated elliptic operator.
- interpret results from advanced textbooks and research papers on the theory of Partial Differential Equations;
- independently appraise and evaluate an advanced topic on Partial Differential Equations, namely the theory of nonlinear elliptic and parabolic equations on the whole space.

Module Content

An indicative list of topics is:

1. Fixed point theorems and applications: the contraction mapping principle, Brouwer and Schauder fixed-point theorems, semi linear parabolic equations.
2. Elements of Fourier analysis: the Fourier transform, the method of stationary phases, singular integrals.
3. Energy estimates and compactness: basic notions, spaces involving time, existence of solutions to nonlinear evolution equations.
4. Applications to equations arising in Mathematical Physics: the Euler and Navier-Stokes equations, dispersive equations, nonlinear heat equations, hyperbolic conservation laws.

MATH70022: Finite Elements: Numerical Analysis and Implementation (A)

Brief Description

Finite element methods form a flexible class of techniques for numerical solution of PDEs that are both accurate and efficient. The finite element method is a core mathematical technique underpinning much of the development of simulation science. Applications are as diverse as the structural mechanics of buildings, the weather forecast, and pricing financial instruments. Finite element methods have a powerful mathematical abstraction based on the language of function spaces, inner products, norms and operators.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the core mathematical principles of the finite element method;

- employ the finite element method to formulate and analyse numerical solutions to linear elliptic PDEs;
- implement the finite element method on a computer;
- compare the application of various software engineering techniques to numerical mathematics;
- generalize the concept of a directional derivative;
- appraise and evaluate techniques for solving nonlinear PDEs using the finite element method.

Module Content

This module aims to develop a deep understanding of the finite element method by spanning both its analysis and implementation. In the analysis part of the module, students will employ the mathematical abstractions of the finite element method to analyse the existence, stability and accuracy of numerical solutions to PDEs. At the same time, in the implementation part of the module students will combine these abstractions with modern software engineering tools to create and understand a computer implementation of the finite element method.

This module is composed of the following sections:

1. Basic concepts: weak formulation of boundary value problems, Ritz-Galerkin approximation, error estimates, piecewise polynomial spaces, local estimates;
2. Efficient construction of finite element spaces in one dimension: 1D quadrature, global assembly of mass matrix and Laplace matrix;
3. Construction of a finite element space: Ciarlet's finite element, various element types, finite element interpolants;
4. Construction of local bases for finite elements: efficient local assembly;
5. Sobolev Spaces: generalised derivatives, Sobolev norms and spaces, Sobolev's inequality;
6. Numerical quadrature on simplices: employing the pullback to integrate on a reference element;
7. Variational formulation of elliptic boundary value problems: Riesz representation theorem, symmetric and nonsymmetric variational problems, Lax-Milgram theorem, finite element approximation estimates;
8. Computational meshes: meshes as graphs of topological entities, discrete function spaces on meshes, local and global numbering;
9. Global assembly for Poisson equation: implementation of boundary conditions, general approach for nonlinear elliptic PDEs;
10. Variational problems: Poisson's equation, variational approximation of Poisson's equation, elliptic regularity estimates, general second-order elliptic operators and their variational approximation;
11. Residual form and the Gâteaux derivative;
12. Newton solvers and convergence criteria.

MATH70023: Computational Dynamical Systems (A)

Brief Description

Nonlinear differential equations arise across the scientific disciplines and often solutions can only be computed numerically. This module entails developing a foundation in the analysis and implementation of numerical methods to obtain these solutions. Along with generating solutions, the module will examine how these methods can also be used to perform bifurcation and stability analyses of steady and time periodic solutions, analyses often required to understand the qualitative features of the system. The module will have equal emphasis on theory, both in method analysis and background stability analysis, as well as implementation. As such assessment with be 50% project and 50% exam. Students will be expected to use Python in their assessments.

Note: this module replaces the module Numerical Solutions of Ordinary Differential Equations, and so may not be taken by students who have already taken the year 3 module MATH60023.

Learning Outcomes

On the successful completion of the module, you will be able to:

- use classical numerical methods to solve initial value problems;
- analyse different properties of numerical methods (e.g. accuracy and stability);
- compare different methods with respect to accuracy, stability, and computational complexity.
- obtain numerically steady state and time periodic solutions to differential equations
- perform numerically linear stability and Floquet analyses
- conduct numerical bifurcation analyses and understand normal forms for bifurcations

Module Content

This module will cover the following topics:

1. Multi-step and multistage methods for solving initial value problems;
2. Stability, accuracy and convergence;
3. Newton methods for finding steady state and time periodic solutions;
4. Algorithms for linear stability and Floquet analyses and associated theory;
5. Jacobian-free algorithms for stability analysis
6. Normal forms for bifurcations, and weakly nonlinear analysis;
7. Applications of the methods to mathematical models arising across science and engineering;

MATH70024: Computational Linear Algebra (A)

Brief Description

Linear systems of equations arise in countless applications and problems in mathematics, science and engineering. Often these systems are large and require a computer to solve. This course provides an overview of the algorithms used to solve linear systems and eigenvalue problems, in terms of their development, stability properties, and application.

Learning Outcomes

On successful completion of this module, you will be able to:

- describe, select and use algorithms for QR decomposition of matrices;
- solve least-squares problems using QR decomposition;
- find numerical solutions to eigenvalue problems;
- critically analyse various iterative methods for solving linear systems;
- combine the techniques you have mastered in order to assess unseen algorithms;

Module Content

The module will cover the following topics:

1. Efficient implementation of numerical algorithms using NumPy.
2. QR factorisation and least squares problems.

3. Conditioning and finite precision arithmetic.
4. Eigenvalue problems: power method and variants, Householder reduction to tridiagonal form, the QR algorithm.
5. Iterative methods: Krylov subspace methods: Lanczos method and Arnoldi iteration, conjugate gradient method, GMRES, preconditioning, the conjugate gradient method, symmetric stationary methods as preconditioners.

MATH70025: Computational Partial Differential Equation (A)

Brief Description

This module will introduce a variety of computational approaches for solving partial differential equations, focusing mostly on finite difference methods, but also touching on finite volume and spectral methods.

Students will gain experience implementing the methods and writing/modifying short programs in Matlab or another programming language of their choice. Applications will be drawn from problems arising in areas such as Mathematical Biology and Fluid Dynamics.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the physical and mathematical differences between different types of PDES;
- design suitable finite difference methods to solve each type of PDE;
- outline a theoretical approach to testing the stability of a given algorithm;
- determine the order of convergence of a given algorithm;
- demonstrate familiarity with the implementation and rationale of multigrid methods;
- develop finite-difference based software for use on research level problems;
- communicate your research findings as a poster, in a form suitable for presentation at a scientific conference.

Module Content

The module will cover the following topics:

1. Introduction to Finite Differences
2. Classification of PDEs
3. Explicit and Implicit methods for Parabolic PDEs
4. Iterative Methods for Elliptic PDEs. Jacobi, Gauss-Seidel, Overrelaxation
5. Multigrid Methods
6. Hyperbolic PDEs. Nonlinear Advection/Diffusion systems. Waves and PMLs

as well as various advanced practical topics from Fluid Dynamics, which will depend on the final project.

MATH70026: Methods for Data Science (A)

Brief Description

This module provides an hands-on introduction to the methods of modern data science. Through interactive lectures, the student will be introduced to data visualisation and analysis as well as the fundamentals of machine learning.

Learning Outcomes

On successful completion of this module, you will be able to:

- Visualise and explore data using computational tools;
- Appreciate the fundamental concepts and challenges of learning from data;
- Analyse some commonly used learning methods;
- Compare learning methods and determine suitability for a given problem;
- Describe the principles and differences between supervised and unsupervised learning;
- Clearly and succinctly communicate the results of a data analysis or learning application;
- Appraise and evaluate new algorithms and computational methods presented in scientific and mathematical journals;
- Design and implement newly-developed algorithms and methods.

Module Content

The module is composed of the following sections:

1. Introduction to computational tools for data analysis and visualisation;
2. Introduction to exploratory data analysis;
3. Mathematical challenges in learning from data: optimisation;
4. Methods in Machine Learning: supervised and unsupervised; neural networks and deep learning; graph-based data learning;
5. Machine learning in practice: application of commonly used methods to data science problems. Methods include: regressions, k-nearest neighbours, random forests, support vector machines, neural networks, principal component analysis, k-means, spectral clustering, manifold learning, network statistics, community detection;
6. Current research questions in data analysis and machine learning and associated numerical methods.

MATH70031: Probability Theory 2

Brief Description

This module builds on Probability Theory 1, building a theory of continuous time stochastic processes (with Brownian Motion as a key example) along with introducing students to rigorous stochastic calculus. This module gives crucial background for students interested in stochastic differential equations, stochastic partial differential equations, and more generally stochastic analysis.

Learning Outcome

On successful completion of this module, you should be able to:

- demonstrate your understanding of the concepts and results associated with the elementary theory of Markov processes, including the proofs of a variety of results
- apply these concepts and results to tackle a range of problems, including previously unseen ones apply your understanding to develop proofs of unfamiliar results
- demonstrate additional competence in the subject through the study of more advanced material
- combine ideas from across the module to solve more advanced problems
- communicate your knowledge of the area in a concise, accurate and coherent manner

Module Content

1. Definition and construction of Brownian Motion, properties of sample Brownian paths, strong Markov property.

2. Continuous time local martingales.
3. Stochastic integrals with Brownian motion, and general continuous local martingales
4. Itˆo's formula
5. Stochastic differential equations, martingale problems, applications to PDE.
6. Markov semigroups, functional inequalities, and convergence to equilibrium.

MATH70049: Introduction to Statistical Learning

Brief Description

This module provides an introduction to methods of statistical learning. That is, using statistical and artificial intelligence (AI) methods to learn from data, often when the data set is large, complex or of high dimension. We will consider both supervised and unsupervised learning. For the former we use a training set of data to learn patterns within data and then use our knowledge of those patterns to devise methods for predicting the outcomes of those patterns for new data. For the latter, there is not (usually) an outcome measure, but we seek to learn about the patterns within the data set itself. The methods in this module are of immense interest in academic and business circles and underpins much of the modern data tech industry to achieve tasks such as suggesting movies you might like to watch given information on those that you have watched, and from people with similar viewing patterns, developing machine methods for identifying cancer and seeking new ways to understand the economy. It is strongly recommended that students will have already passed the module Statistical Modelling I, or similar.

Learning Outcomes

On successful completion of this module, you will be able to:

- Appreciate the structure and likely distribution(s) of various forms of data and which kinds of methods might be suitable for their analysis.
- Understand the concept of multivariate data and situations where supervised and unsupervised learning might be employed.
- Know and deploy a variety of important statistical and AI methods, using the R language to solve data related problems.
- Understand the limitations of methods and how their application can be checked and or corrected.

Module Content

An indicative list of sections and topics is:

1. Linear Models for Regression (variable selection, Lasso, Ridge Regression);
2. Linear methods for classification (Logistic Regression);
3. Basis expansions (piecewise polynomials and splines; smoothing splines, wavelets);
4. Kernel regression (local linear and local polynomial);
5. Additive models and Trees (boosting);
6. Projection pursuit regression and Neural Networks; Support Vector Machines;
7. Cluster Analysis and Multidimensional Scaling;
8. Random Forests;
9. Data Ethics. .

MATH70051: Vortex Dynamics

Brief Description

This is an advanced module in applied mathematical methods applied to the subfield of fluid dynamics called vortex dynamics. The module will focus on the mathematical study of the dynamics of vorticity in an ideal fluid in two and three dimensions. The material will be pitched in such a way that it will be of interest to those specializing in fluid dynamics but can also be viewed as an application of various techniques in dynamical systems theory.

Learning Outcomes

On successful completion of this module, you will be able to:

- interpret the role of vorticity within a range of problems in fluid mechanics;
- derive and compare a range of vortex models, from the point vortex models to distributed models, including vortex patches;
- combine your knowledge of different branches of mathematics (e.g. vector calculus, complex analysis and the theories of Hamiltonian dynamical systems and partial differential equations) in order to describe the dynamics of vorticity;
- choose from an array of applied mathematical techniques to explicitly solve for vorticity distributions;
- appraise the role that vortex structures play in modelling physical systems.

Module Content

The module will cover the following topics:

1. Eulerian description of fluid flows;
2. Incompressible flows and stream functions;
3. Vorticity, vortex lines and vortex tubes;
4. Biot-Savart law;
5. Euler's equations and the vorticity equation;
6. Kelvin's circulation theorem;
7. Bernoulli theorems;
8. Point vortex model, complex potentials;
9. Point vortex equilibria;
10. Dynamics of point vortices;
11. Vortex dynamics on a spherical surface;
12. Vortex patch models;
13. Vortex patch equilibria;
14. Vortex patch dynamics and contour dynamics;
15. Other distributed vortex models.

MATH70052: Hydrodynamic Stability

Brief Description

Fluid flows may exist in two distinct forms: the simple laminar state which exhibits a high degree of order and the turbulent state characterised by its complex chaotic behaviours in both time and space. The transition from a laminar state to turbulence is due to hydrodynamic instability, which refers to the phenomenon that small disturbances to a simple state amplify significantly thereby destroying the latter. This is of profound scientific and technological importance because of its relevance to mixing and transport in the atmosphere and oceans, drag and aerodynamic heating experienced by air/spacecrafts, jet noise, combustion in engines and even the operation of proposed nuclear fusion devices.

Learning Outcomes

On successful completion of this module you will be able to

- construct the basic concepts underpinning hydrodynamic stability theory;
- predict linear stability properties based on eigenvalue analysis;
- compare and contrast various different instability mechanisms;
- derive various theorems that help us decide whether a flow is stable or unstable;
- model the effects of nonlinearity within an asymptotic framework.

Module Content

Topics covered will be a selection from the following list.

1. Basic concepts of stability; linear and nonlinear stability, initial-value and eigenvalue problems, normal modes, dispersion relations, temporal/spatial instability.
2. Buoyancy driven instability: Rayleigh-Benard instability, formulation of the linearised stability problem, Rayleigh number, Rayleigh-Benard convection cells, discussion of the neutral stability properties.
3. Centrifugal instability: Taylor-Couette flow, formulation of the linear stability problem, Taylor number, Taylor vortices; inviscid approximation, Rayleigh's criterion; viscous theory and solutions, characterization of stability properties; boundary layers over concave walls, Görtler number, Görtler instability, Görtler vortices.
4. Inviscid/viscous shear instabilities of parallel flows: Inviscid/Rayleigh instability, Rayleigh equation, Rayleigh's inflection point theorem, Fjortoft's theorem, Howard's semi-circle theorem, solutions for special profiles, KelvinHelmholtz instability, general characteristics of instability, critical layer, singularity; Viscous/Tollmien- Schlichting instability, Orr-Sommerfeld (O-S) equation, Squire's theorem, numerical methods for solving the linear stability problem, discussion of instability properties.
5. Inviscid/viscous shear instabilities of (weakly) non-parallel flows: local-parallel-flow approximation and application to free shear layers and boundary layers; non-parallel-flow effects, rational explanation of viscous instability mechanism, high-Reynolds-number asymptotic theory, multi-scale approach, parabolised stabilityeq uations; transition process and prediction (correlation); receptivity.
6. Nonlinear instability: limitations of linear theories, bifurcation and nonlinear evolution; weakly nonlinear theory, derivation of Stuart-Landau and Ginzburg-Landau equations; nonlinear critical-layer theory.

MATH70053: Random Dynamical Systems and Ergodic Theory

Brief Description

This is a course on the theory and applications of random dynamical systems and ergodic theory. Random dynamical systems are deterministic dynamical systems driven by a random input. The goal will be to present a solid introduction to the subject and then to touch upon several more advanced developments in this field.

Learning Outcomes

On successful completion of this module, you will be able to:

- describe the fundamental concepts of random dynamical systems;

- summarize the ergodic theory of random dynamical systems;
- select and critically appraise relevant research papers and chapters of research monographs;
- combine the ideas contained in such papers to provide a written overview of the current state of affairs concerning a particular aspect of random dynamical systems theory;
- thoughtfully engage orally in discussions related to random dynamical systems.

Module Content

Introductory lectures include foundational material on:

1. Invariant measures and ergodic theory
2. Random (pullback) attractors
3. Lyapunov exponents
4. Random circle homeomorphisms

Further material is at a more advanced level, touching upon current frontline research. Students select material from research level articles or book chapters.

MATH70054: Introduction to Stochastic Differential Equations and Diffusion Processes (A)

Brief Description

This module provides an introduction to stochastic differential equations (SDEs) and diffusion processes, with an emphasis on solution methods and applications. The course covers the following topics: elements of probability theory for stochastic processes in continuous time, random walks and Brownian motion, Ito stochastic calculus from an applied perspective, the connection between SDEs and Fokker-Planck (FP) PDEs, methods for solving SDEs and the FP equation, first passage time problems, and approximation methods. Applications will include noise-induced escape problems, random search processes, molecular motors, stochastic resetting, diffusion approximations of chemical master equations, and reaction rate theory.

Learning Outcomes

On successful completion of this module, you will be able to:

- formulate the basics of the theory of stochastic processes in continuous time;
- understand the connection between random walks and Brownian motion
- apply Ito's theory of stochastic integration;
- connect SDEs and the forward and backward Fokker-Planck (FP) equation
- construct stationary solutions and solve boundary value problems of the FP equation
- calculate splitting probabilities and mean first passage times for simple random search processes
- analyze noise-induced escape in the weak-noise limit
- carry out a system-size expansion of a Markov chain master equation

Module Content

This module will cover the following topics:

1. Elements of probability theory and continuous stochastic processes;
2. Random walks and Brownian motion;
3. Stochastic differential equations and Ito calculus;

4. The Fokker-Planck equation: connections between SDEs and PDEs; equilibrium and non-equilibrium stationary states; boundary value problems; multiplicative noise
5. First passage time problems: bistable potentials; weak noise limits; random search processes; extreme statistics;
6. Approximation schemes: system-size expansion, method of multiple scales

MATH70130: Stochastic Differential Equations in Financial Modeling

Brief Description

To deal with valuation, hedging and risk management of financial options, we briefly introduce stochastic differential equations using a Riemann-Stiltjes approach to stochastic integration. We introduce no-arbitrage theory in continuous time based on replicating portfolios, self-financing conditions and Ito's formula. We derive prices as risk neutral expectations. We derive the Black Scholes model and introduce volatility smile models.

We illustrate valuation of different options and introduce risk measures like Value at Risk and Expected Shortfall, motivating them with the financial crises.

Learning Outcomes

On successful completion of this module you will be able to

- work comfortably with stochastic differential equations commonly encountered in finance
- explain what is meant by no-arbitrage markets and why no-arbitrage is important operationally;
- connect no-arbitrage by replication to the existence of a risk neutral measure;
- price and hedge several types of financial options with several SDE models;
- calculate risk measures such as Value at Risk and Expected Shortfall;
- write code to price options according to SDE models covered in the module.
- independently appraise and evaluate SDE models for financial products.
- adapt a range of numerical methods and apply them in a coherent manner to unfamiliar and open problems in finance.

Module Content

1. Recap of key tools from probability theory
2. Brownian motion
3. Ito and Stratonovich stochastic integrals
4. Ito and Stratonovich stochastic differential equations (SDEs)
5. No-arbitrage through replication
6. No arbitrage though risk neutral measure
7. Derivation of the Black Scholes formula
8. Introduction of a few volatility smile models
9. Pricing of several types of options
10. Introduction to crises and risk measures
11. The Barings collapse and the introduction of value at risk (VaR)
12. Problems of VaR and an alternative: expected shortfall (ES)
13. Numerical examples and problems with risk measures, including software code.

MATH70134: Mathematical Foundations of Machine Learning (A)

Brief Description

Machine learning techniques such as deep learning have recently achieved remarkable results in a very wide variety of applications such as image recognition, self-driving vehicles, partial differential equation solvers, trading strategies. However, how and why the recent (deep learning) models work is often still not fully understood. In this course we will begin with a general introduction into machine learning and continue to deep learning. We will focus on better some observed phenomena in deep learning aiming to gain insight into the impact of the optimization algorithms and network architecture through mathematical tools.

Learning Outcomes

On successful completion of this module you will be able to:

- Demonstrate working familiarity with machine learning principles,
- Design models using a variety of deep learning architectures
- Implement neural network models in code
- Select appropriate optimization algorithms to train deep learning models
- Evaluate the ability of models to generalize by comparing their training and test performance
- Independently evaluate new methodologies in deep learning

Module Content

1. The preliminaries: pre-processing: data cleaning, dimensionality reduction, clustering
2. Regression (linear, Bayesian) and classification (a basic overview)
3. Neural networks and a variety of architectures (fully-connected, convolutional)
4. Generalisation and overfitting
5. Training methods for neural networks and their impact on performance
6. The role of noise
7. Flat minima and escape times
8. Links of neural networks to Gaussian processes
9. Explainability in neural networks through reconstruction

MATH70135: Advanced Partial Differential Equations 1

Brief Description

The focus of this course is on the concepts and techniques for solving partial differential equations (PDEs) that permeate various scientific disciplines. It is designed for a diverse audience in pure and applied mathematics, emphasizing rigor and the development of analytical proofs and techniques. The course places a strong emphasis on the theory of weak solutions of elliptic and parabolic equations and their regularity, involving distributions, Sobolev spaces, and the calculus of variations.

Learning Outcomes

On successful completion of this module you should be able to:

- demonstrate an overview of a variety of partial differential equations, the behaviour of their solutions, and techniques to study them.
- understand and communicate some of the deep connections of PDEs to physics and geometry.
- state and prove well-posedness theorems for a variety of PDEs and explain their relevance.
- apply elliptic regularity theory in the theory of elliptic PDEs.
- apply suitable techniques to hyperbolic equations (wave equations).

- recognise and engage with current research in PDEs

Module content

An indicative list of topics is:

1. Introduction, examples of PDEs that appear in applications. Elliptic, parabolic, hyperbolic PDEs. Course overview.
2. Distributions: definitions and examples, convergence and differentiability, support and convolution.
3. H¨older and Sobolev spaces: definitions and examples, approximation, traces, compactness (Rellich-Kondrachov Theorem), Sobolev inequalities (Gagliardo-Nirenberg-Sobolev, Morrey, Poincar´e) duality. Spaces involving time.
4. Elliptic PDEs: Basic existence-uniqueness theory: Strong/uniform ellipticity; weak formulation; Lax-Milgram; energy estimates. Elliptic regularity theory: Difference quotients; interior and boundary regularity. Elements of calculus of variations.
5. Parabolic PDEs: initial boundary value problems, weak formulation, Galerkin method. Parabolic Regularity theory. Maximum principle, Harnack's inequality. Semigroups, Hille-Yosida theorem.

MATH70137: Mathematical Biology 2: Systems Biology

Brief Description

This module will provide an introduction to the interdisciplinary field of mathematical systems biology. Drawing on analogies between biological and engineered systems, we will learn about mathematical approaches to model functional aspects of biological systems. We will discuss a wide range of topics, including control, memory, and computation in biological systems. Each topic will be discussed in the context of specific experimental systems.

Learning Outcomes

On successful completion of this module you will:

- be able to describe the major concepts and principles of systems biology, including design principles and emergent properties in biological systems
- be able to develop mathematical theories on functional aspects of biological systems, including biochemical and cellular circuits
- appreciate the role of feedback regulation in biological systems, and acquire tools to analyze feedback systems
- critically evaluate the relation between theory and experiment in systems biology
- acquire an understanding of a range of mathematical and computational motifs that play an important functional role in a wide range of biological systems
- develop an appreciation for the complexity and diversity of biological systems, and an understanding of the role of interdisciplinary approaches in advancing our understanding of these systems
- demonstrate an integrated understanding of the concepts of the module by critical, independent study of research articles and books.

Indicative Module Content

1. Introduction to biological circuits
2. Negative and positive feedback (responses, oscillations, memory, differentiation)
3. Integral feedback (adaptation, scale invariance)

4. Gradients: sampling and optimization
5. Bifurcations and feedback tuning to bifurcations
6. Hopfield networks
7. Optimal control and learning

MATH70139: Spatial Statistics

Brief Description

Data collected in space are common in many applications including climate science, epidemiology, and economics. This module covers theoretical and methodological statistical foundations for spatial data. The module is structured to cover in detail the three fundamental forms in which spatial data are collected: gridded data, network data, and point pattern data. For each data type, stochastic models will be defined and explored including random fields and point processes. Properties including isotropy, stationarity and homogeneity will be formalised and explored in the context of each model and data type. In addition, techniques for spatial interpolation will be studied.

Learning Outcomes

On successful completion of this module students will be able to:

- Recognise the key differences between different types of spatial data (gridded, network and point pattern)
- Apply different classes of spatial covariance models to gridded and network data
- Formulate different intensity functions for point pattern data
- Define the concepts of homogeneity, stationarity and isotropy in the context of spatial models
- Select, derive and apply appropriate methods for spatial interpolation
- Demonstrate an integrated understanding of the concepts of the module by critical, independent study of research articles and books.

Indicative Module Content

1. Introduction to spatial data (gridded, network and point pattern data)
2. Random Fields and Covariance Functions
3. Spatial Interpolation and Kriging
4. Network Data and Markov Random Fields
5. Spatial Point Processes

MATH70140: Geometric Complex Analysis

Brief Description

In this module we look at the subject of complex analysis from a more geometric point of view. We shall look at geometric notions associated with domains in the plane and their boundaries, and how they are transformed under holomorphic mappings. In turn, the behaviour of conformal maps is highly dependent on the shape of their domain of definition.

Learning Outcomes

On successful completion of this module you will be able to:

- identify features of, and develop arguments about, certain holomorphic maps;

- state, apply, and explain aspects of the Riemann Mapping Theorem for arbitrary simply connected plane domains;
- explain the automorphisms of the disk and the upper half plane;
- Explain hyperbolic geometry, and basic notions of length, geodesics, isometries;
- apply area theorem, and derive distortion estimates for arbitrary conformal mappings;
- acquire deeper understanding of holomorphic mappings through generalisation to quasi-conformal mappings;
- appreciate significance of universal bounds in geometric function theory;
- explain the statement of the Beltrami-equation, and generalisation of the Riemann mapping theorem;
- demonstrate additional competence in the subject through the study of more advanced material.
- combine results from across the module to solve advanced problems
- work independently and with peers to understand abstract concepts in complex analysis

Indicative Module Content

Complex analysis is the study of the functions of complex numbers. It is employed in a wide range of topics, including dynamical systems, algebraic geometry, number theory, and quantum field theory, to name a few. On the other hand, as the separate real and imaginary parts of any analytic function satisfy the Laplace equation, complex analysis is widely employed in the study of two-dimensional problems in physics such as hydrodynamics, thermodynamics, Ferromagnetism, and percolations.

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An indicative list of topics is:

1. Schwarz lemma and automorphisms of the disk,
2. Riemann sphere and rational maps,
3. Conformal geometry on the disk, Poincare metric, Isometries, Hyperbolic contractions,
4. Conformal Mappings, Conformal mappings of special domains, Normal families, Montel's theorem, General form of Cauchy integral formula, Riemann mapping theorem,
5. Growth and Distortion estimates, Area theorem,
6. Quasi-conformal maps and Beltrami equation, Linear distortion, Dilatation quotient, Absolute continuity on lines, Quasi-conformal mappings, Beltrami equation, application of MRMT,

There will also be extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

MATH70141: Introduction to Game Theory

Brief Description

This module will give students an insight into the wide variety of mathematics and its many applications within the area of game theory. The module aims to promote an active learning style, involving many classroom games as well as games to be played as homework.

The module will cover the classical theory of games involving concepts of dominance, best response and equilibria, where we will prove Nash's Theorem on the existence of equilibria in games. We will see the concept of when a game is termed zero-sum and prove the related Von Neumann's Minimax Theorem. We will briefly discuss cooperation in games and investigate the interesting Nash bargaining solution which arises beautifully from reasonable bargaining axioms.

Broadening our scope, we will look at the area of combinatorial game theory, building up our intuition through investigating the classical game of Nim in detail. We will also see the concept of a congestion game, often applied to situations involving traffic flow, where we will see the counter intuitive Braess paradox emerge and prove Nash's theorem in another context.

The module will finish with a small tour through some other areas and applications of game theory.

Learning Outcomes

On successful completion of this module you will be able to:

- define the concepts of dominance, best-response and equilibria in a variety of competitive scenarios (games);
- solve (determine all equilibria or find optimal strategies) small games via a variety of techniques: iterated deletion of dominated strategies, finding equaliser strategies, use of subgames;
- determine when a game may be termed zero-sum, and be able to recognise, find and apply minimax and maximin strategies in these games;
- apply game theory to traffic flow or flow of information through networks, appreciating the differences and importance of optimal societal routing as compared with selfish individual routing;
- calculate bargaining solutions in simple co-operative games;
- determine whether communication is beneficial or not in different strategic situations;
- demonstrate an integrated understanding of the concepts of the module by critical, independent study of research articles and books.

Indicative Module Content

1. Recap of some basic notions in probability, calculus and analysis, some recap of induction in a game theoretic context.
2. Motivational/illustrative classroom games.
3. Dominance, best-response and equilibria.
4. Nash's theorem on equilibria in games.
5. Zero-sum games and Von Neumann's minimax theorem.
6. Subgame solutions as extensions to full game solutions.
7. Cooperative games, the Nash arbitration procedure and bargaining solutions.
8. Congestion games; Braess paradox, selfish routing vs optimal societal routing, existence of equilibria.
9. Combinatorial games; Nim, Nim sums and Nim values, sums of games.

MATH70143: Dynamics, Symmetry and Integrability

Brief Description

This module on Dynamics, Symmetry and Integrability is a friendly and fast-paced introduction to the geometric approach to proving integrability of classical Hamiltonian systems, at the level suitable for advanced undergraduates and first-year graduate students in mathematics. It fills a gap between traditional classical mechanics texts and advanced mathematical treatments of the geometric approach to integrability. The key idea is to use the momentum maps (e.g. from Noether's theorem) to find enough conservation laws to prove integrability. The main examples of integrable PDEs discussed are those that model shallow water waves, particularly the Korteweg-de Vries and Camassa-Holm equations.

Learning Outcome

On successful completion of this module, you will be able to:

- describe Hamiltonian motion on a smooth finite-dimensional manifold and demonstrate familiarity with the cotangent bundle (T*M, phase space) and the definition of canonical Poisson brackets, as well as Hamiltonian vector fields, symplectic forms, symplectic transformations and solutions as characteristic flows of Hamiltonian vector fields on T*M;
- define Liouville integrability for finite-dimensional Hamiltonian dynamical systems and appreciate that Liouville integrability requires a sufficient number of functionally independent conservation laws in involution;
- select and use several other methods (introduced via worked examples) for acquiring the conservation laws necessary to prove integrability including: reduction to elliptic curves, isospectral reformulation in Lax pair form using covariant derivatives with zero curvature and transformation of variables to the momentum maps which arise in Noether's theorem;
- determine momaps for Hamiltonian systems with symmetry for a variety of classic finite-dimensional problems including: rigid body motion in Rn, coupled nonlinear oscillators in C2 and C3 and the reduction of the CH equation to finite dimensions which results from a singular momap;
- interpret the Lax pair form of isospectral dynamics as coadjoint motion of a cotangent lift momentum map leading to the Lie-Poisson bracket which features widely in establishing the integrability of Hamiltonian systems;
- derive Magri's theorem for establishing integrability via isospectrality of bi-hamiltonian systems in infinite dimensions through the examples of the KdV and CH water wave equations with soliton solutions;
- appraise and interpret Hamiltonian methods of geometric mechanics for ideal fluid dynamics by applying V.I. Arnold's reformulation of Euler's 2D and 3D fluid solutions as geodesic flow on the manifold of smooth volume preserving flows with respect to the fluid kinetic energy which sets the stage for investigation of nonlinear stability of fluid equilibrium solutions.

Module Content

The module is composed of the following sections:

1. Dynamics
   The main ideas of the course are illuminated by considering cases when the solution dynamics on the configuration manifold may be lifted to a (non-Abelian) Lie group symmetry of the Hamiltonian. With an emphasis on applications in ODEs of finite-dimensional mechanical systems, such as the rigid body SO(3) and coupled resonant oscillations U(2), and PDEs of nonlinear waves, such as the KdV and CH equation in infinite dimensions, the properties and results for integrability which are inherited from the geometrical formulation of dynamics induced by Lie group actions are discussed.
2. Symmetry
   Symmetries of the Hamiltonian under Lie group transformations and their associated momentum maps are emphasised, both for reducing the number of independent degrees of freedom and in finding conservation laws by Noether's theorem.
3. Integrability
   Definition: According to Liouville, a Hamiltonian system on a 2N-dimensional symplectic manifold M2N is completely integrable, if it possesses N functionally independent

conservation laws which mutually commute under canonical Poisson brackets. What makes a dynamical system integrable, then? Enough conservation laws!

The course develops a series of geometrical methods for finding mutually Poisson-commuting conservation laws and thereby solving a sequence of integrable Hamiltonian problems ranging from rigid body motion to nonlinear wave PDEs. These methods include isospectral Lax pair formulations and algebraic geometry of elliptic curves for rigid body motion, as well as Lax equations and isospectrality principles due to bi-Hamiltonian structures for the KdV and CH water wave equations. In developing the solvability algorithms for this sequence of problems, the momentum map for the cotangent lift action of a Lie group on a manifold M plays a central role in representing the equations, their solutions and the analysis of their solution behaviour.

MATH70146: Advanced Dynamical Systems

Brief Description

The aim of this course concerns the in-depth study of an advanced topic in the theory of dynamical systems. The precise topic covered will vary from year to year depending on the most exciting recent research directions in the field and on the research directions undertaken in the research of MSc and PhD students in dynamical systems.

Learning Outcomes

On successful completion of this module, you will have:

- developed an understanding of an advanced topic in the theory of dynamical systems;
- worked independently to understand a research paper or a chapter of a research monograph;
- worked independently to produce an essay in which you report on the material that you have mastered;
- the ability to thoughtfully engage in a discussion about an advanced topic in dynamical systems.

Module Content

This course deals with topics in dynamical systems at an advanced level, touching upon current frontline research. Each year a selection will be made of material from the area of local bifurcation theory, global bifurcation theory, ergodic theory of dynamical systems or dynamical systems methods for PDEs/FDEs/networks, etc, in line with current interests of researchers in the dynamical systems group who moderate this course. In recent years, topics that were treated include homoclinic bifurcation theory (2019), circle homeomorphisms (2018) and stochastic approximation (2017). During the academic year the module will be focused on numerical methods in dynamical systems and data.

MATH70147: Statistical Mechanics

Brief Description

This module is an introduction to the principles and applications of statistical mechanics, with a special emphasis on phase transitions. We will study in particular how macroscopic behaviors emerge from microscopic interactions, providing insights into systems at equilibrium and the critical phenomena associated with phase transitions.

Phase transitions are ubiquitous in nature and fundamental to understanding a wide array of emergent phenomena in the physical sciences (including the magnetization of materials, order-disorder transitions in solids, topological phase transitions, the liquid-liquid phase separation taking place inside biological cells, transition to collective motion in a flock of starlings) and beyond (including percolation on networks, the dynamics of opinion formation, financial market crashes, neural network phase transitions...).

This module aims to equip students with a solid understanding of the mechanisms underlying phase transitions. By the end of the course, you will be able to analyze and predict the behavior of complex systems. Through a blend of theoretical lectures and computational labs, you will gain a solid grasp of key concepts and mathematical tools in statistical mechanics, a deep appreciation of the universality of critical phenomena, and the ability to tackle advanced problems in a variety of fields.

Learning Outcomes

On successful completion of this module students will be able to:

- Understand the fundamental principles of statistical mechanics including the concepts of ensembles, partition functions and phase transitions;
- classify and analyse different types of phase transitions, understand the concept of order parameters and describe symmetry breaking;
- apply mean-field theory and renormalization group theory to explain critical phenomena and phase transitions in various systems;
- connect the theoretical concepts introduced in lecture to real-world applications in physical and social sciences;
- implement numerical simulations to study the emergence properties of complex systems and compare your results to analytical arguments.
- demonstrate a holistic understanding of the concepts introduced in the course and show mastery of further advanced concepts in statistical mechanics through a critical analysis of reference textbooks and research articles in the field.

Module Content

Topics to be covered in lecture may include:

1. Principles of equilibrium statistical mechanics (ensembles, fundamental postulate of equilibrium statistical mechanics, elements of thermodynamics);
2. Introduction to spin models (including the Ising model and the XY model);
3. Notion of phase transitions, discontinuous and continuous phase transitions;;
4. Mean-field approaches;
5. Transfer matrix approach to the Ising model;
6. Landau theory of phase transitions - Fluctuations and the breakdown of Landau theory;
7. Critical points, scaling and critical exponents;
8. Renormalization group (Kadanoff approach);
9. Kosterlitz-Thouless transition and spontaneous continuous symmetry breaking;
10. Phase separation and Cahn-Hilliard equation.

Computational labs may include the following topics:

1. Short introduction to Monte Carlo methods;
2. Hard disks simulations – computation of partition function, Maxwell-Boltzmann velocity
3. distribution, and macroscopic observables, liquid-solid transition;
4. Order-disorder transitions in spin systems and Glauber dynamics;
5. Generalized Ising models and spin glasses;

6. Dynamic Monte Carlo methods;
7. Agent-based models (illustrated via order-disorder transition in the Vicsek model).

MATH70148: Probabilistic Generative Models (A)

Brief Description

Probabilistic generative models (PGM) are at the forefront of statistical machine learning research and are central in contemporary AI applications. This module develops a foundation for the design, analysis and implementation of PGMs. The module starts from the fundamentals of training and inference and then studies different PGMs, including parametric, autoregressive, explicit, implicit and (Bayesian) nonparametric models. Dissimilarity metrics are also examined, all to establish a comprehensive setting for the analysis of the models under study. Emphasising theory and practice, the module's assessment features two courseworks (20% each) and an exam (60%). Students are expected to be familiar with classic machine learning models and use Python in their assessments.

Learning Outcomes

On the successful completion of the module, you will be able to:

- select which generative model is appropriate for a given data analysis setup;
- train different types of generative models using dedicated Python toolboxes;
- calculate the similarity between different models;
- obtain samples from a trained model in an efficient and accurate manner;
- determine whether a training procedure has concluded successfully
- demonstrate an integrated understanding of the concepts of this module by independent study of related material.

Module Content

This module will cover the following topics:

1. Fundamentals: graphical models, exact and approximate Bayesian inference;
2. Dissimilarity measures: optimal transport, maximum mean discrepancy, notions of information theory;
3. Explicit models: normalising flows;
4. Variational autoencoders & generative adversarial networks;
5. Autoregressive models: linear filters, recurrent neural networks, transformers;
6. Bayesian nonparametric models: Dirichlet process & Gaussian process;
7. Diffusion models & score-based models